

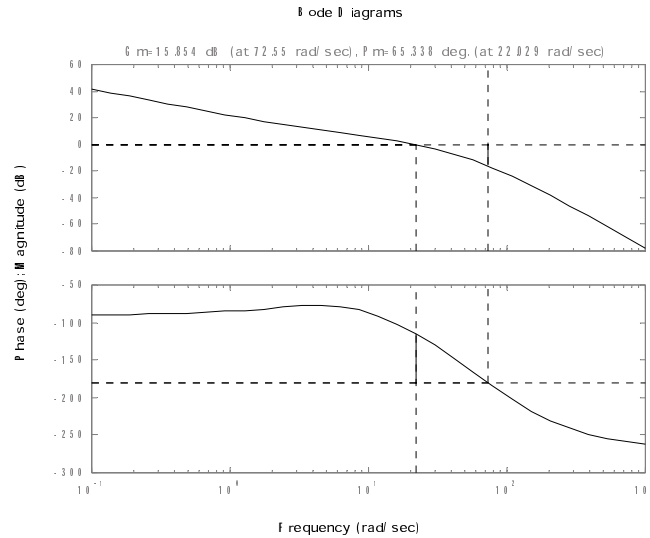
Therefore by applying the lead compensator with some gain adjustments :

$$D(s) = 0.12 \times \frac{\frac{s}{4.5} + 1}{\frac{s}{90} + 1}$$

we get the compensated system with :

$$PM = 65^\circ, \omega_c = 22 \text{ rad/sec, so that } \omega_{BW} \gtrsim 25 \text{ rad/sec.}$$

The Bode plot with designed compensator is :



45. For the system shown in Fig. 6.104, suppose that

$$G(s) = \frac{5}{s(s+1)(s/5+1)}.$$

Design a lead compensation $D(s)$ with unity DC gain so that $PM \geq 40^\circ$ using Bode plot sketches, then verify and refine your design using MATLAB. What is the approximate bandwidth of the system?

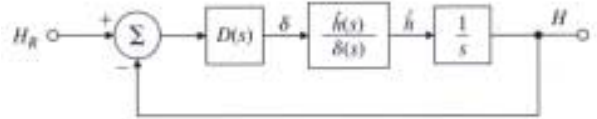
Solution :

Start with a lead compensator design with :

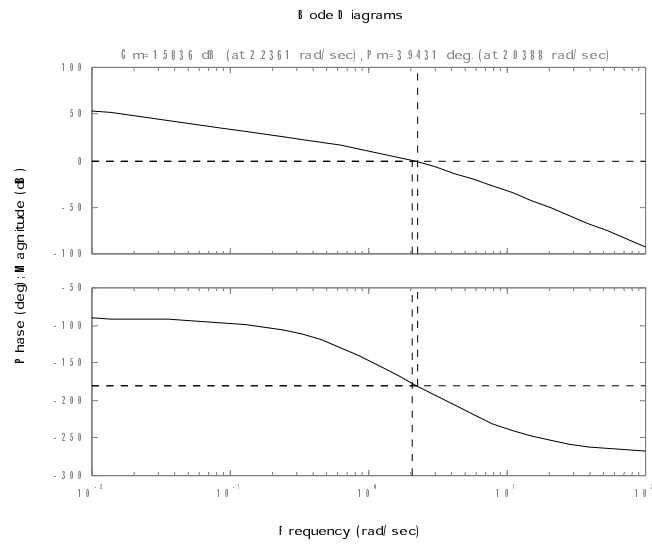
$$D(s) = \frac{Ts + 1}{\alpha Ts + 1}$$

which has unity DC gain with $\alpha < 1$.

