Due at 1700, Fri. Nov. 16 in gradescope. .

Note: up to 2 students may turn in a single writeup. Reading Nise 12

1. (24 pts) State Feedback/Pole placement (Nise 12.2)

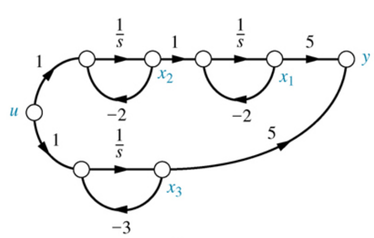
Consider the plant, where $G(s) = Y(s)/U(s)$:

$$G(s) = \frac{100}{(s+2)^2(s+4)^2}.$$

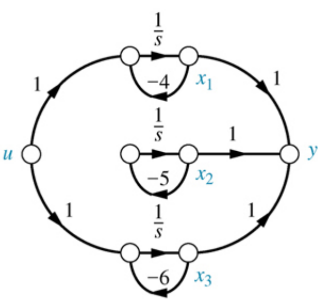
[6pts] a. Draw the signal graph in phase variable form and write the corresponding state equations.
[10pts] b. Find $K = [k_1\ k_2\ k_3\ k_4]$ such that feedback $u = r - K\mathbf{x}$ yields an equivalent second order step response with $\zeta = \sqrt{2}/2$ and $\omega_n = 5\sqrt{2}$. (Place third and fourth pole with real part 5 times further from $j\omega$ axis as the dominant pole pair).
[8pts] c. With zero initial conditions, use Matlab to plot the step response $y(t)$ and also $u(t)$, and each individual component $k_1x_1(t)$, $k_2x_2(t)$, $k_3x_3(t)$, $k_4x_4(t)$. Which state contributes most to $u(t)$?

2. (30 pts) Controllability and Observability (Nise 12.3, 12.6)

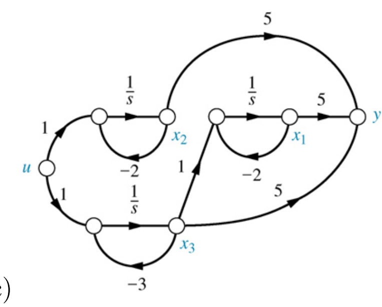
For each of the systems below, input is $u(t)$, output is $y(t)$, the difference in positions, states x_1, x_2, x_3.
[6pts] a) Write state and output equations for the graph.
[2pts] b) Determine if the system is controllable.
[2pts] c) Determine if the system is observable.



a)



b)



c)

3. (20 pts) Control Form transformation (Nise 12.4, 5.8)

Given the following:

$$\dot{\mathbf{z}} = A\mathbf{z} + Bu = \begin{bmatrix} -21 & 5 \\ -15 & 1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{z}$$

[10pts] a) Find the transformation P such that $(\bar{A}, \bar{B})$ is in ~~parallel~~ diagonal form, where $\bar{A} = P^{-1}AP$ and $\bar{B} = P^{-1}B$.
[10pts] b) Find $\bar{A}, \bar{B}, \bar{C}$ such that $\dot{\bar{\mathbf{x}}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{x}$.

4. (26 pts) Observer (Nise 12.5)

Given the plant $G(s) = \frac{Y(s)}{U(s)}$:

$$G(s) = \frac{125}{(s+3)(s+3)(s+10)}$$

where state variables are not available.
[6pts] a. Express $G(s)$ in observer canonical form, $\dot{\mathbf{x}} = A\mathbf{x} + Bu$, $y = C\mathbf{x}$.
[14pts] b. Design an observer: $\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L(y - \hat{y})$ for the observer canonical variables to yield a 2nd order transient response with $\zeta = 0.5$ and $\omega_n = 40$. (The third pole should be placed 10 times further from the imaginary axis than the dominant poles.)
[8pts] c. Using Matlab, compare the state variables in G for a step input with the observer estimate. That is, plot $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$. Let $\mathbf{x}(0) = [1\ 1\ 1]^T$, and $\hat{\mathbf{x}}(0) = [0\ 0\ 0]^T$.