UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Homework 3
Spring 2024

1. **Stock trader**

   A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

   $$-2 \leq x \leq 4, \quad -1 \leq y - x \leq 1.$$

   (a) Find the marginal PMFs and the means of X and Y.

   (b) Find the mean of the trader's profit.

2. **Poisson Practice**

Suppose X is a Poisson random variable with parameter λ. Find the following:

a. $\mathbb{E}(X^2)$.

b. $\mathbb{P}(X \text{ is even})$. (*Hint*: Use the Taylor series expansion of e^x.)

3. **Poisson Properties**

a. Suppose X and Y are independent Poisson random variables with means λ and μ respectively. Prove that $X+Y$ has the Poisson distribution with mean $\lambda+\mu$. **Note**: It is *not* enough to use linearity of expectation to say that $X+Y$ has mean $\lambda+\mu$. You are asked to prove that the *distribution* of $X+Y$ is Poisson.

b. Given X and Y as above, what is the distribution of X conditioned on $X+Y=z$, $z\in\mathbb{N}$?

4. **PDF Practice**

Let X be a random variable with PDF

$$f_X(x) = \begin{cases} \frac{x}{4}, & \text{if } 1 < x \leq 3, \\ 0, & \text{otherwise,} \end{cases}$$

and let A be the event $\{X \geq 2\}$.

(a) Find $\mathbb{E}[X]$, $P(A)$, $f_{X|A}(x)$, and $\mathbb{E}[X|A]$.

(b) Let $Y = X^2$. Find $\mathbb{E}[Y]$ and $\text{var}(Y)$.

5. **Joint Density for Exponential Distribution**

a. If $X \sim \text{Exponential}(\lambda)$ and $Y \sim \text{Exponential}(\mu)$ are independent, compute $\mathbb{P}(X < Y)$.

b. If $X_1, \ldots, X_n$ are independent and Exponentially distributed with parameters $\lambda_1, \ldots, \lambda_n$, show that $\min_{1 \leq k \leq n} X_k \sim \text{Exponential}(\sum_{j=1}^n \lambda_j)$.

c. Deduce that

$$\mathbb{P}\left(X_i = \min_{1 \leq k \leq n} X_k\right) = \frac{\lambda_i}{\sum_{j=1}^n \lambda_j}.$$

6. **Waiting time**

Calamity Jane goes to the bank to make a withdrawal, and is equally likely to find 0 or 1 customers ahead of her. The service time of the customer ahead, if present, is exponentially distributed with parameter λ. What is the CDF of Jane's waiting time? Is this random variable discrete or continuous?