

Name: \_\_\_\_\_

SID: \_\_\_\_\_

- Closed book. One page formula sheet. No calculators.
- There are 5 problems worth 100 points total.

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Problem	Points	Score
1	22	
2	22	
3	27	
4	15	
5	14	
TOTAL	100	

Tables for reference:

$\tan^{-1} \frac{1}{10} = 5.7^\circ$	$\tan^{-1} \frac{1}{5} = 11.3^\circ$
$\tan^{-1} \frac{1}{4} = 14^\circ$	$\tan^{-1} \frac{1}{3} = 18.4^\circ$
$\tan^{-1} \frac{1}{2} = 26.6^\circ$	$\tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$
$\tan^{-1} \frac{2}{3} = 33.7^\circ$	$\tan^{-1} \frac{3}{4} = 36.9^\circ$
$\tan^{-1} 1 = 45^\circ$	$\tan^{-1} \sqrt{3} = 60^\circ$
$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$
$\cos 45^\circ = \frac{\sqrt{2}}{2}$	$\sin 45^\circ = \frac{\sqrt{2}}{2}$

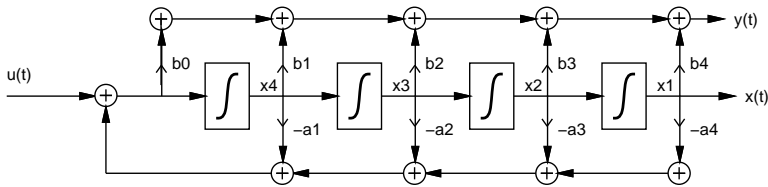
$\log_{10} 2 = 0.30$	$\log_{10} 3 = 0.48$	$\log_{10} 5 \approx 0.7$
$20 \log_{10} 5 = 20dB - 6dB = 14dB$	$20 \log_{10} \sqrt{10} = 10 \text{ dB}$	$\log_{10} 6 \approx 0.78$
$20 \log_{10} 1 = 0dB$	$20 \log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20 \log_{10} \sqrt{2} = 3dB$	$20 \log_{10} \frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$\sqrt{17} \approx 4.12$	$\sqrt{3.5} \approx 1.87$	$\pi/2 \approx 1.57$
$\sqrt{13} \approx 3.61$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e \approx 0.37$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^2 \approx 0.14$	$\sqrt{5} \approx 2.24$	$\sqrt{7} \approx 2.65$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

**Problem 1 (22 pts)**

Each part is independent.

[6 pts] a) Consider a single-input single-output system with input  $u(t)$  and output  $y(t)$  described by the block diagram below with coefficients as given:

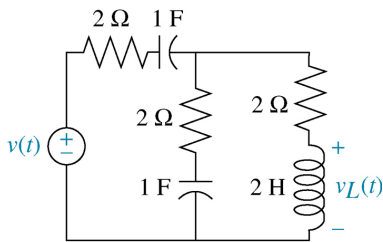
$$\begin{array}{cccccc}
 & a_1 = \underline{0} & a_2 = \underline{0} & a_3 = \underline{3} & a_4 = \underline{4} & \\
 b_0 = \underline{1} & b_1 = \underline{2} & b_2 = \underline{0} & b_3 = \underline{0} & b_4 = \underline{0} & 
 \end{array}$$



Write the transfer function for the system:

$$\frac{Y(s)}{U(s)} = \underline{\hspace{2cm}}$$

[8 pts] b) Draw the equivalent mechanical circuit for this electrical system, with voltage corresponding to force and current to velocity. Input force is  $v(t)$ .



c) Consider a system with a **step** input which has transfer function:

$$Y(s) = \frac{10}{s(s+5)(s^2+2s+2)} = \frac{A}{s} + \frac{B}{s+5} + \frac{Cs+D}{s^2+2s+2}$$

[4 pts] i) Find the partial fraction expansion coefficients for  $Y(s)$ :

