Oral quiz on HW7,8 week of 11/10-11/12 **Due at 1800, Fri. 11/20 in gradescope** .
Note: up to 2 students may turn in a single writeup. Reading Nise 5, 12.

1. (20 pts) Control Form transformation (Nise 12.4)

Given the following: $\dot{\mathbf{z}} = A\mathbf{z} + Bu = \begin{bmatrix} -11 & 1 \\ 5 & -7 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{z}$

[10pts] a) Find the transformation P such that $(\bar{A}, \bar{B})$ is in phase variable form, where $\bar{A} = P^{-1}AP$ and $\bar{B} = P^{-1}B$.
[5pts] b) Find $\bar{A}, \bar{B}, \bar{C}$ such that $\dot{\bar{\mathbf{x}}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{x}$.
[10pts] c) Find state feedback in phase variable form $u = r - k_x\mathbf{x}$ such that closed loop poles are at $-5, -15$. Then find corresponding state feedback for the original system $k_z = [k_{z1} \quad k_{z2}]$.

2. (20 pts) Observer (Nise 12.5)

Given the plant $G(s) = \frac{Y(s)}{U(s)}$:

$$G(s) = \frac{144(s+1)}{(s+2)(s+3)(s+8)}$$

where state variables are not available.
[4pts] a. Express $G(s)$ in observer canonical form, $\dot{\mathbf{x}} = A\mathbf{x} + Bu$, $y = C\mathbf{x}$.
[12pts] b. Design an observer: $\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L(y - \hat{y})$ for the observer canonical variables to yield poles at -10, -10, -50.
[4 pts] c. Let the error dynamics be given by $\mathbf{e_x} = \mathbf{x} - \hat{\mathbf{x}}$. Let $\mathbf{x}(0) = [1\ 2\ 1]^T$, and $\hat{\mathbf{x}}(0) = [0\ 0\ 0]^T$. Using Matlab, plot the error between the true state and the estimate (that is, plot $\mathbf{e_x}(t)$). Matlab command `[Y,T,E] = lsim(sys,r,t,e0)` can be used to take care of initial condition.

3. (30 pts) Steady State Error/Integral Control (Nise 12.8)
Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A_1\mathbf{x} + B_1u = \begin{bmatrix} 0 & 1 \\ -15 & -8 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [1\ \ 2]\ \mathbf{x} \tag{1}$$

[8pts] a) Given error $e(t) = r(t) - y(t)$ where $r(t)$ is a scalar, evaluate the steady state error $\lim_{t\to\infty} e(t)$ for input $r(t)$ a unit step, with state feedback, that is, $u = -K_1\mathbf{x} + r$, where $K_1 = [k_1\ k_2]$ is chosen such that the closed loop poles are at $s_i = -4 \pm 2j$.
[15pts] b) Add an integrator to the plant, using a new state vector $\mathbf{x} = [x_1\ x_2\ x_N]^T$, write the new state and output equations, and find gains such that the closed-loop poles are at $s_i = -4 \pm 2j, -10$. Evaluate the steady-state error for a step input $r(t)$.
[7pts] c) Plot the step response for both systems in Matlab, and compare. For the augmented system, plot $x_1(t), x_2(t), x_N(t), u(t)$ and explain why $e(t) \to 0$.

4. (30 pts) State, and Observer Feedback (Separation principle handout)
Given the following system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [3\ 1]\ \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

[6pts] a) Design a state feedback controller $u = r - [k_1 \quad k_2]\mathbf{x}$ such that the closed loop system has poles $s = -3 \pm 5j$. Plot the step response $y(t)$ using Matlab.
[8pts] b) Design a critically damped observer $\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L(y - \hat{y})$ with both observer poles at $s = -10$.
[8pts] c) Write the state space equations for the controller with $u = r - K\hat{\mathbf{x}}$, such that the closed loop poles are the same as in part a). (This should have 4 state variables, either $\mathbf{x}, \hat{\mathbf{x}}$ or $\mathbf{x}, \mathbf{e}$.) From the separation principle, what are the eigenvalues of the combined system with observer and feedback control? Plot the step response $y_{obs}(t)$ (with Matlab) of the system with state feedback control using the observer estimated states.
[4pts] d) Use Matlab to plot the states $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ for $t > 0$ for the closed loop system of part c) for a step input $r(t)$. (Suggestion, use `sys = ss(Ac,Bc,Cc,Dc)` and `[Y,T,X] = lsim(sys,r,t,x0)`, where `Ac, Bc, Cc, Dc` are the matrices for the system with observer).
[4pts] e) Compare the output responses $y(t)$ to the step input responses from part a) and c). What differences are there? (quantify).