Figure 6.104: Control system for Problem 45



The Bode plot of the given system is :



Since $PM = 3.9^\circ$, let's add phase lead $\geq 60^\circ$. From Fig. 6.53,

$$\frac{1}{\alpha} \simeq 20 \Rightarrow \text{choose } \alpha = 0.05$$

To apply maximum phase lead at $\omega = 10$ rad/sec,

$$\omega = \frac{1}{\sqrt{\alpha T}} = 10 \Rightarrow \frac{1}{T} = 2.2, \quad \frac{1}{\alpha T} = 45$$

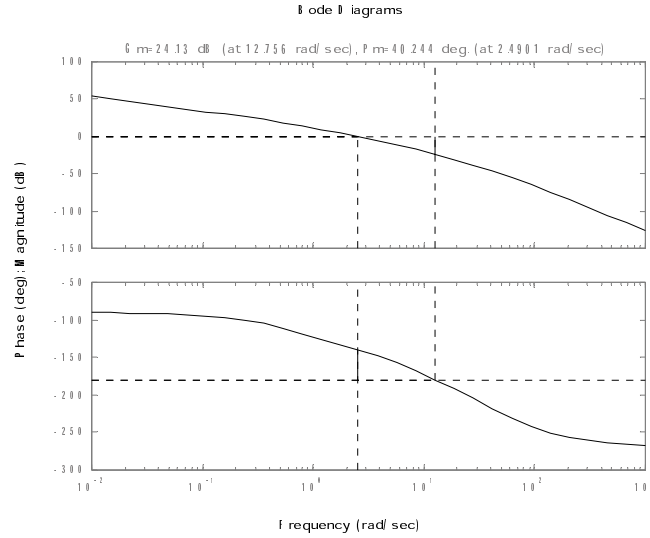
Therefore by applying the lead compensator :

$$D(s) = \frac{\frac{s}{2.2} + 1}{\frac{s}{45} + 1}$$

we get the compensated system with :

$$PM = 40^\circ, \omega_c = 2.5$$

The Bode plot with designed compensator is :



From Fig. 6.50, we see that $\omega_{BW} \simeq 2 \times \omega_c \simeq 5$ rad/sec.

46. Derive the transfer function from T_d to θ for the system in Fig. 6.68. Then apply the Final Value Theorem (assuming $T_d = \text{constant}$) to determine whether $\theta(\infty)$ is nonzero for the following two cases:

- (a) When $D(s)$ has no integral term: $\lim_{s \rightarrow 0} D(s) = \text{constant}$;
- (b) When $D(s)$ has an integral term:

$$D(s) = \frac{D'(s)}{s},$$

where $\lim_{s \rightarrow 0} D'(s) = \text{constant}$.

Solution :

The transfer function from T_d to θ :

$$\frac{\Theta(s)}{T_d(s)} = \frac{\frac{0.9}{s^2}}{1 + \frac{0.9}{s^2} \frac{2}{s+2} D(s)}$$

where $T_d(s) = |T_d|/s$.

- (a) Using the final value theorem :

$$\begin{aligned}\theta(\infty) &= \lim_{t \rightarrow \infty} \theta(t) = \lim_{s \rightarrow 0} s\Theta(s) = \lim_{s \rightarrow 0} \frac{\frac{0.9}{s^2}}{\frac{s^2(s+2)+1.8D(s)}{s^2(s+2)}} \frac{|T_d|}{s} \\ &= \frac{|T_d|}{\lim_{s \rightarrow 0} D(s)} = \frac{|T_d|}{\text{constant}} \neq 0\end{aligned}$$

Therefore, there will be a steady state error in θ for a constant T_d input if there is no integral term in $D(s)$.

- (b)

$$\begin{aligned}\theta(\infty) &= \lim_{t \rightarrow \infty} \theta(t) = \lim_{s \rightarrow 0} s\Theta(s) = \lim_{s \rightarrow 0} \frac{\frac{0.9}{s^2}}{\frac{s^3(s+2)+1.8D'(s)}{s^3(s+2)}} \frac{|T_d|}{s} \\ &= \frac{0}{1.8 \lim_{s \rightarrow 0} D'(s)} = 0\end{aligned}$$

So when $D(s)$ contains an integral term, a constant T_d input will result in a zero steady state error in θ .

47. The inverted pendulum has a transfer function given by Eq. (2.35), which is similar to

$$G(s) = \frac{1}{s^2 - 1}.$$

- (a) Design a lead compensator to achieve a PM of 30° using Bode plot sketches, then verify and refine your design using MATLAB.
- (b) Sketch a root locus and correlate it with the Bode plot of the system.
- (c) Could you obtain the frequency response of this system experimentally?