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

$$C = \underline{\hspace{2cm}}$$

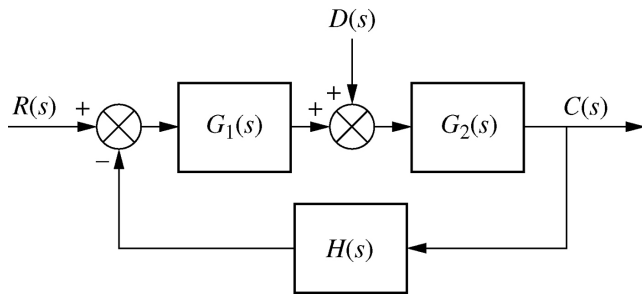
$$D = \underline{\hspace{2cm}}$$

[2 pts] ii) Find  $y(t)$ , the inverse Laplace transform of  $Y(s)$ , using the partial fraction expansion above.

$$y(t) = \underline{\hspace{2cm}}$$

[2 pts] iii) For the system above, the real pole is 5 times more negative than the real part of the complex pole pair. Is it reasonable to ignore the pole at  $s = -5$  and treat the step response as the step response of a second order system? Briefly explain using the results above.

**Problem 2 Steady State Error (22 pts)**



[6 pts] a) Let  $D(s) = 0$ . Let  $e(t) = r(t) - c(t)$ . Find  $\frac{E(s)}{R(s)}$  in terms of  $G_1, G_2, H$ .

$$\frac{E(s)}{R(s)} = \underline{\hspace{2cm}}$$

[8 pts] b) For the system above, let  $G_1(s) = k$ ,  $G_2(s) = \frac{s}{s+2}$ ,  $H(s) = \frac{1}{(s+1)s}$ . Let  $D(s) = 0$ . For  $r(t) = tu(t)$ , a unit ramp, find the steady state expression for  $e(t)$  for large  $t$ .

$$\lim_{t \rightarrow \infty} e(t) = \underline{\hspace{2cm}}$$

[8 pts] c) For the system above, let  $G_1(s) = k$ ,  $G_2(s) = \frac{s}{s+2}$ ,  $H(s) = \frac{1}{(s+1)s}$ . For  $d(t) = tu(t)$ , a unit ramp, and  $r(t) = 0$ , find the steady state expression for  $c(t)$  for large  $t$ .

$$\lim_{t \rightarrow \infty} c(t) = \underline{\hspace{2cm}}$$

**Problem 3. Root Locus Plotting (27 pts)**

For the root locus  $(1 + kG(s) = 0)$  with  $k > 0$ , and given open loop transfer function  $G(s)$ :

$$G(s) = \frac{(s + 3)}{(s + 1)^2(s + 2)^2}$$

[1 pts] a) Determine the number of branches of the root locus = \_\_\_\_

[2 pts] b) Determine the locus of poles on the real axis \_\_\_\_\_

[3 pts] c) Determine the angles for each asymptote: \_\_\_\_\_

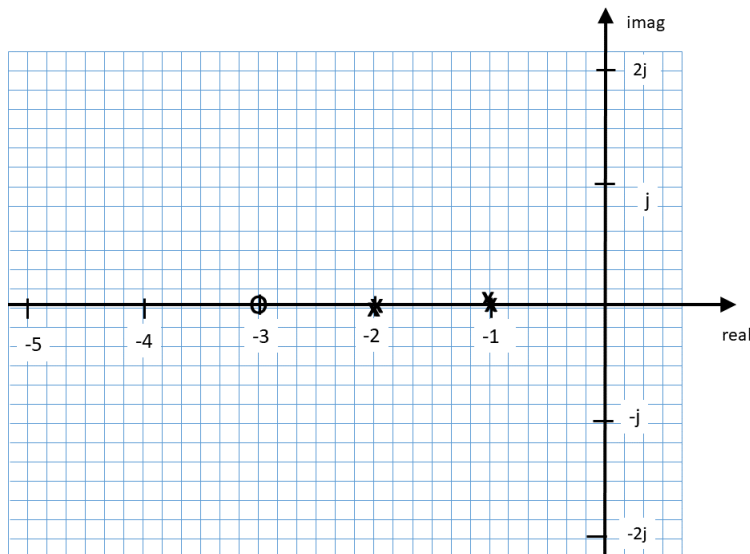
[4 pts] d) determine the real axis intercept for the asymptotes  $s =$  \_\_\_\_\_

