EECS C128/ ME C134
Final
Thu. May 14, 2015
1510-1800 pm

Name:___________________________
SID:_____________

- Closed book. One page, 2 sides of formula sheets. No calculators.

- There are 8 problems worth 100 points total.

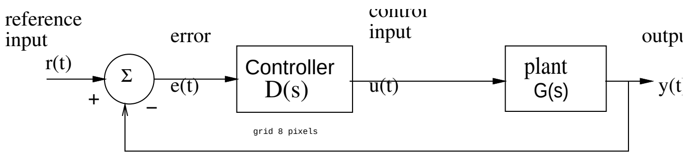
Problem	Points	Score
1	14	
2	14	
3	16	
4	8	
5	13	
6	13	
7	14	
8	8	
Total	100	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of 'F' and a letter will be written for your file and to the Office of Student Conduct.

$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1} 1 = 45°$
$\tan^{-1}\frac{1}{3} = 18.4°$	$\tan^{-1}\frac{1}{4} = 14°$
$\tan^{-1}\sqrt{3} = 60°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\sin 30° = \frac{1}{2}$	$\cos 60° = \frac{\sqrt{3}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB
$1/e \approx 0.37$	$1/e^2 \approx 0.14$
$1/e^3 \approx 0.05$	$\sqrt{10} \approx 3.16$

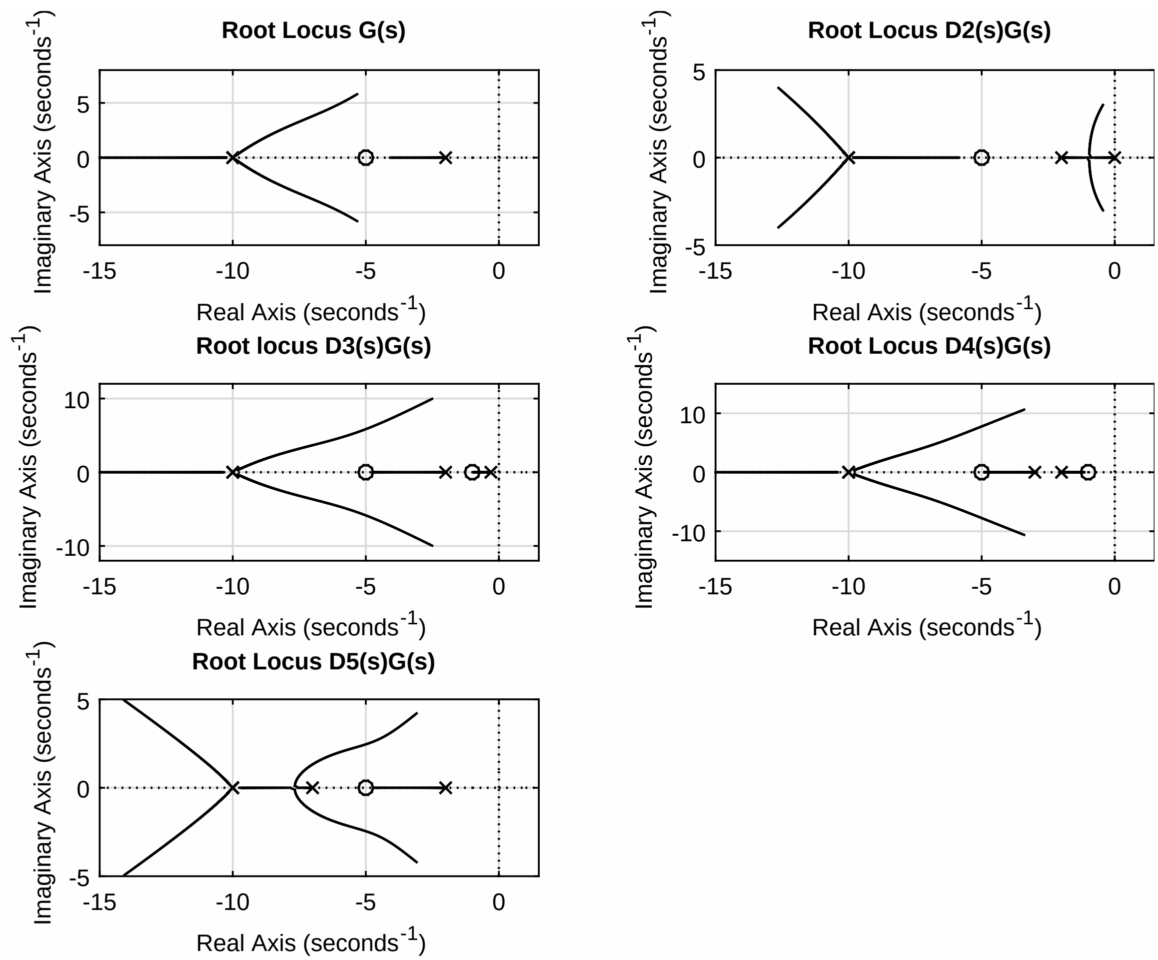
Problem 1 (14 pts)



