

**Due at 17:00 Berkeley time, Fri. Sep. 11 in Gradescope .**

Note: up to 2 students may turn in a single writeup. Reading Nise 2, 3

1. (15 pts) Equivalent Circuit (Nise 2.9)

Draw the equivalent mechanical circuits for the two systems shown in Fig. 1. (Note input  $v_{in}$  and output  $v_{out}$ ).

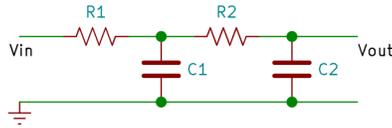


Fig.1a

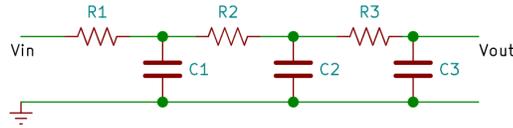


Fig.1b

2. (20 pts) Equivalent Circuit (Nise 2.9)

Draw the equivalent electrical circuit for the system in Fig. 2 below.

3. (25 pts) State Space for Mechanical System (Nise 2.6, 2.7, 3.4, 3.5)

Consider the system in Fig. 2, with input force  $f(t)$  and output  $y(t) = x_2(t)$ .

[10pts] a. Find the transfer function for the system shown in Fig. 2,  $\frac{Y(s)}{F(s)}$ .

[10pts] b. Write the state space equations for this system in phase-variable form and find  $A, B, C, D$ .

[5pts] c. Draw the equivalent block diagram of the system in phase variable form using integrator, scale, and summing blocks.

4. (20 pts) Linearization (Nise 2.11)

A maglev system uses an electromagnet to levitate a steel sphere of mass  $m$ . The vertical displacement  $x$  of the sphere is described by the following non-linear differential equation:

$$m \frac{d^2 x}{dt^2} = mg - k \frac{u^2}{x^2}$$

where  $g$  is gravity constant,  $k$  is a positive constant, input  $u(t)$  is coil current, and  $x(t)$  is the gap between electromagnet and sphere.

[5pts] a) Write the dynamic equations in state space form  $\dot{\mathbf{x}} = f(\mathbf{x}, u)$ , with  $x$  and  $\dot{x}$  as the states.

[2pts] b) Find the equilibrium point in terms of a nominal current  $u_o = I_o$  and nominal position  $x_o$ .

[13pts] c) Write the dynamic equations in state space form  $\dot{\mathbf{x}} = A\mathbf{x} + Bu$  for the system linearized about a non-zero equilibrium point found in part b). Use  $x(t) = x_o + \delta_x(t)$  and  $u(t) = u_o + \delta_u(t)$

5. (20 pts) State Space (Nise 3.4, 3.5, Lec 3. Phase Variable Form handout)

Given  $\frac{Y(s)}{U(s)} = \frac{s^4 + s}{s^4 + s^3 + 45s + 10}$

[8pts] a. Write the state space equations for this system in phase-variable form and find  $A, B, C, D$ .

[4pts] b. Write the differential equation relating  $y(t)$  to  $u(t)$ .

[6pts] c. Draw the block diagram (in phase variable form using integrators) corresponding to this differential equation.

[2pts] d. Explain how  $\frac{d^4 x(t)}{dt^4}$  could be found from signals in the block diagram.

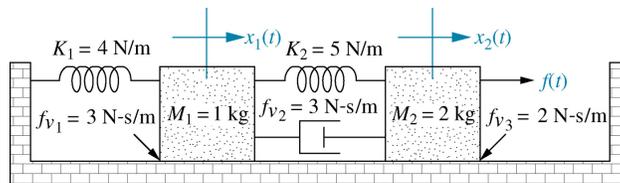


Fig. 2.