

Discussion 09

Spring 2024

1. Poisson Process Warmup

Give an interpretation of the following fact in terms of a Poisson process with rate λ . If N is Geometric with parameter p and $(X_k)_{k \in \mathbb{N}}$ are i.i.d. $\text{Exponential}(\lambda)$, then $X_1 + \cdots + X_N$ has an Exponential distribution with parameter λp .

2. Illegal U-Turns

Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal, and police cars drive by according to a Poisson process with rate λ . You decide to make a U-turn once you see that the road has been clear of police cars for time $\tau > 0$. Let N be the number of police cars you see before you make a U-turn.

- a. Find $\mathbb{E}(N)$.
- b. Let $n \geq 2$. Find the conditional expectation of the time elapsed between police cars $n - 1$ and n , given that $N \geq n$.
- c. Find the expected time that you wait until you make a U-turn.

3. Bus Arrivals at Cory Hall

Starting at time 0, the 52 line makes stops at Cory Hall according to a Poisson process of rate λ . Students arrive at the stop according to an independent Poisson process of rate μ . Every time the bus arrives, all students waiting get on.

- a. Given that the interarrival time between bus $i - 1$ and bus i is x , find the distribution for the number of students entering the i th bus. Here, x is a given number, not a random quantity.
- b. Given that a bus arrived at 9:30 AM, find the distribution for the number of students that will get on the next bus.