You are given the open-loop plant:

$$G(s) = \frac{s+5}{(s+2)(s+10)^3}.$$

For the above system, the partial root locus is shown for 5 different controller/plant combinations, $G(s), D_2(s)G(s), ..., D_5(s)G(s)$. (Note: the root locus shows open-loop pole locations for $D(s)G(s)$, and closed-loop poles for $\frac{DG}{1+DG}$ are at end points of branches).



[5 pts] a) For each set of open-loop poles and zeros given above, choose the best corresponding open-loop Bode plot V,W,X,Y, or Z from the next page:

(i) $G(s)$: Bode Plot ____

(ii) $D_2(s)G(s)$: Bode plot ____

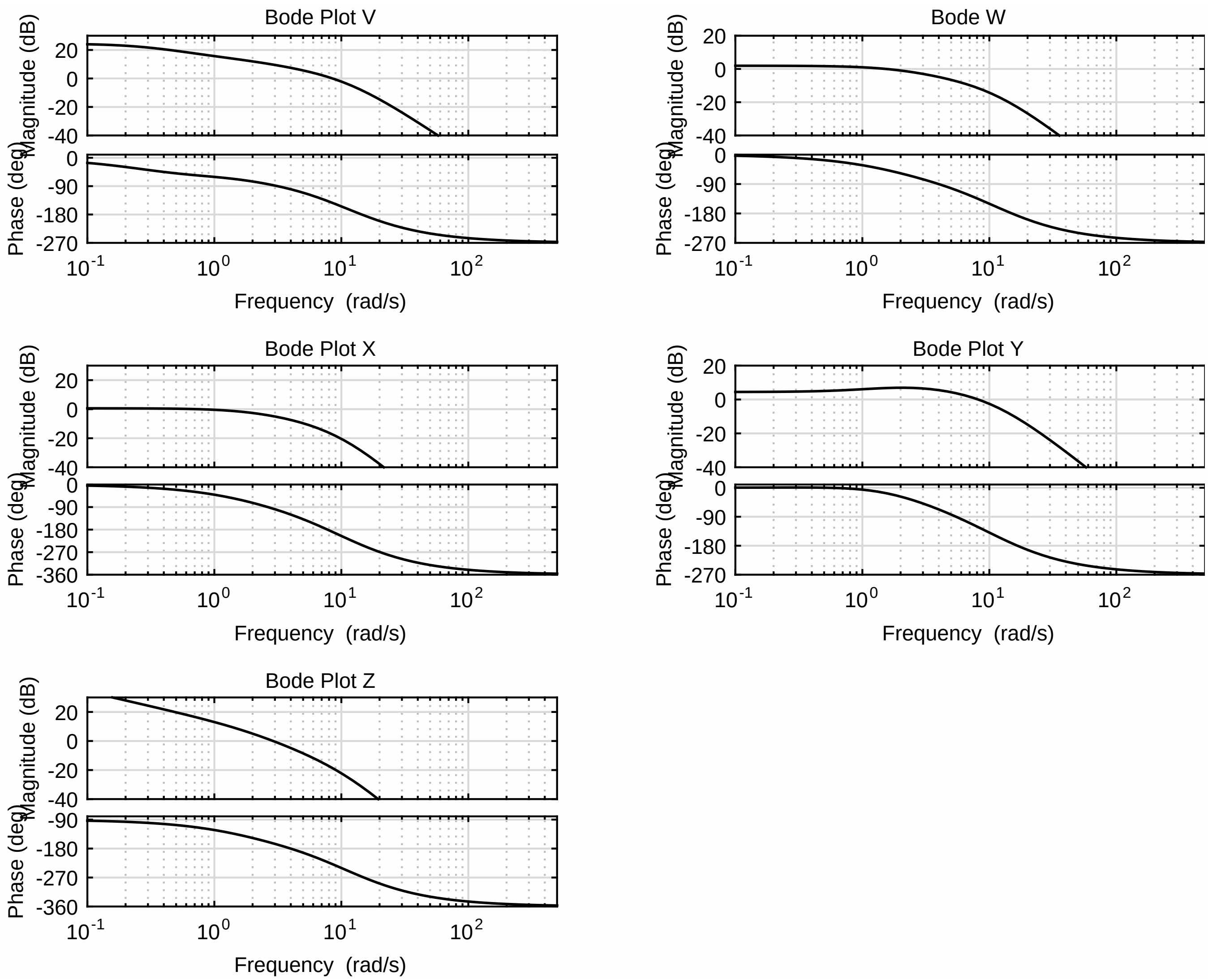
(iii) $D_3(s)G(s)$: Bode plot ____

(iv) $D_4(s)G(s)$: Bode Plot ____

(v) $D_5(s)G(s)$: Bode Plot ____

Problem 1, cont.

The open-loop Bode plots for 5 different controller/plant combinations, $D_1(s)G(s), ..., D_5(s)G(s)$ are shown below.



[4 pts] b) For the listed Bode plots, estimate the phase and gain margin:

(i) Bode plot V: phase margin ____ (degrees) at $\omega =$ ____
Bode plot V: gain margin ___ dB at $\omega =$ ____

(ii) Bode plot Z: phase margin ____ (degrees) at $\omega =$ ____
Bode plot Z: gain margin ___ dB at $\omega =$ ____

Problem 1, cont.

[5 pts] c) For each closed loop controller/plant with root locus as given in part a), choose the best corresponding closed-loop step response (A-E)

