Due at 1700, Fri. Sep 27 on gradescope.
Note: up to 2 students may turn in a single writeup. Reading Nise 5-5.6, 6

1. (20 pts) Pole-Zero cancellation (Nise 4.8)
Find the partial fraction expansion for the following transfer functions $C_1(s), C_2(s)$. Determine if pole-zero cancellation is a reasonable approximation for the step response by comparing coefficients of the terms of the partial fraction expansion. Using MATLAB, plot the step response for the complete system and the approximate system assuming pole-zero cancellation. (Note that the `step()` command should be on the systems $H(s)$ and $\tilde{H}(s)$). (Normalize amplitude so that final value is 1.) For C_1, C_2, briefly discuss if the step response approximation is reasonably close to the step response $c_i(t)$ of the original system.

a. $C_1(s) = \frac{1}{s} \cdot H_1(s) = \frac{(s+5.8)}{s(s+1)(s+3)(s+6)} \quad \tilde{C}_1(s) = \frac{1}{s} \cdot \tilde{H}_1(s) \approx \frac{1}{s(s+1)(s+3)}$

b. $C_2(s) = \frac{1}{s} \cdot H_2(s) = \frac{(s+2.8)}{s(s+1)(s+3)(s+6)} \quad \tilde{C}_2(s) = \frac{1}{s} \cdot \tilde{H}_2(s) \approx \frac{1}{s(s+1)(s+6)}$

2. (25 pts) Time Domain Solution - Convolution (Nise 4.11)
For the system below,
`[ 5 pts]` a. Find e^{At}.
`[15 pts]` b. Find $\mathbf{x}(t)$ and $y(t)$ using convolution (4.109) for the system below with unit step input $u(t)$:
`[ 5 pts]` c. Show that the $\mathbf{x}(t)$ found above is a solution to $\dot{\mathbf{x}} = A\mathbf{x} + Bu$ (by direct substitution).

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } y = [2 \quad 1]\mathbf{x}$$

3. (15 pts) Time Domain Solution - Laplace Transform (Nise 4.10)

For the system $\{A, B, C, D\}$ above, find $\mathbf{x}(t)$ and $y(t)$ using the Laplace transform method with unit step input $u(t)$.

4. (20 pts) Block Diagram Equivalence (Nise 5.2)
Find and draw the unity feedback system that is equivalent to the system in Fig. 1 below.

5. (20 pts) Routh Array (Section 6.4)
In the control system in Fig. 2, $D(s) = 0$, $G_1(s) = k$, $H(s) = 1$, and

$$G_2(s) = \frac{1}{(s+49)(s^2+4s+5)}$$

`[2pts]` a. Determine the closed loop transfer function $\frac{C(s)}{R(s)}$.
`[18pts]` b. Using the Routh-Hurwitz table, find the range of k for the system to have all closed loop poles in the LHP.

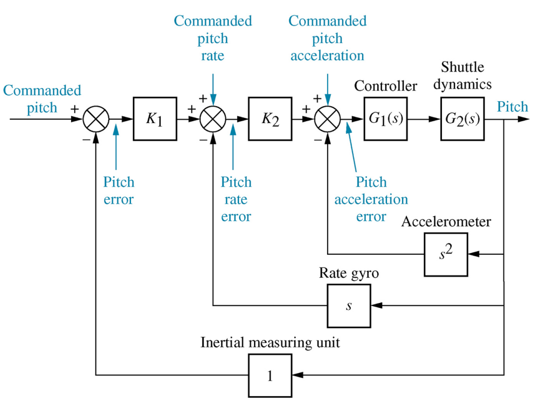

Fig. 1. Block Diagram.

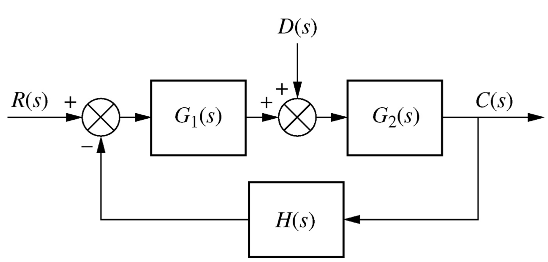

Fig. 2. Control System