Solution :

- (a) Design the lead compensator :

$$D(s) = K \frac{Ts + 1}{\alpha Ts + 1}$$

such that the compensated system has $PM \simeq 30^\circ$ & $\omega_c \simeq 1$ rad/sec. (Actually, the bandwidth or speed of response was not specified, so any crossover frequency would satisfy the problem statement.)

$$\alpha = \frac{1 - \sin(30^\circ)}{1 + \sin(30^\circ)} = 0.32$$

To apply maximum phase lead at $\omega = 1$ rad/sec,

$$\omega = \frac{1}{\sqrt{\alpha T}} = 1 \Rightarrow \frac{1}{T} = 0.57, \quad \frac{1}{\alpha T} = 1.77$$

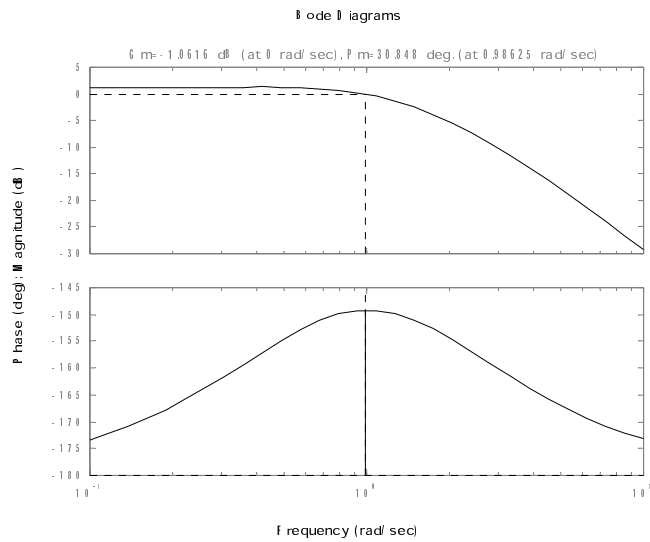
Therefore by applying the lead compensator :

$$D(s) = K \frac{\frac{s}{0.57} + 1}{\frac{s}{1.77} + 1}$$

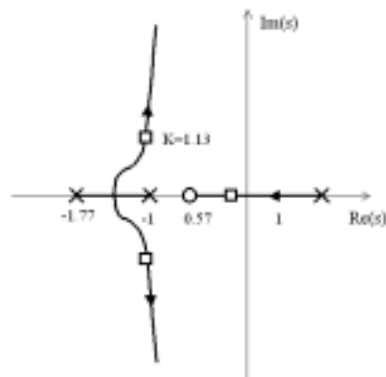
By adjusting the gain K so that the crossover frequency is around 1 rad/sec, $K = 1.13$ results in :

$$PM = 30.8^\circ$$

The Bode plot of compensated system is :



(b) Root Locus of the compensated system is :



and confirms that the system yields all stable roots with reasonable damping. However, it would be a better design if the gain was raised some in order to increase the speed of response of the slow real root. A small decrease in the damping of the complex roots will result.

- (c) No, because the sinusoid input will cause the system to blow up because the open loop system is unstable. In fact, the system will "blow up" even without the sinusoid applied. Or, a better description would be that the pendulum will fall over until it hits the table.

48. The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s/5 + 1)(s/50 + 1)}.$$

- (a) Design a lag compensator for $G(s)$ using Bode plot sketches so that the closed-loop system satisfies the following specifications:
- i. The steady-state error to a unit ramp reference input is less than 0.01.
 - ii. $PM \geq 40^\circ$
- (b) Verify and refine your design using MATLAB.

