EECS C128/ ME C134
Final
Fri. Dec. 18, 2015
1910-2200 pm

Name:___________________________
SID:____________

- Closed book. One page, 2 sides of formula sheets. No calculators.

- There are 6 problems worth 100 points total.

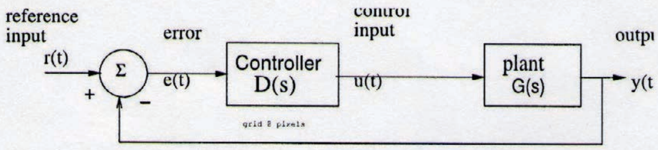
Problem	Points	Score
1	15	
2	16	
3	18	
4	20	
5	16	
6	15	
Total	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of 'F' and a letter will be written for your file and to the Office of Student Conduct.

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

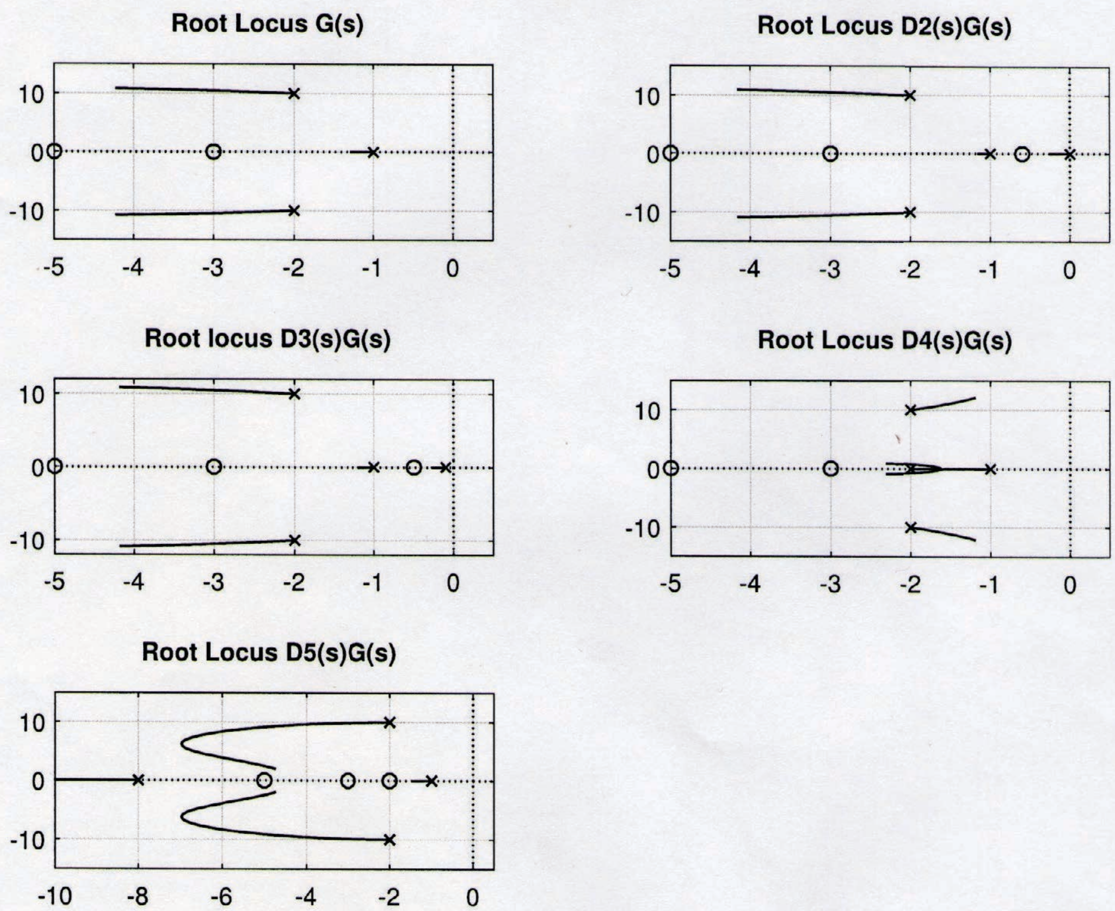
Problem 1 (15 pts)



You are given the open-loop plant:

$$G(s) = \frac{5(s+5)(s+3)}{(s+1)(s^2+4s+104)}.$$

For the above system, the partial root locus is shown for 5 different controller/plant combinations, $G(s), D_2(s)G(s), \ldots, D_5(s)G(s)$. (Note: the root locus shows open-loop pole locations for $D(s)G(s)$, and closed-loop poles for $\frac{DG}{1+DG}$ are at end points of branches).



