Professor Fearing EECS128/MEC134 Problem Set 11 v1.02 Fall 2020

Due at 1700, Fri. Dec. 4 in gradescope. Note: up to 2 students may turn in a single writeup.

1. (30 pts) Continuous vs Discrete Time Control (Handout and Matlab)

For each part, hand in relevant Matlab code as well as plots. Use `hold on` *to superimpose plots.*

Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -17 & -8 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t), \quad y = [15\ 5\ 0]\ \mathbf{x}$$

[4pts] a) Find the corresponding discrete time (DT) system $x[n+1] = Gx[n] + Hu[n]$, $y[n] = Cx[n]$

which can be found using the Matlab function `c2d(ctsys,T,'zoh')`, with sampling period $T_s = 0.1$ sec. (Note `ctsys` can be found from `ss(A,B,C,D)`.) Compare eigenvalues for CT A and DT G; are both systems stable?

[8pts] b) The continuous time system uses output feedback such that $u(t) = r(t) - kC\mathbf{x}(t)$. Find k such that damping factor associated with dominant poles $\zeta \approx 0.5$ (using `rlocus()` is sufficient). Plot the closed-loop step response using Matlab.

[6pts] c) Consider the CT system having output feedback applied in discrete time using a D/A converter such that $u[n] = u(nT) = r[n] - k_d C\mathbf{x}[n]$. (Use `rlocus()` on the discrete time system to find k_d to give $\zeta \approx 0.5$). Thus the closed loop DT system has state equation $x[n+1] = (G - Hk_dC)x[n] + Hr[n]$, $y[n] = Cx[n]$. Plot the step response for the closed loop step response on the same axes as the CT step response of part b).

[6pts] d) Consider the step response for the DT converted system (using `c2d()`) for $\dot{\mathbf{x}} = (A - BkC)\mathbf{x} + Br$, and plot. Explain why the step response using output feedback for the DT system from part c) does not look like this DT version of the step response.

[6pts] e) Use Matlab (iteratively if necessary) to find a sampling period T_s which gives a closed-loop step response for DT that is "reasonably close" (within 5%) to the CT closed-loop step response (using $k_d = k$). Determine DT closed-loop pole locations, and plot the DT step response on same axes as part c. How does the T_s found compare to the fastest eigenvalue of the openloop CT system?

2. (12 pts) Laplace to Z conversion (Nise 13.3)

Given $H(s) = \frac{1}{(s+\alpha)(s+\beta)}$ and sample rate T, find $H(z)$ using the definition of Z transform, i.e.

$H(z) = \Sigma_k h(kT) z^{-k}$.

3. (12 pts) Z transform (Nise 13.3)

For $F(z)$, find $f^*(t) = f^*(kT)$, the ideal time sampled signal, using partial fraction expansion.

$F(z) = \frac{(z+1)}{(z-0.1)(z-0.4)}$

4. (20 pts) SS to TF (Nise 3.6, 13.3, 13.4, 13.7, DT handout)
Given the following discrete time (DT) system, with sample period $T = 1$:

$$\mathbf{x}[k+1] = G\mathbf{x}[k] + Hu[k] = \begin{bmatrix} 0 & 1 \\ -\frac{3}{8} & \frac{5}{4} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \qquad \mathbf{x}[0] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

[8pts] a. Find the transfer function $\frac{\mathbf{X}(z)}{U(z)}$.
[2pts] b. Is the system BIBO stable? Why or why not?
[10pts] c. Find $\lim_{k\to\infty} \mathbf{x}[k]$ for a unit step input.

5. (26 pts) Transient performance using gain compensation (Nise 13.4, 13.8,13.9)
Given a CT plant $kG_1(s) = \frac{k}{(s+1)(s+2)}$.
[14pts] a. With sample period $T_s = -\log_e 0.8$, find $G_1(z)$, the Z transform of $G_1(s)$ (zero order hold in cascade with $G_1(s)$ as in example 13.4).
[4pts] b. Sketch the root locus for $G_1(z)$ in unity gain feedback, and find the range of k for stability (Matlab ok for k).
[4pts] c. With unity gain feedback, find the value of k for damping factor $\zeta = 0.7$ for both CT and DT systems, and note this k value in root locus for CT and DT.
[4pts] d. Plot the step response for the closed-loop CT and DT system in Matlab, and compare.