S o l u t i o n :

Let's design the lag compensator :

$$D(s) = \frac{Ts + 1}{\alpha Ts + 1}, \alpha > 1$$

From the first specification,

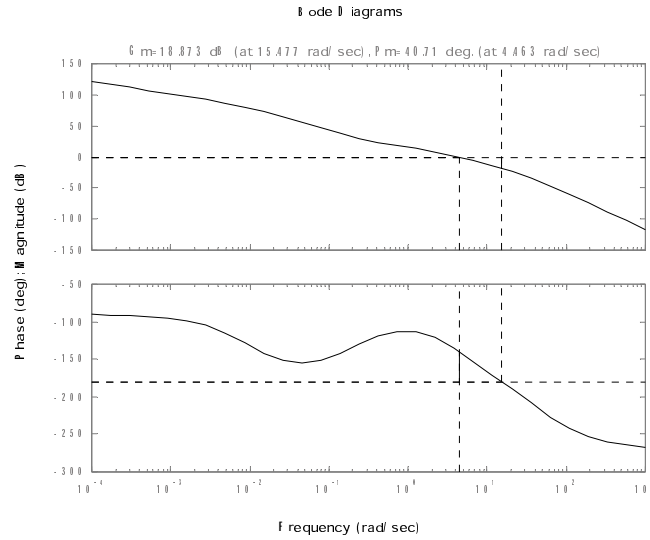
$$\begin{aligned} \text{Steady-state error to unit ramp} &= \lim_{s \rightarrow 0} \left| \frac{D(s)G(s)}{1 + D(s)G(s)} \frac{1}{s^2} - \frac{1}{s^2} \right| < 0.01 \\ &\Rightarrow \frac{1}{K} < 0.01 \\ &\Rightarrow \text{Choose } K = 150 \end{aligned}$$

Uncompensated, the crossover frequency with $K = 150$ is too high for a good PM . With some trial and error, we find that the lag compensator,

$$D(s) = \frac{\frac{s}{0.2} + 1}{\frac{s}{0.01} + 1}$$

will lower the crossover frequency to $\omega_c \simeq 4.46$ rad/sec where the $PM =$

40.7°.



49. The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s/5 + 1)(s/200 + 1)}.$$

- (a) Design a lead compensator for $G(s)$ using Bode plot sketches so that the closed-loop system satisfies the following specifications:
- The steady-state error to a unit ramp reference input is less than 0.01.
 - For the dominant closed-loop poles the damping ratio $\zeta \geq 0.4$.
- (b) Verify and refine your design using MATLAB including a direct computation of the damping of the dominant closed-loop poles.

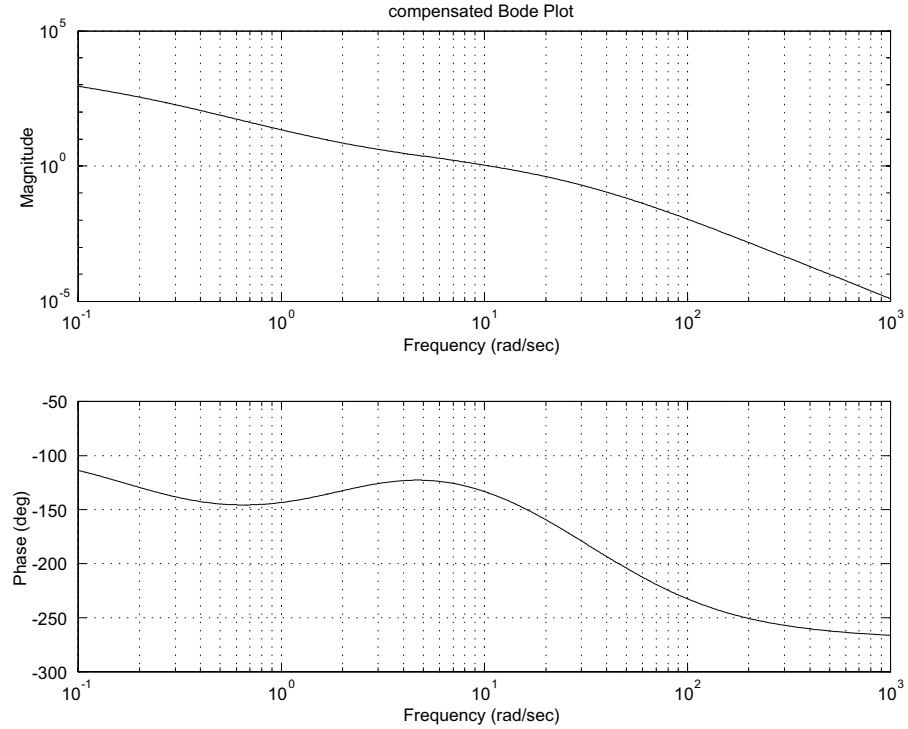
Solution :