[5 pts] a) For each set of open-loop poles and zeros given above, choose the best corresponding open-loop Bode plot V,W,X,Y, or Z from the next page:

(i) $G(s)$: Bode Plot Y

(ii) $D_2(s)G(s)$: Bode plot X

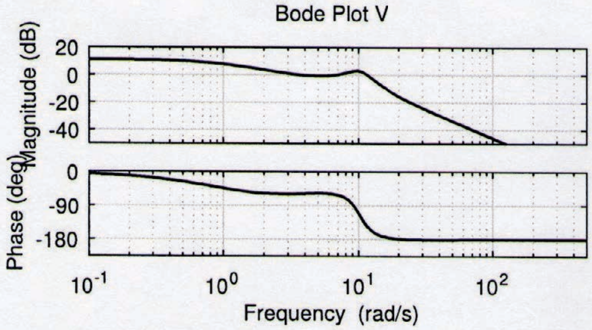
(iii) $D_3(s)G(s)$: Bode plot W

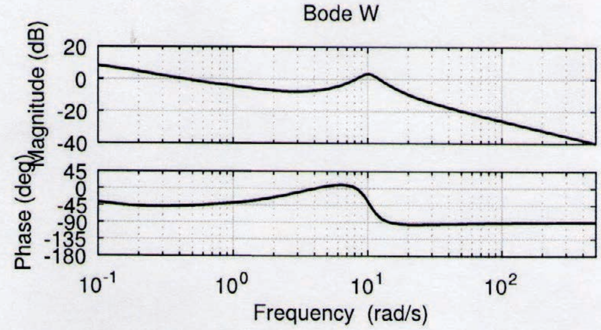
(iv) $D_4(s)G(s)$: Bode Plot V

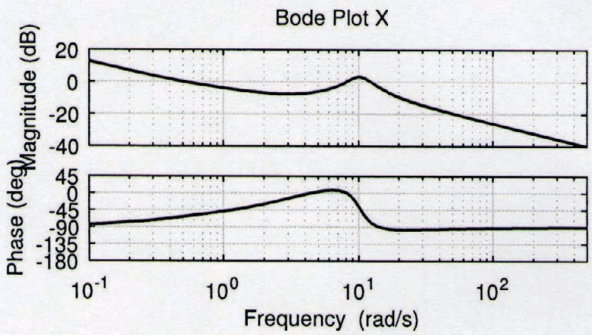
(v) $D_5(s)G(s)$: Bode Plot Z

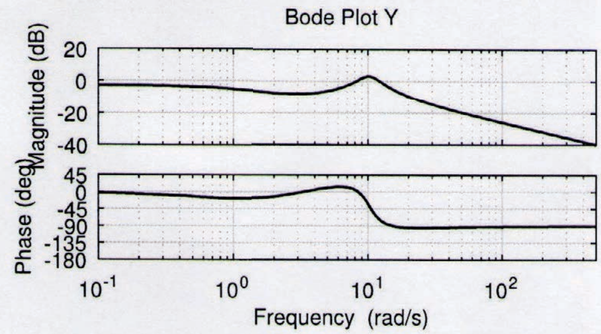
Problem 1, cont.

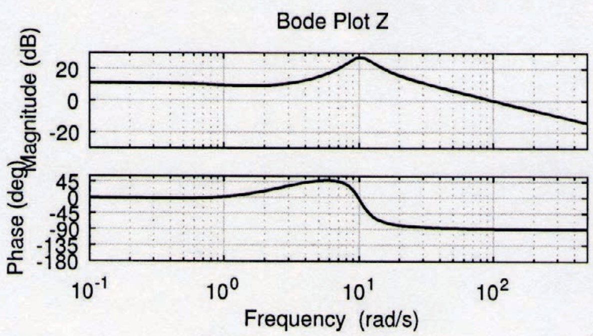
The open-loop Bode plots for 5 different controller/plant combinations, $D_1(s)G(s), \ldots, D_5(s)G(s)$ are shown below.











[5 pts] b) For the Bode plots above:

(i) Bode plot V: phase margin 40 (degrees) at $\omega =$ 11 rad/s
Bode plot V: gain margin 20 dB at $\omega =$ 23 rad/s
Estimate damping factor $\zeta =$ .3

(ii) Bode plot Z: phase margin 90 (degrees) at $\omega =$ 100
Bode plot Z: gain margin ∞ dB at $\omega =$ —

Problem 1, cont.

[5 pts] c) For each closed loop controller/plant with root locus as given in part a), choose the best corresponding closed-loop step response (A-E). (Note: dashed line shows final value.)