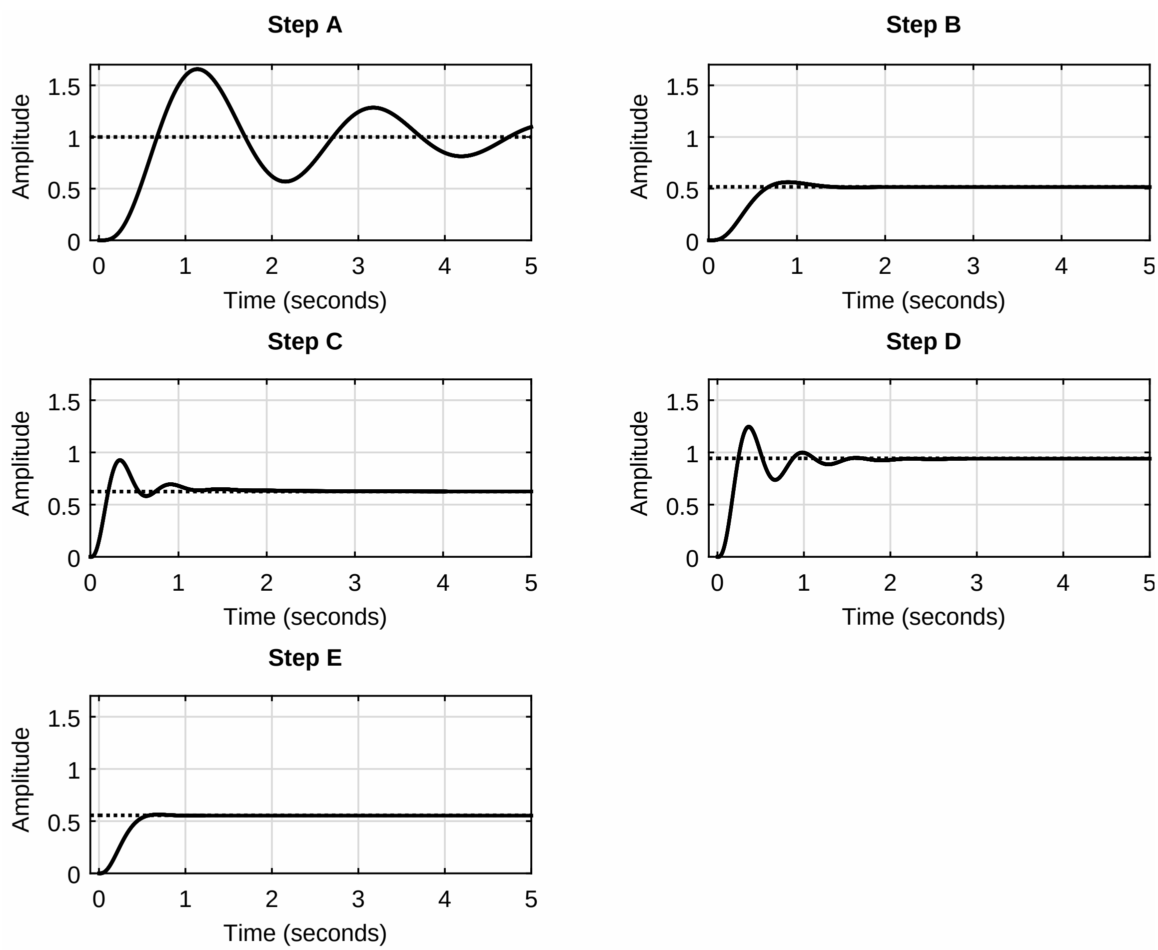
(i) $G(s)$: step response ____

(ii) $D_2(s)G(s)$: step response ____

(iii) $D_3(s)G(s)$: step response ____

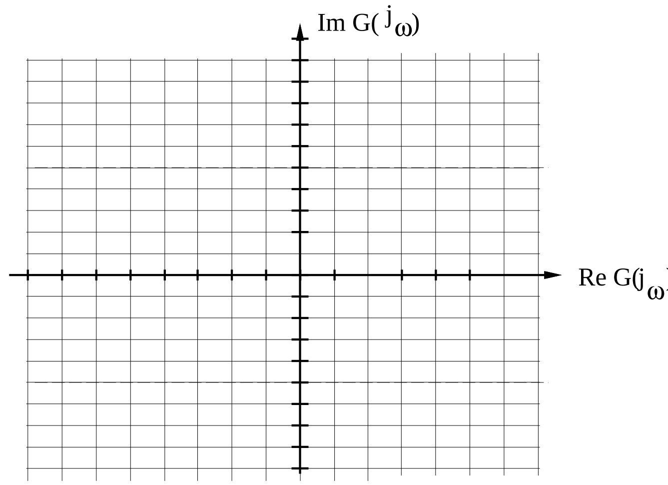
(iv) $D_4(s)G(s)$: step response ____

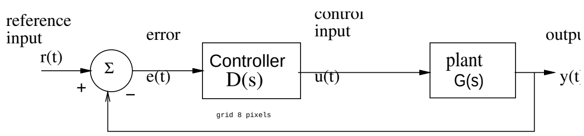
(v) $D_5(s)G(s)$: step response ____



Problem 2 (14 pts)

[4 pts] a. You are given the open loop plant: $G(s) = k\frac{s+6}{s-6}$, with $D(s) = 1$. Sketch Nyquist plot for $G(s)$ with $k = 1$, showing clearly any encirclements.