Let's design the lead compensator :

$$D(s) = \frac{Ts + 1}{\alpha Ts + 1}, \quad \alpha < 1$$

From the first specification,

$$\begin{aligned} \text{Steady-state error to unit ramp} &= \lim_{s \rightarrow 0} s \left| \frac{D(s)G(s)}{1 + D(s)G(s)} \frac{1}{s^2} - \frac{1}{s^2} \right| < 0.01 \\ &\Rightarrow \frac{1}{K} < 0.01 \\ &\Rightarrow \text{Choose } K = 150 \end{aligned}$$



52. Consider a type I unity feedback system with

$$G(s) = \frac{K}{s(s+1)}.$$

Design a lead compensator using Bode plot sketches so that $K_v = 20 \text{ sec}^{-1}$ and $\text{PM} > 40^\circ$. Use MATLAB to verify and/or refine your design so that it meets the specifications.

Solution :

Use a lead compensation :

$$D(s) = \frac{Ts+1}{\alpha Ts+1}, \quad \alpha > 1$$

From the specification, $K_v = 20 \text{ sec}^{-1}$,

$$\Rightarrow K_v = \lim_{s \rightarrow 0} sD(s)G(s) = K = 20$$

$$\Rightarrow K = 20$$

From a hand sketch of the uncompensated Bode plot asymptotes, we see that the slope at crossover is -2, hence the PM will be poor. In fact, an exact computation shows that

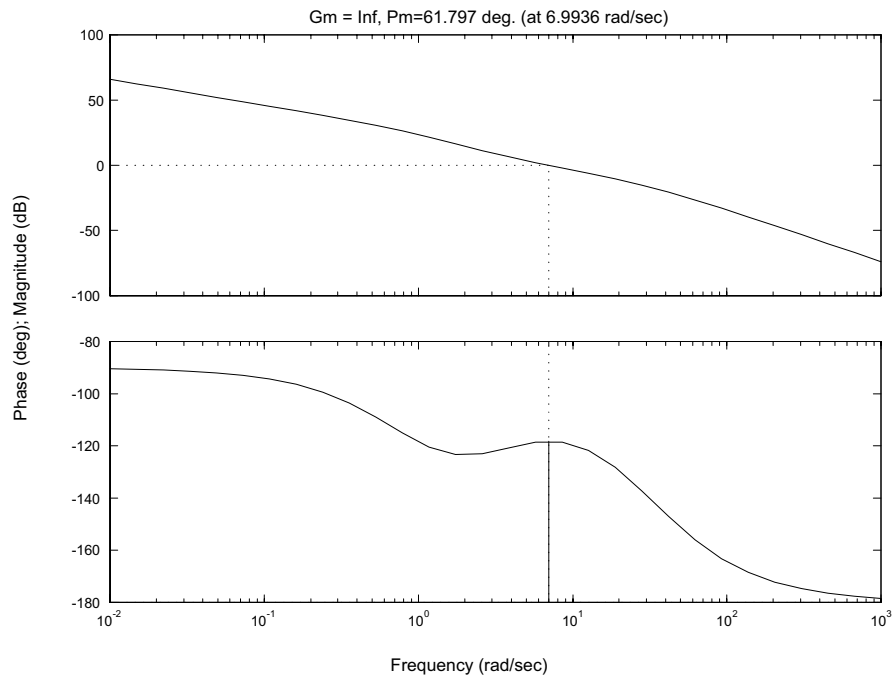
$$PM = 12.75 \text{ (at } \omega_c = 4.42 \text{ rad/sec)}$$

Adding a lead compensation

$$D(s) = \frac{\frac{s}{3} + 1}{\frac{s}{30} + 1}$$

will provide a -1 slope in the vicinity of crossover and should provide plenty of PM. The Bode plot below verifies that indeed it did and shows that the $PM = 62^\circ$ at a crossover frequency $\cong 7$ rad/sec thus meeting all specs.

Bode Diagrams



53. Consider a satellite-attitude control system with the transfer function

$$G(s) = \frac{0.05(s + 25)}{s^2(s^2 + 0.1s + 4)}.$$