(i) $G(s)$: step response E

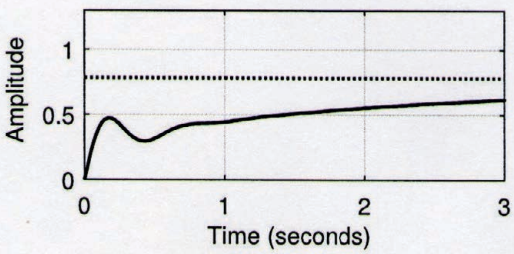
(ii) $D_2(s)G(s)$: step response D

(iii) $D_3(s)G(s)$: step response A

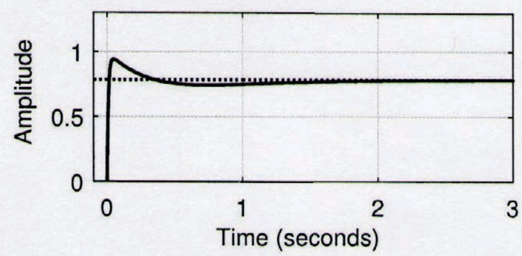
(iv) $D_4(s)G(s)$: step response C

(v) $D_5(s)G(s)$: step response B

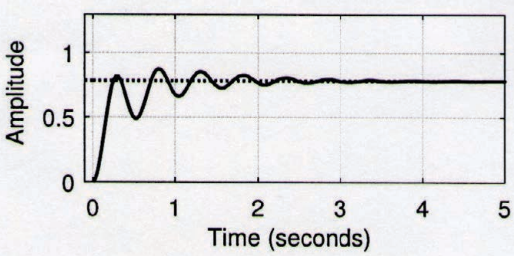
Step A



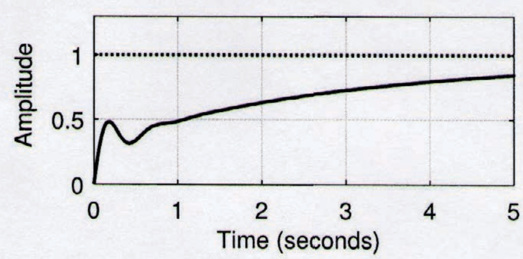
Step B



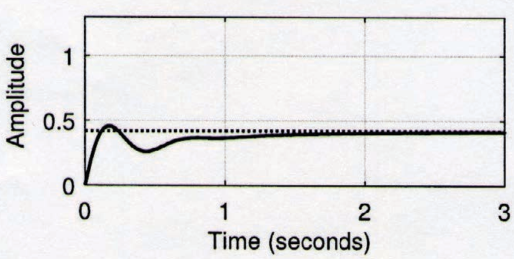
Step C



Step D

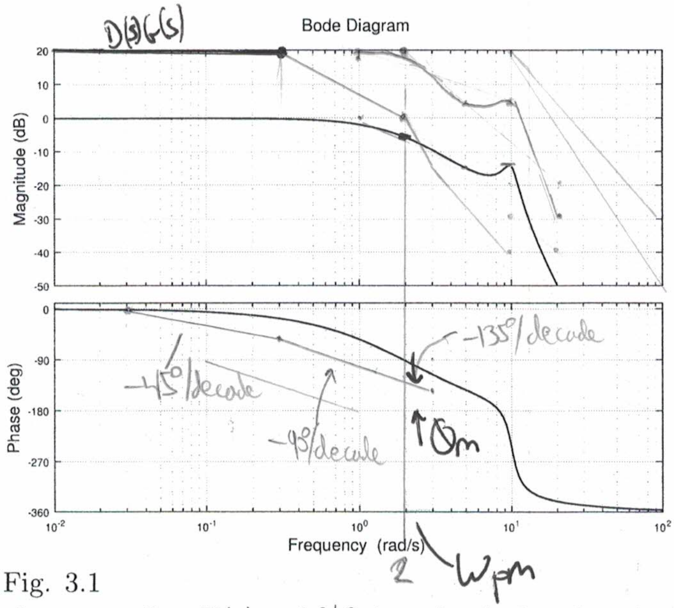


Step E



Problem 2 (16 pts) F15 Key. update ?

The open-loop system is given by $G(s) = \frac{400}{(s+2)^2(s^2+2s+101)}$, and Bode plot for $G(s)$ is here:



