Professor Fearing EEC128/MEC134 Problem Set 5 v1.02 Fall 2019

Due at 1700, Fri. Oct. 4 in gradescope

Note: up to 2 students may turn in a single writeup. Reading Nise 7,8.

1. (20 pts) Routh Array (Nise 6.3)

In the control system in Fig. 1, $D(s) = 0$, $G_1(s) = k$, $H(s) = \frac{1}{s+2}$, and

$$G_2(s) = \frac{s+1}{s^3 + 2s^2 + 5s}$$

[2pts] a. Determine the closed loop transfer function $\frac{C(s)}{R(s)}$.
[16pts] b. Using the Routh-Hurwitz table, find the range of k for the system to have all closed loop poles in the LHP.
[2pts] c. Find the value of k that makes the system have poles on the $j\omega$ axis.

~~3~~ 2. (20 pts) Steady state error (Nise 7)

Consider a PI controller $G_1(s) = k_p + k_i/s$ for plant $G_2(s) = \frac{1}{s+1}$ and $H(s) = 1$, with $r(t) = 0$.
[4pts] a. For a step disturbance $d(t)$, show that the steady state error is zero.
[8pts] b. For a disturbance input $d(t) = \cos(\omega_o t)u(t)$, (e.g. a power line signal) find the steady state error $e(t)$ after all transients have died away. (Note $s = 0$ is not part of region of convergence.) Hint: consider response of system to $e^{j\omega_o t}$, and let $s = j\omega_o$.
[8pts] c. For the same disturbance in b., find a new $G_1(s)$ which will have zero steady state error for a step disturbance, and zero steady state error for $d(t) = \cos(\omega_o t)u(t)$. (Hint: add something else in the controller which copies the disturbance.)

3. (20 pts) Steady state error for non-unity feedback (Nise 7.4,7.6)

For the system in Fig. 1, let $G_1(s) = 12$, $G_2(s) = \frac{(s+8)}{(s+3)(s+4)}$ and $H(s) = \frac{s+1}{s}$, $D(s) = 0$. $E = R - C$.
[5pts] a. Find and draw the equivalent unity gain system.
[3pts] b. Find $E(s)$.
[3pts] c. What is the system type?
[4pts] d. What are static error constants K_p, K_v, K_a for step and ramp inputs?
[5pts] e. What is the steady state error for a unit step input? For a unit ramp input? For $r(t) = t^2 u(t)$?

4. (20 pts) Steady state error (Nise 7.8)

[10pts] a) Find steady state error for $r(t)$ a unit step input, using input substitution.
[10pts] b) Find steady state error for $r(t)$ a unit ramp input, using input substitution.
Given system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} -10 & -5 & -1 \\ 1 & 0 & -2 \\ -3 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} r \text{ , and } \quad y = [1 \quad -2 \quad 4]\mathbf{x}$$

5. (20 pts) Closed-loop poles from geometry (Nise 8.3)

For each part below with open loop transfer function $G_i(s)$ in unity gain feedback (Fig.2):
[5] ~~i~~i) Showing pole-zero sketch with angles, find $j\omega$ axis intercepts.
Verify that $\angle G_i(j\omega) = (2k+1)180°$.
[4] ~~i~~ii) Show in a sketch length of vectors to $j\omega$ axis intercepts, and find k.
Verify that $|kG_i(j\omega)| = 1$.
[1] iii ~~v~~) Verify the values found using MATLAB root locus function and include plot.

a) $G_1(s) = \frac{k}{(s+6)(s+4)(s-2)}$ b) $G_2(s) = \frac{k(s+4)}{(s+6)(s-2)}$

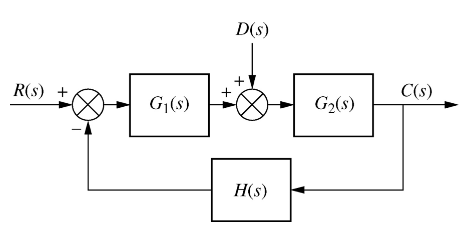

Fig. 1. Control System Block Diagram.

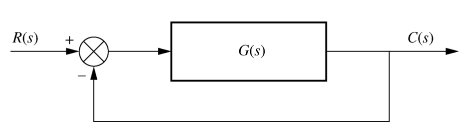

Fig. 2. Unity Gain Feedback.