[6 pts] e) Determine the break-in location on the real axis.  $s =$  \_\_\_\_\_

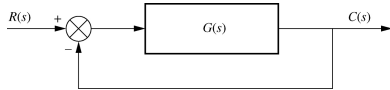
[6 pts] f) Estimate the value of  $k$  for which  $p \approx +1.9j$  is a closed-loop pole. (Show work for full credit).

$k =$  \_\_\_\_\_

[5 pts] g) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. Draw asymptotes.



**Problem 4. Root Locus Compensation (15 pts)**



Given open loop transfer function  $G(s)$ , where  $G_3(s)$  is the open-loop plant:

$$G(s) = G_1(s)G_3(s) = G_1(s)\frac{1}{(s+2)(s+3)}$$

and  $G_1(s)$  is a PI compensation of the form  $G_1(s) = k_p \frac{s+z_c}{s}$ . The closed loop system, using unity gain feedback and the PI controller, should have a pair of poles at  $p \approx -2 \pm 4j$ .

[6 pts] a. To obtain the closed loop pole at  $p$ , estimate the angle contribution (in degrees) for the following open loop poles:

pole	angle
$s = 0$	
$s = -2$	
$s = -3$	

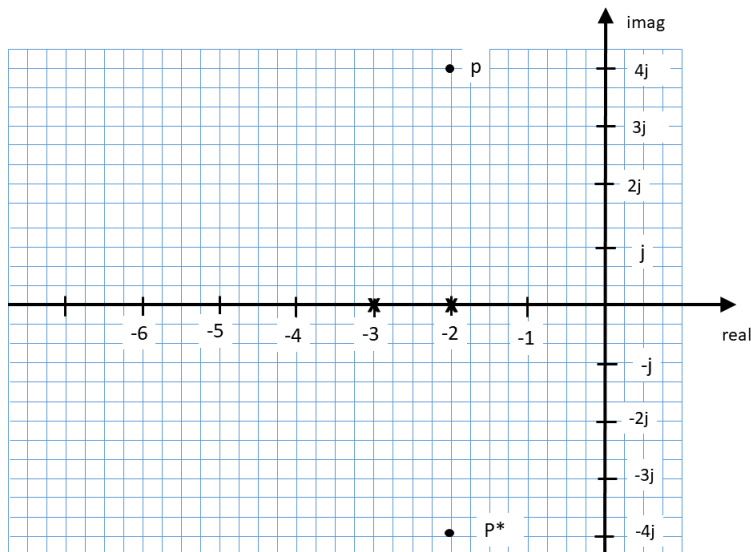
[2 pts] b. What is the necessary angle contribution of the zero  $z_c$  for the closed loop pole  $p$  to be on the root locus?

[9 pts] c. Find  $z_c$  to within  $\pm 0.1$  such that  $p$  is approximately on the root locus, within  $\pm 5$  degrees. (Show work.)

$z_c =$  \_\_\_\_\_

[2 pts] d. Estimate  $k_p$  for the pair of complex conjugate closed loop poles to be at  $p \approx -2 \pm 4j$ .

(Pole-Zero plot below for scratch work. It will not be graded).



**Problem 5. Routh-Hurwitz (14 pts)**

Given system with closed loop transfer function (assuming unity feedback)

$$T(s) = \frac{k(s+3)}{s^4 + 6s^3 + 13s^2 + (12+k)s + 4 + 3k}$$

[10 pts] a. Using the Routh-Hurwitz table, show that the maximum positive  $k$  for which the closed loop system is stable is approximately 10.

[4 pts] b. For  $k \approx 10$ , approximately find the pair of closed loop poles on the imaginary axis. (Show work).

$$s = \pm j\omega_o = \pm j \underline{\hspace{2cm}}$$

page for scratch work