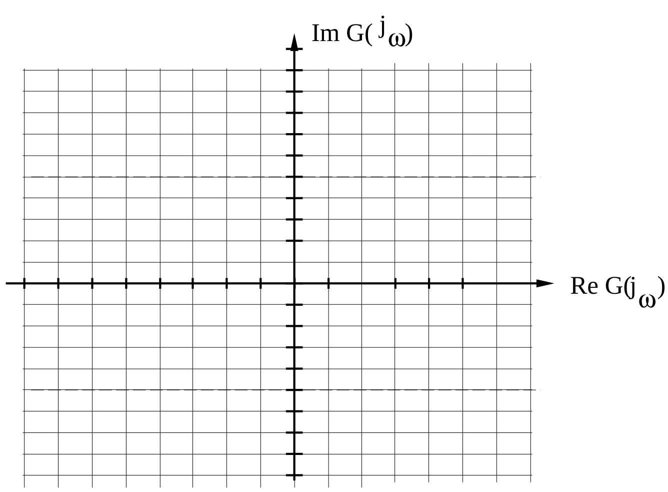




[2 pts] b. Find the bounds on k for the system with unity feedback to be stable.

[4 pts] c. You are given the open loop plant $G(s) = k\frac{(s-6)(s-4)}{(s+6)(s-1)}$.

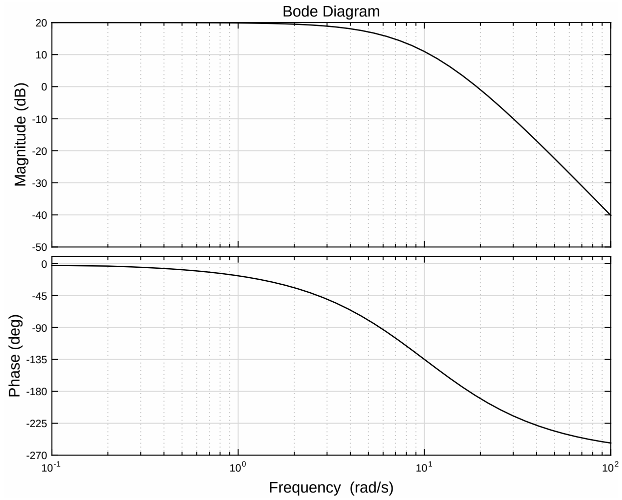
Sketch Nyquist plot for $G(s)$ with $k = 1$, showing clearly any encirclements. Hint: phase of $G(j\omega = 2)$ is $+180°$.



[4 pts] d. Find the bounds on k for the system with unity feedback to be stable.

Problem 3 (16 pts)

The open-loop system is given by $G(s) = \frac{10^4}{(s+10)^3}$, and Bode plot for $G(s)$ is here (Fig. 3.1):

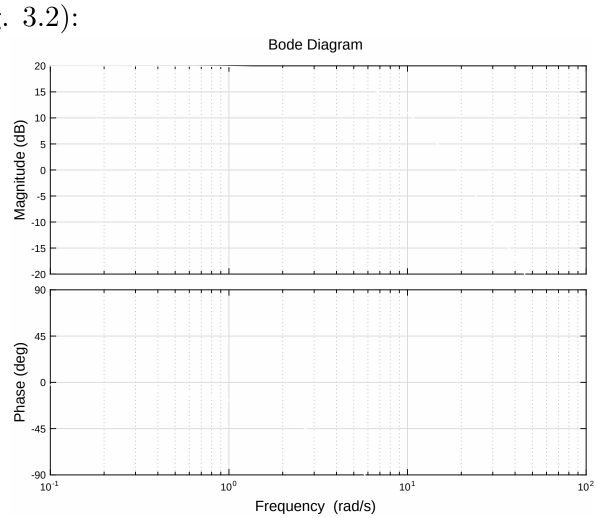


A lag controller $D(s) = k\frac{s+\alpha}{s+\beta}$ is to be designed such that the unity gain feedback system with openloop transfer function $D(s)G(s)$ has the same steady state error as with OLTF $G(s)$ and has a nominal (asymptotic approximation) phase margin $\phi_m = 55°$ at $\omega = 8$ rad s^{-1}. Note $20 \log |G(j\omega = j8)| = 13$ dB.

