Professor Fearing EECS128/ME134 Problem Set 3 v1.01 Fall 2019

Due at 1700, Fri. Sep. 20 on gradescope .

Note: up to 2 students may turn in a single writeup. Reading Nise 4. Note: eqn 4.45: $\frac{4}{\sigma_d}$

1. (25 pts) State Space (Nise 2.6, 2.7, 3.4, 3.5)

Consider the rotary mechanical system shown in Fig.1 with input input $T(t)$ and output $\theta_1(t)$.

a. Draw the equivalent electrical circuit for the system in Fig. 1. (Assume AC circuit operation with a transformer equivalent to a gear ratio. A transformer with a winding turn ratio of 1:N with input voltage V and input current i, will have an output voltage of NV and output current of i/N).

b. Find the transfer function relating input $T(t)$ to output $\theta_1(t)$.

c. Write the state space equations for this system in phase-variable form and find A, B, C, D.

d. Draw the equivalent block diagram of the system in phase variable form using integrator, scale, and summing blocks.

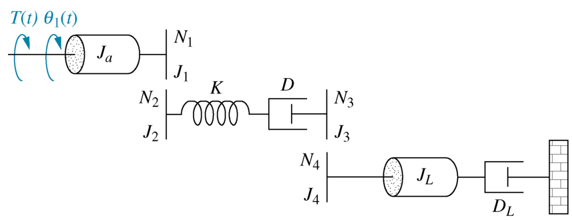

Fig. 1

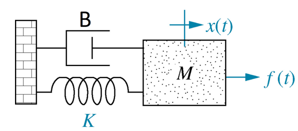

Fig. 2

2. (20 pts) Linearization (Nise 3.7)

For the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 1 - e^{u+x_1} \\ u x_1 x_2 \\ u\frac{x_1}{x_3} + x_4 \\ \cos(\frac{\pi}{2} u x_1) \end{bmatrix} \tag{1}$$

Linearize the system about $x_1 = 1, x_2 = 0, x_3 = 2, x_4 = \frac{1}{2}$, $u = -1$, and express in state space form:

$\dot{\delta\mathbf{x}} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u$.

3. (20 pts) 2nd order step response (Nise 4.6)

[15pts] a) For the system shown in Fig. 2, find the values of M, K and B such that $x(t)$ has a 10% overshoot with a settling time of $10^{-7}s$ with $f(t)$ a step input. Also note ζ and ω_n.

[5pts] b) For a MEMS device, consider Si with density of 2300 $kg \cdot m^{-1}$. With $K = 10^5 Nm^{-1}$ (and appropriate B for the specified settling time and overshoot), find the area of a $1\mu m$ thick Si layer which gives the desired overshoot and settling time.

4. (15 pts) Second order poles (Nise 4.6)

For each pair of second-order system step response specifications, find the location of the second order pair of poles.

a. %OS = 20%; $T_s = 0.2$ seconds.

b. %OS = 20%; $T_r = 0.2$ seconds.

c. %OS = 20%; $T_p = 0.2$ seconds.

5. (20 pts) Dominant poles (Nise 4.7)

A system has transfer function $H(s) = \frac{101a}{(s^2+2s+101)(s+a)}$. Consider the step reponse of the system to be $y(t)$. The claim is made that for some range of a, (with $\omega_d = \omega_n\sqrt{1-\xi^2}$), that

$$y(t) \approx 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

i.e. the pole at $s = -a$ can be neglected. If so, find the range of a for which the approximate step response has peak c_{max} within 2% of the c_{max} of the true step response for $H(s)$. Also provide a plot using Matlab for the true and approximate step response with the a found above.

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