Due at 1700, Fri. Nov. 9 in bcourses. .

Note: up to 2 students may turn in a single writeup. Reading Nise 12

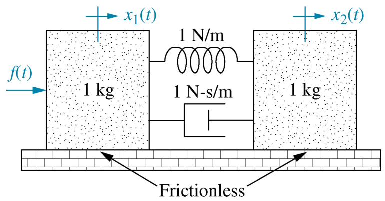
1. (20 pts) Controllability and Observability (Nise 12.3, 12.6)

For the system below, input is force $f(t)$, output is $y = x_1 - x_2$, the difference in positions, states x_1 and x_2.

[8pts] a) Write state and output equations for the circuit.

[6pts] b) Determine if the system is controllable.

[6pts] c) Determine if the system is observable.



2. (20 pts) Control Form transformation (Nise 12.4)

Given the following: $\dot{\mathbf{z}} = A\mathbf{z} + Bu = \begin{bmatrix} -20 & -5 \\ -10 & -25 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{z}$

[10pts] a) Find the transformation P such that $(\bar{A}, \bar{B})$ is in phase variable form, where $\bar{A} = P^{-1}AP$ and $\bar{B} = P^{-1}B$.

[10pts] b) Find $\bar{A}, \bar{B}, \bar{C}$ such that $\dot{\bar{\mathbf{x}}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{x}$.

3. (30 pts) Observer (Nise 12.5)

Given the plant:

$$G(s) = \frac{125(s+2)}{(s+5)(s+5)(s+10)}$$

where state variables are not available.

[6pts] a. Express $G(s)$ in observer canonical form, $\dot{\mathbf{x}} = A\mathbf{x} + Bu$.

[16pts] b. Design an observer: $\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L(y - \hat{y})$ for the observer canonical variables to yield a 2nd order transient response with $\zeta = 0.5$ and $\omega_n = 20$. (The third pole should be placed 10 times further from the imaginary axis than the dominant poles.)

[8pts] c. Using Matlab, compare the state variables in G for a step input with the observer estimate. (That is, plot $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$.)

4. (30 pts) Steady State Error/Integral Control (Nise 12.8)

Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A_1\mathbf{x} + B_1 u = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [1\ \ 2]\ \mathbf{x} \tag{1}$$

[8pts] a) Given error $e(t) = r(t) - y(t)$ where $r(t)$ is a scalar, evaluate the steady state error $\lim_{t\to\infty} e(t)$ for input $r(t)$ a unit step, with state feedback, that is, $u = -K_1\mathbf{x} + r$, where $K_1 = [k_1\ k_2]$ is chosen such that the closed loop poles are at $s_i = -8, -15$.

[15pts] b) Add an integrator to the plant, using a new state vector $\mathbf{x} = [x_1\ x_2\ x_N]^T$, write the new state and output equations, and find gains such that the closed-loop poles are at $s_i = -10, -10, -25$. Evaluate the steady-state error for a step input $r(t)$.

[7pts] c) Plot the step response for both systems in Matlab, and compare. Plot $x_1(t), x_2(t), x_N(t), u(t)$ and explain why $e(t) \to 0$.