$K_p = 10 \Rightarrow D(0)G(0) = 20\,dB$

$\frac{K\alpha}{\beta} = 10$

$|D(j2)\,G(j2)| = 1$

$20 \log |G(j2)| \approx -6\,dB \Rightarrow$ need $+6\,dB$

D: +20 dB

+6dB

0dB

.3 2 3

Fig. 3.1

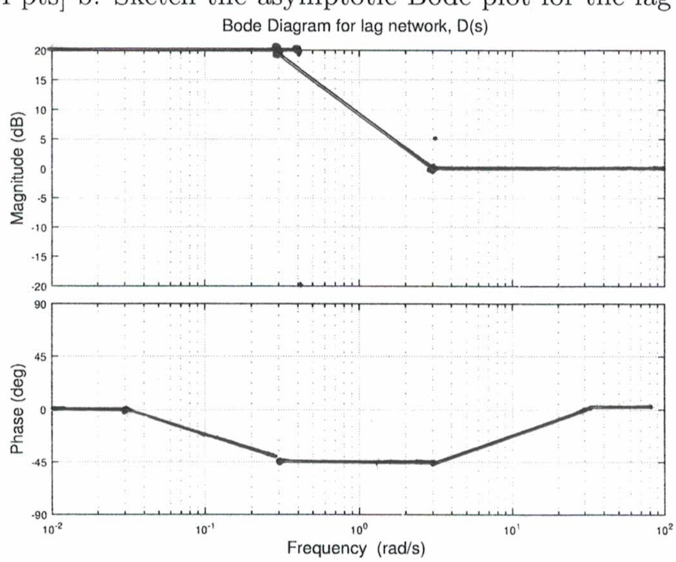
A lag controller $D(s) = k\frac{s+\alpha}{s+\beta}$ is to be designed such that the unity gain feedback system with openloop transfer function $D(s)G(s)$ has static error constant $K_p = 10$. $D(s)G(s)$ should have a nominal (asymptotic approximation) phase margin $\phi_m \approx 40°$ at $\omega_{pm} = 2$ rad s^{-1}.

[6 pts] a. Determine gain, zero, and pole location for the lag network $D(s)$:

$\frac{K(s+3)}{s+.3}$

gain $k =$ 1 zero: $\alpha =$ 3 pole: $\beta =$ .3

[4 pts] b. Sketch the asymptotic Bode plot for the lag network $D(s)$ alone on the plot below:



[4 pts] c. Sketch the asymptotic Bode plot for the combined lag network and plant $D(s)G(s)$ on the plot (Fig. 3.1) above.

[2 pts] d. Mark the phase and phase margin frequency on the plot of $D(s)G(s)$ (Fig. 3.1).

Problem 3 (18 pts)

[2 pt] a. Given the homogeneous linear differential equation $\dot{\mathbf{x}} = A\mathbf{x}$ with initial condition $\mathbf{x}(0) = \mathbf{x}_o$. Show that the solution $\mathbf{x}(t) = e^{At}\mathbf{x}_o$ satisfies both conditions.

$$\frac{d}{dt}(x(t)) = Ae^{At}x_0 = Ax(t) \checkmark \qquad \lim_{t\to 0} x(t) = \lim_{t\to 0} e^{At}x_0 = x_0 \checkmark$$

dynamics initial condition

[2 pt] b. Show that e^{At} must equal $\mathcal{L}^{-1}[sI - A]^{-1}$. (Hint: see part a. above.)

$$\dot{x} = Ax$$

$$\mathcal{L}\{\dot{x} = Ax\} = \quad sIX(s) - x_0 = AX(s)$$

$$(sI-A)X(s) = x_0$$

$$x(t) = \mathcal{L}^{-1}\{(sI-A)^{-1}\}x_0$$

[2 pts] c. Given $\bar{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, find $e^{\bar{A}t}$

$$e^{\bar{A}t} = \left[\begin{array}{c|c} e^{\lambda_1 t} & 0 \\ \hline 0 & e^{\lambda_2 t} \end{array}\right]$$

[4 pts] d. Given $\bar{A}, A, P$ such that $\bar{A} = P^{-1}AP$ is diagonal, and given $e^{\bar{A}t}$. Also given the state vector $x = P\bar{x}$. Show how to find e^{At} given $\bar{A}, A, P, e^{\bar{A}t}$, starting from $\dot{\bar{x}} = \bar{A}\bar{x}$. (Leave in general form.)

$e^{At} =$ $Pe^{\bar{A}t}P^{-1}$

$$\dot{\bar{x}} = \bar{A}\bar{x}$$

$$\bar{x} = e^{\bar{A}t}\bar{x}_0$$

$$P^{-1}x = e^{\bar{A}t}P^{-1}x_0$$

$$x = Pe^{\bar{A}t}P^{-1}x_0$$

from above

$$x = \exp(At)x_0$$

$$\Rightarrow e^{At} = Pe^{\bar{A}t}P^{-1}$$

Problem 3, cont.

Given the two LTI systems

$$\dot{\mathbf{x}}(t) = A\mathbf{x} + Bu = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u, \quad y = C\mathbf{x} = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{\mathbf{z}}(t) = A_z\mathbf{z} + B_z u = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix}u\,, \quad y = C_z\mathbf{z} = [0 \quad 1]\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

[4 pts] e. Find a transformation P such that $A = P^{-1}A_zP$ is diagonal. (Hint: this could be found using the controllability matrix for each system.)

