Professor Fearing EECS128/MEC134 Problem Set 7 v1.01 Fall 2019

Due at 1700, Fri. Oct 18 in gradescope .

Note: up to 2 students may turn in a single writeup. Reading 9, 10-10.7.

Midterm: Thurs. Oct. 24, Location: tba, 1710-1830 pm.

1. (12 pts) Steady state error (Nise 7.8, review)

[6pts] a) Find steady state error for $r(t)$ a unit step input, using input substitution.

[6pts] b) Find steady state error for $r(t)$ a unit ramp input, using input substitution.

Given system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} r \text{ , and } \; y = [1 \quad -2 \quad 4]\mathbf{x}$$

2. (10 pts) Positive Feedback Root locus (Nise 8.9)

The open loop transfer function for a system in unity feedback is given by:

$$G(s) = \frac{k(s^2 + 25s - 100)}{s^2 + 15s + 50}$$

.

`[2 pts]` a) Determine the characteristic equation for the closed loop system.

`[8 pts]` b) Sketch the root locus with respect to **negative** values of k.

3. (18 pts) PD compensation (Nise 9.3)

Consider open loop plant

$$G(s) = \frac{10K}{s(s+10)(s+20)}$$

with unity feedback.

`[2 pts]` a) find K such that overshoot is 10%.

`[8 pts]` b) Design a PD controller (i.e. find zero location) such that $T_p \approx 0.2$ sec, with the same 10% overshoot. (Note K may change from part a)).

`[4 pts]` c) Hand sketch the root locus for the original system and the system with the **PD** compensator from part b), and verify with Matlab.

`[2 pts]` d) Use Matlab to compare the step response for the closed-loop compensated and uncompensated systems, transient and steady state response.

`[2 pts]` e) Find the steady state error for a step for both systems.

4. (23 pts) PID Compensation (Nise 9.4)

Consider open loop plant

$$G(s) = \frac{K}{(s+4)^2(s+2)}$$

with unity feedback.

[2pts] a. Find the gain K for the uncompensated system to have $\zeta = 0.5$ (Matlab ok). What is the setting time T_s?

[14pts] b. Design a PID controller such that $\zeta = 0.5$ and $T_s < 1.5$ sec, with zero steady state error for a step. (Note that PI effect on transient must be considered). Specify open and closed-loop poles, zeros and gains.

[5pts] c. Hand sketch the root locus of the original and compensated system, and verify with Matlab (rules1-5,9).

[2pts] d. Show before and after compensation step response using Matlab on same plot.

5. (15 pts) Bode Plot (Nise 10.2)

Sketch (by hand) the asymptotes of the Bode plot magnitude and phase for each of the following open-loop transfer functions. For second order poles, note peak magnitude in dB. (For the approximation, be sure to label the trends as $j\omega \to 0$ and $j\omega \to \infty$, as well as the slopes, and the frequency at which the slopes change.) Verify sketch using MATLAB plot with same axes scales, and turn in (log frequency, and magnitude in dB).

$$G_1(s) = \frac{s}{(s+1)^2(s+100)} \quad G_2(s) = \frac{s+1}{s(s+100)} \quad G_3(s) = \frac{1}{s^2+2s+101}$$

6. (22 pts) Nyquist Plot (Nise 10.5)

Consider a closed loop system with unity feedback and gain k. The open loop transfer function is:

$$G(s) = \frac{1000\sqrt{10}(s-10)}{(s+10)^2(s+10\sqrt{10})^2}$$

[6pts] a) Hand sketch the asymptotes of the Bode plot magnitude and phase for the open-loop transfer function.

[10pts] b) Hand sketch Nyquist diagram.

[4pts] c) From Nyquist diagram, determine range of k for stability.

[2pts] d) Verify sketches with MATLAB (`bode()` and `nyquist()`) and hand in.