

Name: _____

SID: _____

- Closed book. One page formula sheet. No calculators.
- There are 5 problems worth 100 points total.

Problem	Points	Score
1	22	
2	22	
3	15	
4	17	
5	23	
TOTAL	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

$\tan^{-1} \frac{1}{2} = 26.6^\circ$	$\tan^{-1} 1 = 45^\circ$
$\tan^{-1} \frac{1}{3} = 18.4^\circ$	$\tan^{-1} \frac{1}{4} = 14^\circ$
$\tan^{-1} \sqrt{3} = 60^\circ$	$\tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$
$\sin 30^\circ = \frac{1}{2}$	$\cos 60^\circ = \frac{\sqrt{3}}{2}$

$20 \log_{10} 1 = 0dB$	$20 \log_{10} 2 = 6dB$
$20 \log_{10} \sqrt{2} = 3dB$	$20 \log_{10} \frac{1}{2} = -6dB$
$20 \log_{10} 5 = 20dB - 6dB = 14dB$	$20 \log_{10} \sqrt{10} = 10 \text{ dB}$
$1/e \approx 0.37$	$1/e^2 \approx 0.14$
$1/e^3 \approx 0.05$	$\sqrt{10} \approx 3.16$

Problem 1 (22 pts)

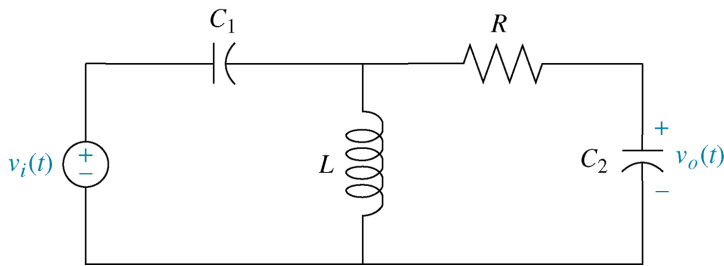
A system described by a linear differential equation has input $u(t)$ and output $y(t)$:

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 20y = -u + \frac{du}{dt}$$

[5 pts] a) Assuming zero initial conditions:

Find the transfer function $\frac{Y(s)}{U(s)} = \underline{\hspace{2cm}}$

[8 pts] b) Draw the equivalent mechanical system for this circuit, with voltage corresponding to force and current to velocity. Let $C_1 = \frac{1}{K_1}$, $L = M$, $R = B$, $C_2 = \frac{1}{K_2}$, $v_i(t) = F_i(t)$.



[9 pts] c) A nonlinear system with input V_{in} and output V_{out} is described by the differential equation

$$\frac{V_{out}}{R} + C\frac{dV_{out}}{dt} = e^{V_{in}-V_{out}} - 1$$

For small V_{in} and V_{out} , find the transfer function for the linearized system:

$\frac{V_{out}(s)}{V_{in}(s)} = \underline{\hspace{2cm}}$

Problem 2 Steady State Error (22 pts)

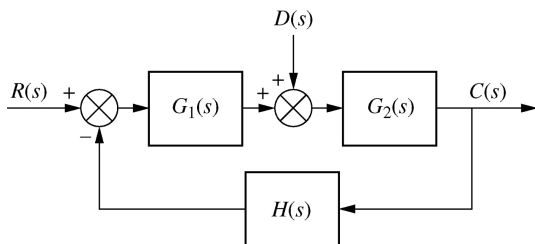
For the system below, let $H(s) = \frac{s}{s+10}$, $G_1(s) = \frac{s+4}{s}$, and $G_2(s) = \frac{2}{s+1}$.