$$\mathcal{C}_x = \begin{bmatrix} 1 & -1 \\ 2 & -6 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \qquad \mathcal{C}_z = \begin{bmatrix} -1 & 5 \\ 1 & -1 \end{bmatrix}$$

$$P = \mathcal{C}_z \mathcal{C}_x^{-1}$$

$$\hookrightarrow \mathcal{C}_x^{-1} = \frac{-1}{4}\begin{bmatrix} -6 & 1 \\ -2 & 1 \end{bmatrix}$$

$$P = \frac{-1}{4}\begin{bmatrix} -1 & 5 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -6 & 1 \\ -2 & 1 \end{bmatrix}$$

$$= -1/4 \begin{bmatrix} -4 & 4 \\ -4 & 0 \end{bmatrix}$$

[4 pts] f. Show that both systems have the same input-output behavior. That is, for the same input $u(t)$, the output $y(t)$ will be identical for both systems. Use P from part e, and also verify B_z and C_z are correct.

note $z = Px$

$$P^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$$y = C_z z$$

$$y = C_z P x$$

$$= [0 \quad 1]\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} x$$

$$y = [1 \quad 0]x \checkmark$$

$$\dot{z} = A_z z + B_z u$$

$$P\dot{x} = A_z P x + B_z u$$

$$\dot{x} = P^{-1}A_z P x + P^{-1}B_z u$$

$$= Ax + \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}u$$

$$\dot{x} = Ax + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u \checkmark$$

Problem 4. (20 pts)

Given the LTI system

$$\dot{\mathbf{x}}(t) = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad y = C\mathbf{x} = [1 \quad 0]\mathbf{x},$$

[3 pts] a. Find $\mathbf{k} = [k_1 \; k_2]$ such that with state feedback $u = r - \mathbf{kx}$, the closed-loop poles of the system are at λ_1, λ_2.

$\Delta^{des}(s) = (s-\lambda_1)(s-\lambda_2)$
$= s^2 - (\lambda_1+\lambda_2)s + \lambda_1\lambda_2$

$k_1 =$ $\lambda_1\lambda_2$ $\quad k_2 =$ $-(\lambda_1+\lambda_2)$

$\Delta^{cl}(s) = \begin{vmatrix} s & -1 \\ +k_1 & s+k_2 \end{vmatrix}$
$= s^2 + k_2 s - k_1$

[1 pts] b. The initial condition is $\mathbf{x}(0) = [0 \;\; 0]^T$. For $r(t)$ a unit step input, it is required that $x_1(t) < 1 \; \forall t$, that is over shoot is not allowed.

What is range of λ_1, λ_2 to avoid over shoot?

$\lambda_i < 0, \quad \lambda_i \in \mathbb{R}$

[3 pts] c. Assume $\mathbf{k} = [k_1 \; k_2] = [4 \;\; 5]$. Let $e(t) = r(t) - C\mathbf{x}$. For $r(t)$ a unit step input, find the steady state error.

$\lim_{t\to\infty} e(t) =$ $3/4$

$e_{ss} = 1 + C A_{cl}^{-1} B$

$A_{cl} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \qquad A_{cl}^{-1} = \frac{1}{4}\begin{bmatrix} -5 & -1 \\ 4 & 0 \end{bmatrix}$

$e_{ss} = 1 + [1 \;\; 0]\frac{1}{4}\begin{bmatrix} -5 & -1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

[3 pts] d. For $\mathbf{k} = [k_1 \; k_2] = [4 \;\; 5]$, with $u = r - \mathbf{kx}$, find $\frac{Y(s)}{R(s)}$. (Express the transfer function as a ratio of polynomials, not as matrix operations.)

$\frac{Y(s)}{R(s)} = \frac{1}{s^2+5s+4}$

$\frac{Y(s)}{R(s)} = C(sI - A_{cl})^{-1}B$

$(sI - A_{cl})^{-1} = \frac{1}{\Delta^{cl}(s)}\begin{bmatrix} s+5 & 1 \\ -4 & s \end{bmatrix}$

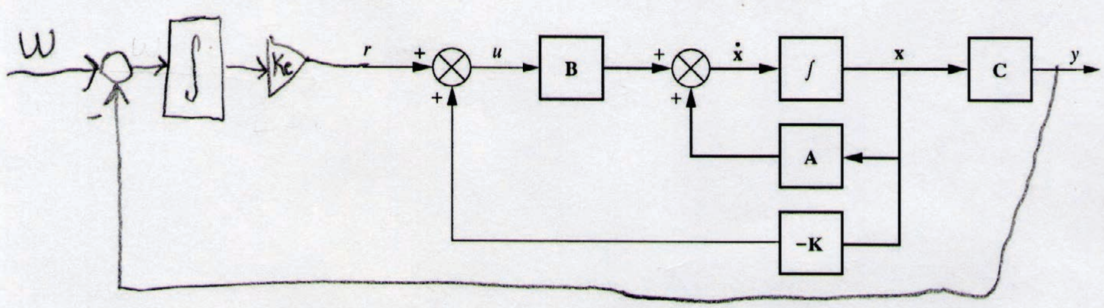
$C(sI - A_{cl})^{-1}B = \frac{1}{s^2+5s+4}$

Problem 4, cont. (20 pts)

[4 pts] e. Define $e_w(t)$ to be the error between an input $w(t)$ and output $y(t)$. That is, $e_w(t) = w(t) - y(t)$. We desire to find an input $r(w, y)$ to the state feedback system shown below in part f such that $\lim_{t\to\infty} e_w(t) = 0$ for a step input $w(t)$ of any amplitude.

$$r(w,y) = K_e \int_0^t (w(\tau) - y(\tau))\,d\tau$$

[2 pts] f. Using the controller from part e, expand the block diagram below to include the controller and input w.



