UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Homework 09
Spring 2024

1. **System Shocks**

   For a positive integer n, let $X_1, \ldots, X_n$ be independent Exponentially distributed random variables, each with mean 1. Let $\gamma > 0$. A system experiences shocks at times $k = 1, \ldots, n$, and the size of the shock at time k is X_k.

   a. Suppose that the system fails if any shock exceeds the value γ. What is the probability of system failure?

   b. Suppose instead that the effect of the shocks is cumulative, i.e. the system fails when the total amount of shock received exceeds γ. What is the probability of system failure?

2. **Random Telegraph Wave**

   Let $(N_t)_{t\geq 0}$ be a Poisson process with rate λ, let X_0 be a Bernoulli random variable independent of $(N_t)_{t\geq 0}$, and define $X_t = X_0(-1)^{N_t}$.

   a. Does the process $(X_t)_{t\geq 0}$ have independent increments?

   b. Calculate $\mathbb{P}(X_t = 1)$ if $\mathbb{P}(X_0 = 1) = p$.

   c. Assume that $p = \frac{1}{2}$. Calculate $\mathbb{E}(X_{t+s}X_s)$ for $s, t \geq 0$.

3. **Poisson Process Practice**

Let $(N_t)_{t\geq 0}$ be a Poisson process with rate λ. Let T_k, $k \geq 1$ denote the time of the kth arrival. Given $0 \leq s < t$, we write $N(s,t) := N(t) - N(s)$. Compute the following:

a. $\mathbb{P}(N(1) + N(2,4) + N(3,5) = 0)$.

b. $\mathbb{E}(N(1,3) \mid N(1,2) = 3)$.

c. $\mathbb{E}(T_2 \mid N(2) = 1)$.