[6 pts] a. Determine gain, zero, and pole location for the lag network $D(s)$:

gain $k =$ ____ zero $\alpha =$ -_____ pole $\beta =$ -_____

[4 pts] b. Sketch the asymptotic Bode plot for the lag network $D(s)$ alone on the plot below (Fig. 3.2):



[4 pts] c. Sketch the asymptotic Bode plot for the combined lag network and plant $D(s)G(s)$ on the plot (3.1) above.

[2 pts] d. Mark the phase and phase margin frequency on the plot of $D(s)G(s)$ (Fig. 3.1). Explain briefly (1 sentence) how does the actual phase margin compare to the asymptotic predction?

Problem 4 (8 pts)

You are given the following plant

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu, \qquad y = C\mathbf{x}$$

where A is $N \times N$, u is scalar, B is $N \times 1$, C is $1 \times N$, and $\mathbf{x}$ is $N \times 1$. The system is observable and controllable.

[2 pts] a. Consider a controller $u = r - K\mathbf{x}$ where r is a reference input, and K is $1 \times N$.

Determine the transfer function $\frac{Y(s)}{R(s)} =$ ______

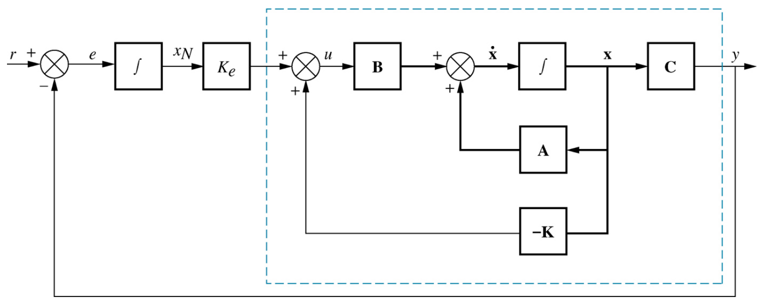
[2 pts] b. Consider a controller $u = K(r - \mathbf{x})$ where r is a reference input, and K is $1 \times N$.

Determine the transfer function $\frac{Y(s)}{R(s)} =$ ______

[2 pts] c. Draw a block diagram of the controlled system using integrators, summing junctions, and scaling functions.

[3 pts] d. If the same K is used in part a. and b. above, briefly explain any difference in transfer function or behavior:

Problem 5. (13 pts)
Consider the following control system:



[3 pts] a. Write the state and output equations for the system, in terms of A, B, C, K, K_e.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \quad & & & \quad \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix} r(t), \qquad y = [\qquad\qquad] \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} \quad (1)$$

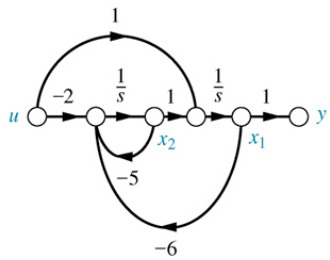
[6 pts] b. Given $C = [1 \quad 0]$, $\quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\quad A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$

find K and K_e such that the closed loop poles are at $s = -1, -2, -4$.

$K = [\qquad\qquad\qquad]$ $\qquad\qquad$ $K_e =$______

[4 pts] c. Show, with $r(t)$ a unit step input, that $e = 0$ in steady state (with the K, K_e found above). (Hint: do not use matrix inverse.)

Problem 6. 13 pts



Given the following system model:

[3 pts] a. Write the state and output equations for the system above.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \; = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \qquad & \qquad \\ & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \qquad \\ \\ \end{bmatrix} u(t), \qquad y = C\mathbf{x} = [\qquad] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

[2 pts] b. Determine if the system A, B, C is controllable and observable.

[2 pts] c. Provide state equations for an observer which takes as inputs $u(t), y(t)$, and provides an estimate of the state $\hat{\mathbf{x}}(t)$.