[4 pts] g. Assume the overall control system is stable, and refer to the expanded block diagram above. Describe in words why $\lim_{t\to\infty} e_w(t) = 0$ for a step input $w(t)$.

Suppose a constant error $e_w(t) \neq 0$, Over time r will grow as this error accumulates, increasing the resulting reference passed to state fb system, driving the output higher or lower. $Y(t)$ as $t \to \infty$ must be bounded and constant (stability shows this), thus contradiction

$\Rightarrow e_w(t) = 0$

Problem 5. 13 pts
Given the following system model:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \;=\; \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -k_2 & 1 \\ 0 & -k_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \qquad y = C\mathbf{x} = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

[2 pts] a. Determine if the system A, B, C is controllable, and restrictions if any on k_1, k_2 for controllability.

$$\mathcal{C} = [B \mid AB] = \begin{bmatrix} 0 & 1 \\ 1 & -k_1 \end{bmatrix}$$

$\therefore$ Completely Controllable $\forall\, k_1, k_2 \in \mathbb{R}$

[2 pts] b. Determine if the system A, B, C is observable, and restrictions if any on k_1, k_2 for observability.

$$\mathcal{O} = \left[\frac{C}{CA}\right] = \begin{bmatrix} 1 & 0 \\ -k_2 & 1 \end{bmatrix}$$

$\therefore$ Completely Observable $\forall\, k_1, k_2 \in \mathbb{R}$

[2 pts] c. Provide state equations for an observer which takes as inputs $u(t), y(t)$, and provides an estimate of the state $\hat{\mathbf{x}}(t)$.

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$

[6 pts] d. Given $k_1 = 1, k_2 = 4$, find observer gain L such that the observer has closed loop poles at $s_1 = -10, s_2 = -10$.

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 81 \end{bmatrix} =$$

$$\bar{A} = A - LC = \begin{bmatrix} -4 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} [1 \quad 0] = \begin{bmatrix} -l_1 - 4 & 1 \\ -l_2 & -1 \end{bmatrix}$$

$$|sI - \bar{A}| = (s + (4 + l_1))(s + 1) + l_2 = s^2 + (l_1 + 5)s + (l_1 + l_2 + 4)$$

$$(s+10)(s+10) = s^2 + 20s + 100$$

$\therefore$ $l_1 = 15 \quad (l_1 + 5 = 20)$

$l_2 = 81 \quad (l_1 + l_2 + 4 = 100)$

Problem 5, cont.

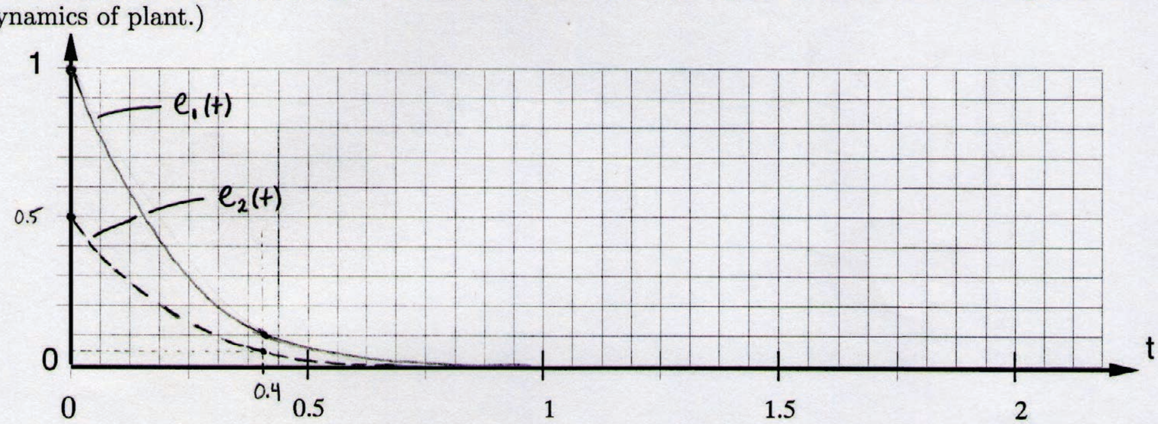
[4 pts] e. Let the error between the estimated state and the true state be given by $\mathbf{e}(t) = \hat{\mathbf{x}} - \mathbf{x}$. Find the dynamics of the error in terms of A, B, C, L.