[5 pts] a) For $d(t) = 0$, and $r(t)$ a unit step, determine $C(s)$. $C(s) = \underline{\hspace{2cm}}$

[6 pts] b) For $d(t) = 0$, and $r(t)$ a unit step, find $\lim_{t \rightarrow \infty} c(t) = \underline{\hspace{2cm}}$

[5 pts] c) For $d(t)$ a unit step and $r(t) = 0$, determine $C(s)$. $C(s) = \underline{\hspace{2cm}}$

[6 pts] d) For $d(t)$ a unit step and $r(t) = 0$, find $\lim_{t \rightarrow \infty} c(t) = \underline{\hspace{2cm}}$



Problem 3. Routh-Hurwitz (15 pts)

Given open loop transfer function:

$$G(s) = \frac{k}{(s+3)^3}$$

and closed loop transfer function (assuming unity feedback)

$$T(s) = \frac{k}{s^3 + 9s^2 + 27s + 27 + k}$$

[10 pts] a. Using the Routh-Hurwitz table, find the range of k for which the closed loop system is stable.

$$\text{_____} < k < \text{_____}$$

[5 pts] b. For the positive value of k found above, find the pair of closed loop poles on the imaginary axis.

$$s = \pm j\omega_o = \pm \text{_____}$$

Problem 4. Root Locus (17 pts)

Given open loop transfer function $G(s)$:

$$G(s) = \frac{(s + 8)}{(s + 1)(s + 2)}$$

For the root locus ($1 + kG(s) = 0$):

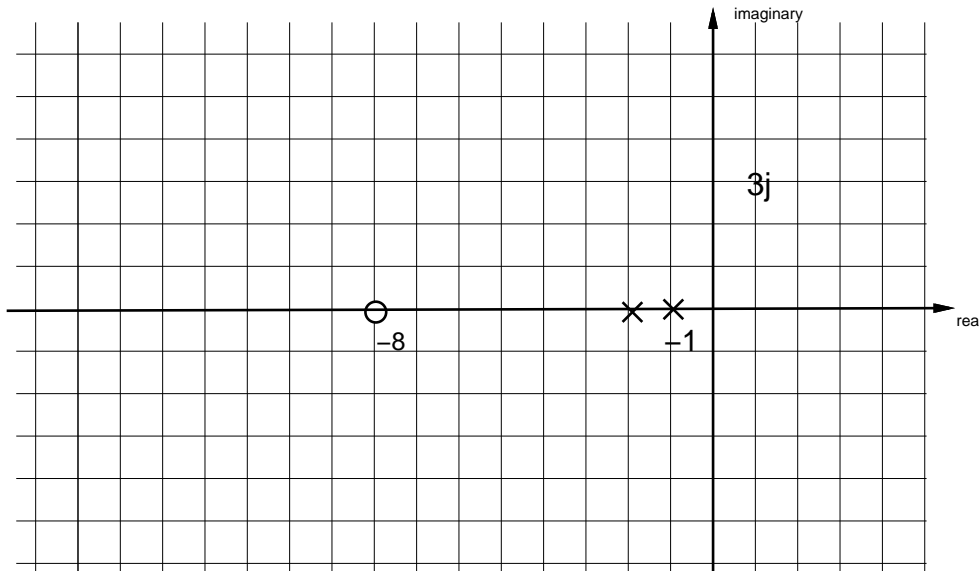
[2 pts] a) Determine the number of branches of the root locus = ____

[2 pts] b) Determine the locus of poles on the real axis _____

[2 pts] c) Determine the angles for each asymptote: _____

[6 pts] d) The break away and break in points are at $s =$ ____ and $s =$ ____

[5 pts] e) Sketch the root locus below using the information found above.



Problem 5. Root Locus Compensation (24 pts)

Given open loop transfer function $G(s)$:

$$G(s) = G_1(s) \frac{1}{(s+3)^2(s+1)^2}$$

Where $G_1(s)$ is a PD control of the form $G_1(s) = k(s + \alpha)$.

The closed loop system, using unity gain feedback and the PD controller, should have a pair of poles at $p = -1 \pm j2\sqrt{3}$.

[14 pts] a. Use the angle criteria to determine the zero location α for p to be on the root locus. Specify the angle contributions from each open loop pole. Mark the calculated zero on the pole-zero diagram below.

$\alpha = \underline{\hspace{2cm}}$

[10 pts] b. For the determined zero location, sketch the root locus, considering real-axis segments, real-axis intercept, and asymptotes.