[6 pts] d. Find observer gain L such that the observer has closed loop poles at $s_1 = -10, s_2 = -10$.

Problem 7 (14 pts)

[3 pts] a. Given $G(s) = \frac{1}{s+2}$. Let $m(t)$ be the step response of $g(t)$, i.e. $M(s) = \frac{1}{s(s+2)}$. Let $x_1(t) = m(t) - m(t-T)$ where T is the sampling period. Find $X_1(z)$ the Z transform of $x_1(t)$.

$X_1(z) =$ ____

[3 pts] b. Given $\dot{x}_2(t) = -2x_2(t) + u(t)$. Find the discrete time equivalent system using zero-order hold for input $u(t)$ and sampling period T: $x_2((k+1)T) = Gx_2(kT) + Hu(kT)$.

$G =$ ____ $\qquad H =$ ____

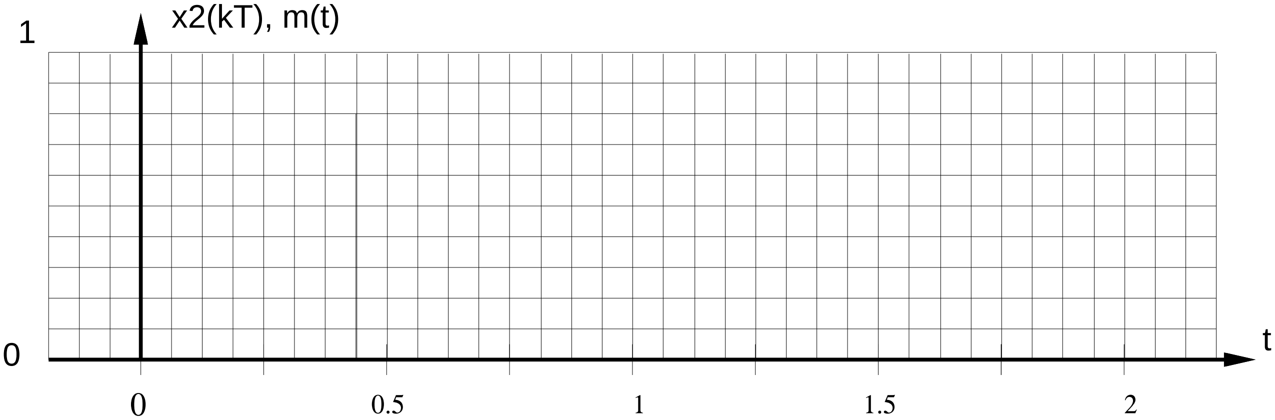
[2 pts] c. Find the $\frac{X_2(z)}{U(z)}$ the discrete time transfer function from input u to state x_2 using the state-space form.

$\frac{X_2(z)}{U(z)}$ ____

Problem 7, cont.

[2 pts] d. Does $\frac{X_2(z)}{U(z)} = X_1(z)$? Why or why not?

[4 pts] e. With zero initial conditions (ZSR), $T = 0.25$, and a unit step input for $x_2(kT)$, sketch $m(t)$ and $x_2(kT)$ on the plot below in the interval shown:



Problem 8 Short Answers (8 pts)

[4 pts] a. Given the discrete time system below, find $\lim_{k\to\infty} x(k)$ for a unit step input $u(k) = 1$.

$$x(k+1) = \begin{bmatrix} 1/6 & 1/2 \\ 2/3 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$\lim_{k\to\infty} x(k) = \begin{bmatrix} \quad \end{bmatrix}$.

[4 pts] b. Given $\dot{x}(t) = -10x(t) + u(t)$. The discrete time equivalent system using zero-order hold for input $u(t)$ and sampling period T is of the form $x((k+1)T) = Gx(kT) + Hu(kT)$. The discrete time system has a state feedback controller $u(kT) = r(kT) - 20x(kT)$ applied.

i. Find the eigenvalue for the closed loop system: ____

ii. Find the largest value of T for which the system will be stable. (May be left in terms of ln.) : $T <$ ____

Blank page for scratch work.