$\dot{\mathbf{e}} =$ $\underline{(A - LC)\,\underline{e}}$

[4 pts] f. Given initial conditions

$$\mathbf{x} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \text{ and } \hat{\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Sketch approximately $e_1(t), e_2(t)$ for $t \geq 0$. (Hint: consider dynamics of observer compared to dynamics of plant.)



Given:

$\dot{\underline{e}} = (A - LC)\,\underline{e}$ with closed loop poles @ $s = -10, -10$

↳ Critically damped

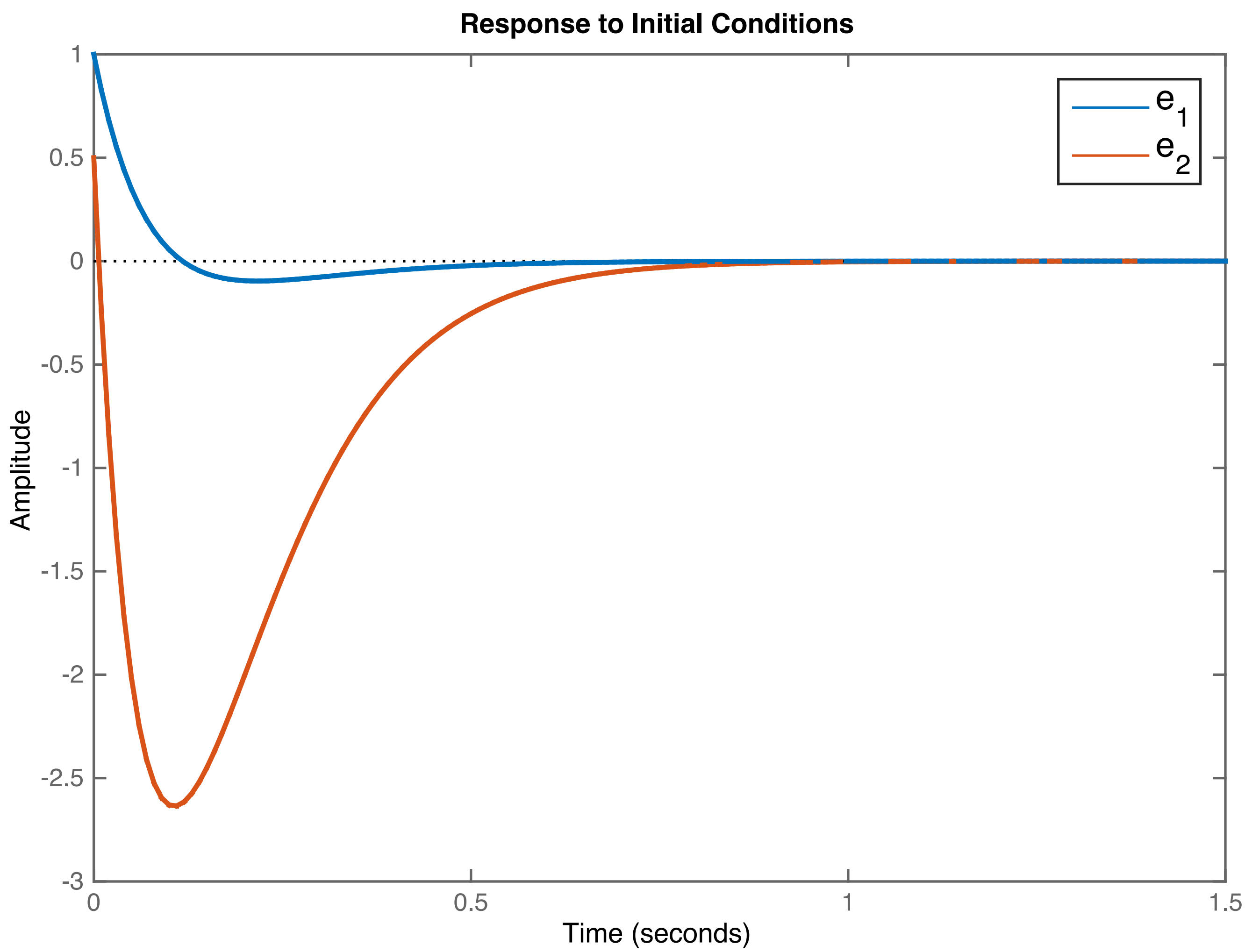
↳ $e(0) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

$T_s \cong 4/\sigma_D = 4/10 = 0.4$ sec to 90% settle

(Dynamics of $\underline{e}(t)$ are independent from plant dynamics by separability)

⊛ ⇓ Accepted for Full Credit...

