EECS 127

Today: Quadratic Programs.
Second-Order Cone Programs.

Quadratic Programs.

$$\min. \quad \frac{1}{2}\vec{x}^T H \vec{x} + \vec{c}^T \vec{x}.$$
$$\text{s.t.} \quad A\vec{x} \le \vec{b} \quad \text{ineq.}$$
$$F\vec{x} = \vec{d} \quad \text{eq.}$$

$H = H^T$
PSD
objective is convex.

Equality constrained quad. programs

$\Longleftrightarrow$ Unconstrained programs.

Minimia of unconstrained quad. programs

→ • Unique point
- Multiple points
- Infinitely many.
- $-\infty$

Example : Linear Control Problem.

Linear Quadratic Regulator (LQR).

state : $\vec{x}(t)$ $\quad$ t: time.

How does $\vec{x}(t)$ evolve in time.

t as my time index ?

$$\vec{x}(t+1) = \underset{\substack{\uparrow \\ \text{matrix} \\ \text{dynamics of} \\ \text{system.}}}{A} \cdot \overbrace{\vec{x}(t)}^{\text{state.}} + \underset{\substack{\uparrow \\ \text{matrix} \\ \text{or} \\ \text{vector.} \\ \text{"Control gain"}}}{B} \cdot \underset{\substack{\uparrow \\ \text{vector or scalar}}}{\overset{\swarrow \ \text{what I choose "control"}}{\vec{u}(t)}}$$

We want to choose $u(t)$ so that my $x(t)$ can reach a "goal"

$\vec{x}(0)$

$$\vec{x}(1) = A \cdot \vec{x}(0) + B \cdot \vec{u}(0)$$

$$\vec{x}(2) = A \cdot \vec{x}(1) + B \vec{u}(1)$$

$$= A(A \cdot \vec{x}(0) + B \cdot \vec{u}(0)) + B \cdot \vec{u}(1).$$

$$= A^2 \cdot \vec{x}(0) + AB \vec{u}(0) + B \cdot \vec{u}(1).$$

$$\vec{x}(t) = A^t \cdot \vec{x}(0) + \sum_{i=0}^{t-1} A^{t-i-1} \cdot B \cdot u(i)$$

$$t \geq 0$$

T: final time.

$$\min_{\substack{x(t):\, t=0,\dots T \\ u(t):\, t=0,\dots T}} \underbrace{\|\vec{x}(T) - \vec{x}_d\|_2^2} + \overbrace{\sum_{t=0}^{T} \|u(t)\|_2^2}$$

s.t. $$x(t) = A^t x(0) + \sum_{i=0}^{t-1} A^{t-i-1} \cdot B \cdot u(i)$$

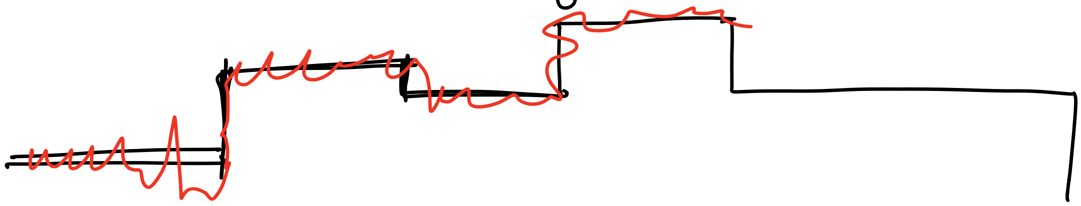
x_d : destination $\forall t = 0, 1, \dots n$.

constraint

Objective : Quadratic

Constraints : Linear

Example: Trying to fit a piecewise constant function.



You want to use data

$$\underset{\text{obs}}{\vec{y}} = \underset{\text{true}}{\vec{x}} + \text{noise.}$$

→ Side info: Your signal is piecewise constant

<u>Find</u>: $\vec{\hat{x}}$ such that $\vec{\hat{x}}$ does not change on consecutive timesteps as much as possible.

Consider:

$$\vec{z} = \overset{\text{SPARSE}}{[\hat{x}_2 - \hat{x}_1, \ \hat{x}_3 - \hat{x}_2 \cdots, \ \hat{x}_n - \hat{x}_{n-1}]}.$$

$$\vec{\hat{x}} = [\hat{x}_1, \hat{x}_2 \cdots \hat{x}_n].$$

$$\vec{z} = D \cdot \vec{\hat{x}}$$

$$= \begin{bmatrix} -1 & +1 & 0 & \cdots & & 0 \\ 0 & -1 & +1 & & \cdots & 0 \\ 0 & 0 & -1 & +1 & \cdots & 0 \\ & & & & & \\ \cdots & & \cdots & & -1 & +1 \end{bmatrix}$$

$$\text{Formulate:} \quad \left.\begin{array}{l} \min \ \|\vec{y} - \vec{\hat{x}}\|_2^2 \\ \text{s.t. cardinality}(\underline{D\hat{x}}) \leq K \end{array}\right]$$

l_0 : norm constraints.

Combinatorial Problems.

Relax:

$$\left.\begin{aligned} &\min \; \|\vec{y} - \vec{\hat{x}}\|_2^2 \\ &\text{s.t.} \; \underline{\|D\hat{x}\|_1 \leq \alpha} \end{aligned}\right] \begin{matrix} \text{LASSO} \\ \text{QP} \end{matrix}$$
