

Due at 1700, Fri. Sep. 28 in bcourses

Note: up to 2 students may turn in a single writeup. Reading Nise 7,8.

1. (10 pts) Stability in state space (Nise 6.5)

For the following system, use the Routh array to determine how many eigenvalues are in the RHP, on imaginary axis, and in LHP. Check with Matlab.

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & -12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t), \text{ and } y = [0 \ 0 \ 1]\mathbf{x}$$

2. (20 pts) Steady state error (Nise 7.4,7.5)

For the system in Fig. 1, let $G_1(s) = \frac{k_1(s+2)}{(s+3)}$, $G_2(s) = \frac{k_2}{s(s+4)}$ and $H(s) = 1$.

[6pts] a. Find transfer functions $\frac{E(s)}{R(s)}$ and $\frac{E(s)}{D(s)}$.

Let $e(t) = r(t) - c(t)$. Find the values of k_1, k_2 such that:

[7pts] b. The steady state error $|e(t)|$ due to a unit step disturbance $d(t) = u(t)$ is $< 10^{-4}$;

[7pts] c. The steady state error $|e(t)|$ due to a unit ramp input $r(t) = tu(t)$ is $< 10^{-4}$;

3. (20 pts) Steady state error for unity feedback (Nise 7.4,7.6)

For the system in Fig. 1, let $G_1(s) = 2$, $G_2(s) = \frac{10(s+10)}{s(s+2)}$ and $H(s) = s + 1$, $D(s) = 0$. $E = R - C$.

[5pts] a. Find and draw the equivalent unity gain system.

[3pts] b. Find $E(s)$.

[3pts] c. What is the system type?

[4pts] d. What are static error constants K_p, K_v, K_a for step and ramp inputs?

[5pts] e. What is the steady state error for a unit step input? For a unit ramp input? For $r(t) = t^2u(t)$?

4. (20 pts) Steady state error (Nise 7.8)

[10pts] a) Find steady state error for $r(t)$ a unit step input, using input substitution.

[10pts] b) Find steady state error for $r(t)$ a unit ramp input, using input substitution.

Given system:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} = \begin{bmatrix} -9 & -5 & -1 \\ 1 & 0 & -2 \\ -3 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} r, \text{ and } y = [1 \ -2 \ 4]\mathbf{x}$$

5. (30 pts) Root locus sketching (Nise 8.6)

For each part below with open loop transfer function $G(s)$ in unity gain feedback (Fig.2):

[8] i) Apply root locus rules (1-8): specify real axis segments, asymptotes and real axis intercept, break-away and break-in locations on real axis, and angle of departure from complex poles.

[2] ii) Find $j\omega$ axis intercepts if any.

[2] iii) Hand sketch root locus.

[2] iv) Specify range of k for stability.

[1] v) Verify your root locus using MATLAB.

a) $G(s) = \frac{k(s+20)}{(s+5)(s+15)}$ b) $G(s) = \frac{k(s+20)}{(s^2+10s+50)(s+5)}$

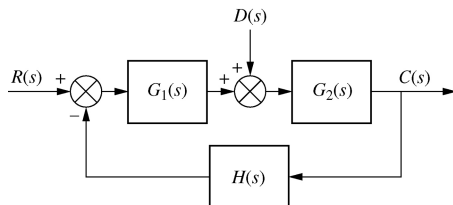


Fig. 1. Control System Block Diagram.

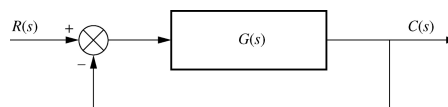


Fig. 2. Unity Gain Feedback.