Due at 1700, Fri. 11/22 in gradescope .

Note: up to 2 students may turn in a single writeup. Reading Nise 5, 12.

1. (30 pts) Steady State Error/Integral Control (Nise 12.8)

Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A_1\mathbf{x} + B_1 u = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t), \quad y = [2 \;\; 1]\;\mathbf{x} \tag{1}$$

[8pts] a) Given error $e(t) = r(t) - y(t)$ where $r(t)$ is a scalar, evaluate the steady state error $\lim_{t\to\infty} e(t)$ for input $r(t)$ a unit step, with state feedback, that is, $u = -K_1\mathbf{x} + r$, where $K_1 = [k_1 \; k_2]$ is chosen such that the closed loop poles are at $s_i = -4 \pm 4j$.

[15pts] b) Add an integrator to the plant, using a new state vector $\mathbf{x} = [x_1 \; x_2 \; x_N]^T$, write the new state and output equations, and find gains such that the closed-loop poles are at $s_i = -5, -10, -15$. Evaluate the steady-state error for a step input $r(t)$.

[7pts] c) Plot the step response for both systems in Matlab, and compare. For the augmented system, plot $x_1(t), x_2(t), x_N(t), u(t)$ and explain why $e(t) \to 0$.

2. (35 pts) State, and Observer Feedback (Separation principle handout)

Given the following system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t), \quad y = [2 \; 1]\;\mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

[6pts] a) Design a state feedback controller $u = r - [k_1 \;\; k_2]\mathbf{x}$ such that the closed loop system has poles $s = -3 \pm 5j$. Plot the step response $y(t)$ using Matlab.

[10pts] b) Design a critically damped observer $\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L(y - \hat{y})$ with both observer poles at $s = -15$.

[8pts] c) Write the state space equations for the controller with $u = r - K\hat{\mathbf{x}}$, such that the closed loop poles are the same as in part a). (This should have 4 state variables, either $\mathbf{x}, \hat{\mathbf{x}}$ or $\mathbf{x}, \mathbf{e}$.) From the separation principle, what are the eigenvalues of the combined system with observer and feedback control? Plot the step response $y_{obs}(t)$ (with Matlab) of the system with state feedback control using the observer estimated states.

[6pts] d) Use Matlab to plot the states $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ for $t > 0$ for the closed loop system of part c) for a step input $r(t)$. (Suggestion, use `sys = ss(Ac,Bc,Cc,Dc)` and `[Y,T,X] = lsim(sys,r,t,x0)`, where `Ac, Bc, Cc, Dc` are the matrices for the system with observer).

[5pts] e) Compare the output responses $y(t)$ to the step input from part a) and c). What differences are there? (quantify).

3. (35 pts) Linear Quadratic Regulator (handout)

Consider two cars travelling in a straight line. The dynamics of car 1 are $\dot{x_2} = \ddot{x_1} = 3.5u_1$, and car 2 has a plant model $\dot{x_4} = \ddot{x_3} = 3u_2$ where u_1 and u_2 are the car's thrust due to engine and braking. (x_1 is a point 1.0 m behind car 1, and x_3 is the position of the front bumper of car 2.) The outputs of the system are $y_1 = x_1$ and $y_2 = x_3 - x_1$. Note that if $y_2 > 0$ then car 2 has intruded on the safety zone of car 1.

Initial conditions are car 1 at -200 m, 30 m/sec, and car 2 at -210 m, 45 m/sec.

[4pts] a) Write the system equations in state space form.

[6pts] b) Use the LQR output method (Matlab function `lqry(sys,Q,R)`, with `Q=diag([1,1])` and `R=diag([1,1])` to find an optimal K for the state feedback control $\mathbf{u} = -K_b\mathbf{x}$. Plot $\mathbf{x}(t)$ and $\mathbf{u}(t)$ for the given initial condition (Matlab `initial`) and state feedback with gain K_b. How long does it take car 1 to get to within 1 m of the origin? What is car 1 velocity at 10 sec? Are there any problems with the system performance?

[12pts] c) Find new cost functions Q and R which maintains $y_2 < 0.3$ meter to prevent a collision, minimizes overshoot, and has both cars moving at less than 0.5 m/sec in 10 seconds. Also, max velocity should be less than $100m \cdot s^{-1}$ (which is approximately 200 mph). Plot $\mathbf{x}(t)$ and $\mathbf{u}(t)$ for the given initial condition (Matlab `initial`) and state feedback with new gain K_c.

[4pts] d) Find the solution to the Riccati equation P using Matlab function `care(A,B,Q,R)` and estimate the cost $J = (x^T P x)(0)$ for each of b) and c).

[4pts] e) Briefly compare the tradeoffs between control effort and time response between the two cases.