True response has zeros and the initial response is on the next page.


Response to Initial Conditions
Amplitude
Time (seconds)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
0.5
1
1.5
e_1
e_2

Problem 6 (8 pts)

[4 pts] a. Given the discrete time system below, find $\mathbf{X}(z)$ the z-transform of $\mathbf{x}(k)$, where $u(k) = (\frac{1}{2})^k$ for $k \geq 0$.

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$\mathbf{X}(z) = \begin{bmatrix} 1/(z)(z-1/2) \\ 1/(z-1/2) \end{bmatrix}$

$X(z) = (zI - G)^{-1}(H\,U(z) + x[0]\,z)$, assuming $x[0] = 0$

$= (zI-G)^{-1}(H)(\frac{z}{z-1/2})$

$= \begin{bmatrix} z & -1 \\ 0 & z \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ z/(z-1/2) \end{bmatrix}$

$= \begin{bmatrix} 1/z & 1/z^2 \\ 0 & 1/z \end{bmatrix} \begin{bmatrix} 0 \\ z/(z-1/2) \end{bmatrix} \Rightarrow \begin{bmatrix} 1/(z)(z-1/2) \\ 1/(z-1/2) \end{bmatrix}$

[4 pts] b. Given

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

Determine the response of the system to $u(k)$ a unit step input.

$\mathbf{x}(k) = \begin{bmatrix} u(k-2) \\ u(k-1) \end{bmatrix}$

Using above ...

$X(z) = \begin{bmatrix} 1/z & 1/z^2 \\ 0 & 1/z \end{bmatrix} \begin{bmatrix} 0 \\ z/(z-1) \end{bmatrix} \Rightarrow \begin{bmatrix} (z^{-2})(z/(z-1)) \\ (z^{-1})(z/(z-1)) \end{bmatrix}$

Inverse Z-Transform yields a 1k and 2k delay on the unit step...

$x(k) = \begin{bmatrix} u(k-2) \\ u(k-1) \end{bmatrix}$

[4 pts] c. Given

$$X(z) = \frac{z^2}{(z-\frac{1}{2})(z-1)}$$

find $x(k)$ for $k \geq 0$.

$x(k) = \underline{-(1/2)^k + 2u(k)}$

$\frac{X(z)}{z} = \frac{z}{(z-1/2)(z-1)} = \frac{A}{(z-1/2)} + \frac{B}{(z-1)}$

$z = A(z-1) + B(z-1/2)$

$\hookrightarrow z=1: \quad 1 = B(1/2) \Rightarrow B = 2$

$\hookrightarrow z = 1/2: \quad 1/2 = A(-1/2) \Rightarrow A = -1$

$X(z) = \frac{-z}{(z-1/2)} + \frac{2z}{(z-1)}$

$\Downarrow$

$x(k) = -(1/2)^k + 2u(k)$

Problem 6, cont.)

[4 pts] d. Given

$$X(z) = \frac{z}{(z-\frac{1}{2})(z-1)(z-\frac{2}{5})}$$

find $\lim_{k\to\infty} x(k) =$ $\underline{\quad 10/3 \quad}$

Final Value Thm: $\lim_{k\to\infty} x(k) = \lim_{z\to 1} \left(\frac{(z-1)}{z}\right) X(z)$

$$= \lim_{z\to 1} \frac{\cancel{(z-1)}\cancel{(z)}}{\cancel{(z)}(z-1/2)\cancel{(z-1)}(z-2/5)}$$

$$= 1/((1/2)(3/5)) = 10/3$$

[4 pts] e. Given a mass m, and input force f, $\ddot{x} = f/m$. Let the state x_1 be the position and x_2 velocity of the mass. The continuous time state equations for the system are :

$$\dot{\mathbf{x}} = A\mathbf{x} + Bf = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t),$$

Find the discrete time equivalent system using zero-order hold for input force $f(t)$ and sampling period T: $\mathbf{x}((k+1)T) = G\mathbf{x}(kT) + Hf(kT)$.

$$G = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} T^2/2m \\ T/m \end{bmatrix}$$

$\underline{G = e^{AT}}$

$$e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$$

$$= \mathcal{L}^{-1}\left\{\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix}\right\}$$

$$e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

$\Downarrow$

$$e^{AT} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$H = \left(\int_0^T e^{A\lambda} d\lambda\right) B$$

$$= \left(\int_0^T \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1/m \end{bmatrix} d\lambda\right)$$

$$= \int_0^T \begin{bmatrix} \lambda/m \\ 1/m \end{bmatrix} d\lambda$$

$$= \begin{bmatrix} \lambda^2/2m \\ \lambda/m \end{bmatrix}\Bigg|_0^T$$

$$H = \begin{bmatrix} (\frac{1}{2m}) T^2 \\ (\frac{1}{m}) T \end{bmatrix}$$