Due at 1700, Fri. Oct. 11 in gradescope.
Note: up to 2 students may turn in a single writeup. Reading Nise 8
Midterm: Thurs. Oct. 24. Location: tba, 1710-1830 am.

1. (30 pts) Root locus sketching (Nise 8.6)
For each part below with open loop transfer function $G(s)$ in unity gain feedback (Fig.1):
[8] i) Apply root locus rules (1-8): specify real axis segments, asymptotes and real axis intercept, break-away and break-in locations on real axis, and angle of departure from complex poles.
[2] ii) Find $j\omega$ axis intercepts if any.
[2] iii) Hand sketch root locus.
[2] iv) Specify range of k for stability.
[1] v) Verify your root locus using MATLAB.

a) $G(s) = \frac{k(s+20)}{s(s-20)}$ b) $G(s) = \frac{k(s+20)}{(s(s+6)(s+12))}$

2. (25 pts) Root locus (Nise 8.7)
Given the unity gain feedback system in Fig. 1, where

$$G(s) = \frac{K(800)(s+20)}{s(s^2+20s+200)(s+40)}$$

[14 pts] a) Find and approximately hand sketch the root locus using RL rules 1-8 for $k > 0$.
[4 pts] b) Find the range of K which makes the system stable.
[5 pts] c) Using the second order approximation (assuming dominant 2nd order poles) find the value of K that gives $\zeta \approx 0.30$ for the system's dominant closed-loop poles.
[2 pts] e) Use MATLAB to plot the actual step response for c) and compare to 2nd order poles approximation estimate. How far off is the approximatiom?

3. (26 pts) Root locus (Nise 8.6, 8.9)
Consider the unity gain feedback system in Fig. 1 with $G(s) = \frac{k(s^2+s-2)}{-s^2-2s+8}$. Here $-\infty < k < \infty$
[14 pts] a) Apply root locus rules: specify real axis segments, break-away and break-in locations on real axis, and angle of departure from complex poles.
[6 pts] b) Find the $j\omega$ crossing using Routh-Hurwitz.
[4 pts] c) Hand sketch the closed-loop root locus for positive and negative k.
[2 pts] d) Find the range of k for stability.

4. (19 pts) Generalized Root locus (Nise 8.8)
This problem exams using generalized root locus for tuning of a P+D control ($u = k_p e + k_d \dot{e}$) on a second order plant. (Fig. 2 with $D(s) = 0$, $H(s) = 1$.) Given ${\omega_n}^2 = 5, 2\zeta\omega_n = 2, k = 2, k_p = 1, k_d = 1$ and

$$G_1(s) = k_d s + k_p \qquad G_2(s) = \frac{k\ {\omega_n}^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For each part below, find the characteristic equation in terms of the requested parameter, approximately hand sketch the root locus using RL rules 1-6 with respect to positive values of the changing parameter, and verify with Matlab.
[5] a) Standard root locus as k changes ($k_d = 1, k_p = 1$ fixed).
[5] b) Generalized root locus as k_p changes ($k_d = 1, k = 2$ fixed).
[5] c) Generalized root locus as k_d changes ($k_p = 1, k = 2$ fixed).
[4] d) Consider a system with starting values of $k = 2, k_p = 1, k_d = 1$. Which parameter is best to tune for reducing the settling time? Which gain parameter reduces damping factor?

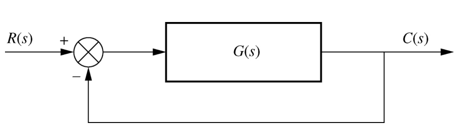

Fig. 1. Unity Gain Feedback.

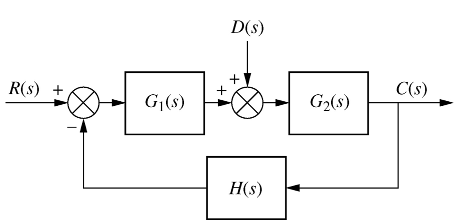

Fig. 2. Control System