EECS 127

Spring 2020

Optimization Models.

- Resource allocation
- Control + Robotics.
- Communications
- Signal Processing.
- Prediction / Classification.

Oil Production

10,000 Crude oil.

Jet Fuel .10 \$	x_1

Gasoline. .20 \$	x_2

Contracts:

$\geq$ 1000 Jet Fuel

$\geq$ 2000 Gasoline.

180,000 barrel-miles

Distributor: Gasoline: 30 miles away.

JF : 10 miles away.

objective function

$$\text{maximize} \quad .10x_1 + .20 \cdot x_2$$

$$\text{subject to.} \quad x_1 \geq 1000$$
$$x_2 \geq 2000.$$
$$10 \cdot x_1 + 30 \cdot x_2 \leq 180,000.$$

constraints.

° General:

$$\text{minimize} \quad f_0(\vec{x}) \leftarrow \text{objective function.}$$

$$\text{subject to.} \quad f_i(\vec{x}) \le b_i \quad \text{for } i = 1, 2, \dots m.$$

$\vec{x} \in \mathbb{R}^n$ optimization variable.

$\vec{x}^*$: optimum value. among feasible set.

Student:

	Classes	Interesting	Workload.
x_i	127	α_1	β_1
	126	α_2	β_2
	189	α_3	β_3
	170	$\vdots$	$\vdots$
	$\vdots$		

$$\text{maximize} \quad \vec{\alpha}^T \cdot \vec{x}$$

$$\text{subject to:} \quad \vec{\beta}^T \vec{x} \le B.$$

total workload.

- Department:

Courses	Size	Credits	Resources
127	x_1	c_1	r_1
126	x_2	c_2	r_2
189	$\vdots$	c_3	r_3
$\vdots$			
	$\uparrow$ $\vec{x}$	$\uparrow$ $\vec{c}$	$\vec{r}$

maximize $\vec{c}^{\,T}\vec{x}$

subject to $\vec{r}^{\,T}\cdot\vec{x}\cdot \leq B.$

Least-squares

A: matrix. $\vec{b}$: vector.

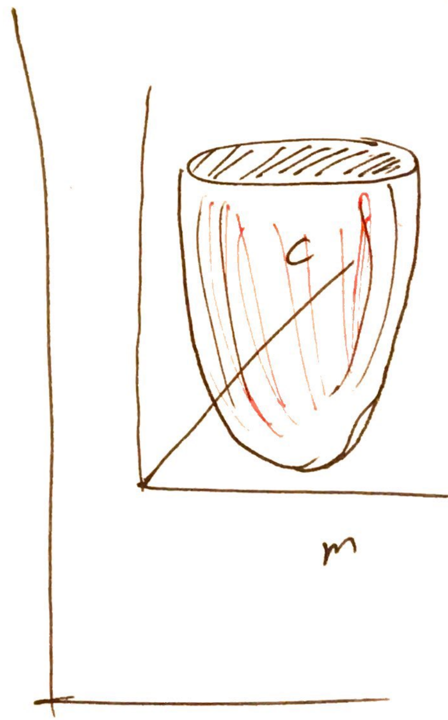
$$\min_{\vec{x}} \quad \|A\vec{x} - \vec{b}\|^2$$

- Linear regression.
- Projection.
- Curve-fitting.

$y = mx + c$

$(x_1, y_1), (x_2, y_2), (x_3, y_3) \cdots (x_n, y_n).$

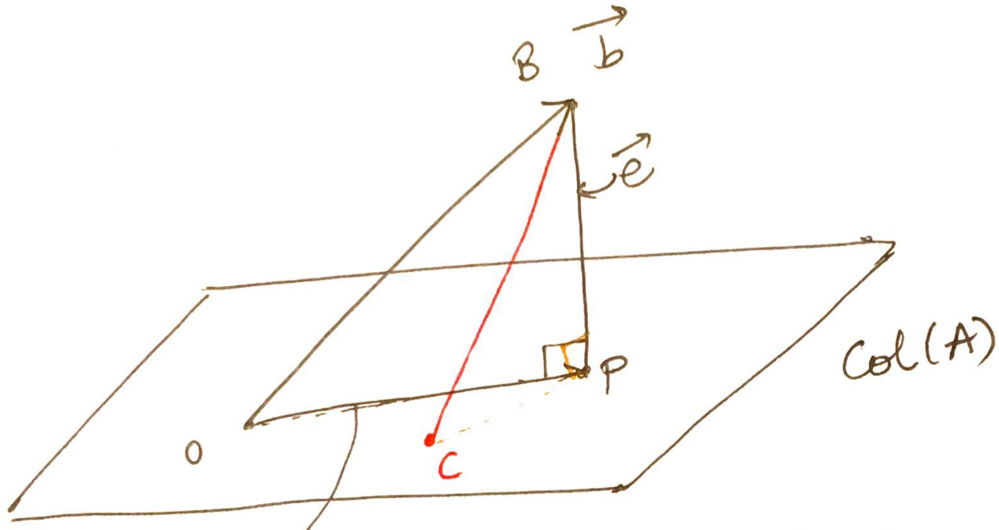
$$\underbrace{\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}}_{A} \begin{bmatrix} m \\ c \end{bmatrix} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{\vec{b}}$$



Find m, c to minimize.

$$\min_{m, c} \sum_{i=1}^{n} (y_i - (mx_i + c))^2$$

- How do you solve this?



Project B onto Col(A).

BPC is a right triangle.
$\angle P = 90°$.

Hyp > Side.

$\therefore$ P is closest point.

$\vec{e} = A\vec{x} - \vec{b}$ to be orthogonal to the columns of Matrix A.

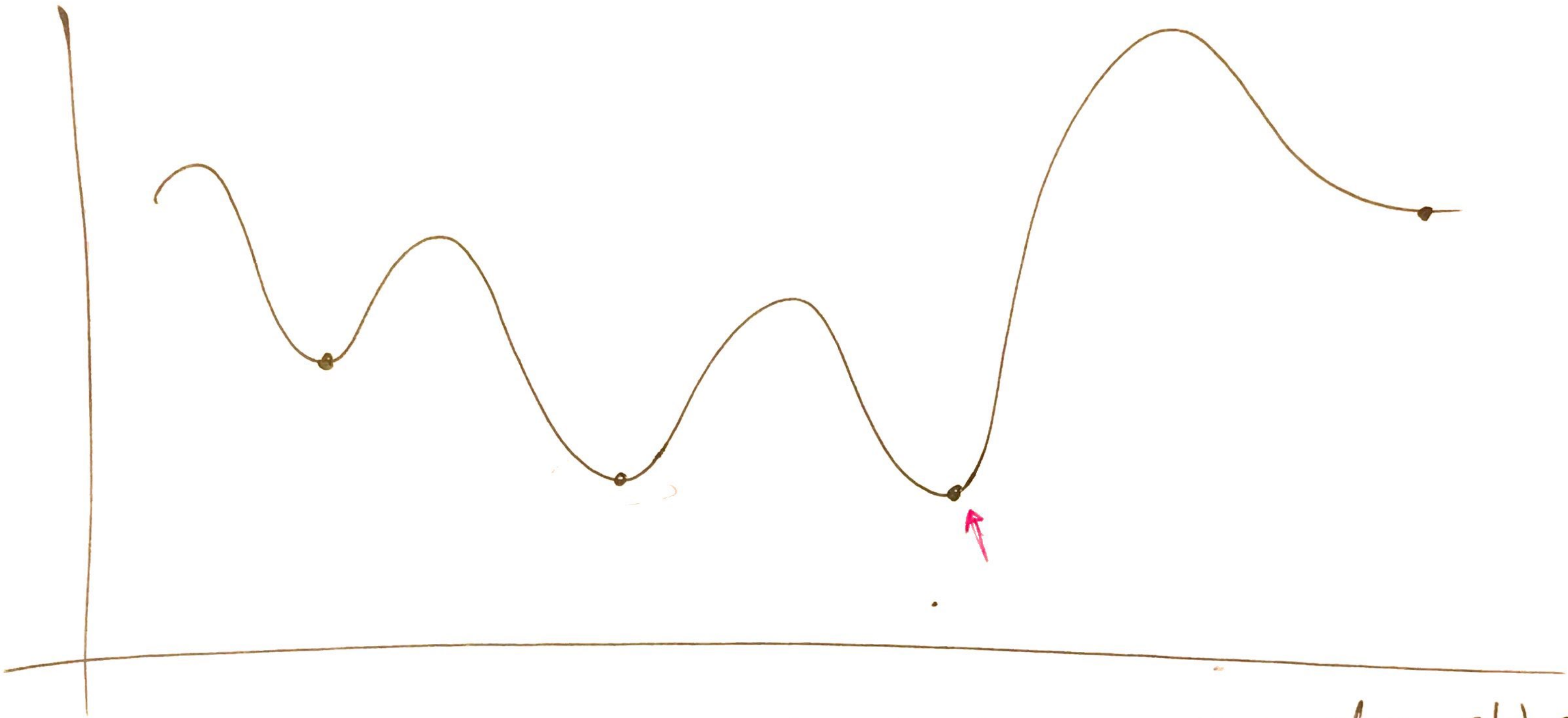
$$A^T(A\vec{x} - \vec{b}) = 0$$

$$A^TA\vec{x} - A^T\vec{b} = 0.$$

$$\vec{x} = (A^TA)^{-1}A^T\vec{b}.$$

(A^TA) is invertible.
$\hookrightarrow$ A is full columnm rank.

Convex: functions.



<u>Convex:</u> Local minima are also global minima.