Professor Fearing EECS128/ME134 Problem Set 3 v1.01 Fall 2020

Due at 1700, Fri. Sep. 18 on gradescope .

Note: up to 2 students may turn in a single writeup. Reading Nise 4. Note: eqn 4.45: $T_s = \frac{4}{\sigma_d}$

1. (25 pts) State Space (Nise 2.6, 2.7, 3.4, 3.5, Lec3 H.O.)

Consider the rotary mechanical system shown in Fig.1 with input input $T(t)$ and output $\theta_2(t)$.

[10pts] a. Draw the equivalent electrical circuit for the system in Fig. 1. (Assume AC circuit operation with a transformer equivalent to a gear ratio. A transformer with a winding turn ratio of 1:N with input voltage V and input current i, will have an output voltage of NV and output current of i/N).

[5pts] b. Find the transfer function relating input $T(t)$ to output $\theta_2(t)$.

[5pts] c. From the transfer function, write the state space equations for this system in phase-variable form and find A, B, C, D.

[5pts] d. Draw the equivalent block diagram of the system in phase variable form using integrator, scale, and summing blocks.

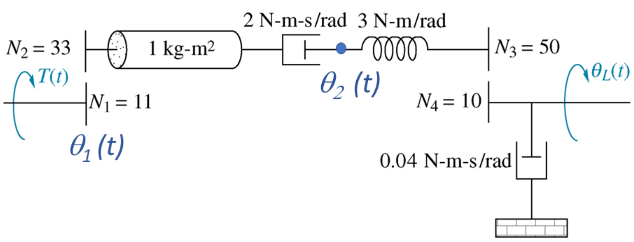

Fig. 1

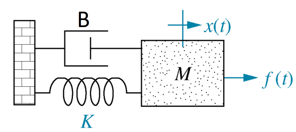

Fig.2

2. (20 pts) Linearization (Nise 3.7)

For the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 + u \\ u \sin(\pi x_3 x_4) \\ 2x_4 - e^{x_1 x_2} \\ u + x_1 + x_2 + x_3 - 4x_4 \end{bmatrix} \tag{1}$$

Linearize the system about $x_1 = 1, x_2 = 0, x_3 = 2, x_4 = \frac{1}{2}$, $u = -1$, and express in state space form:

$\dot{\delta}\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u.$

3. (20 pts) 2nd order step response (Nise 4.6)

The system shown in Fig. 2 is a model for an extensional oscillating beam. Given damping factor of the structure $\xi = 10^{-5}$ (obtainable with single crystal silicon).

[5pts] a. Find % overshoot.

[10pts] b. Find M, K and B such that $x(t)$ has $T_p = 10^{-8}s$ and final displacement is $10^{-9}m$ with $f(t) = 10^{-5}Nu(t)$. Note ω_n.

[5pts] c. For a MEMS device, consider Si with density $\rho = 2300 kg \cdot m^{-3}$, and elastic modulus $Y = 180 \times 10^9 Nm^{-2}$. With $K = \frac{Y \cdot w \cdot t}{l}$ find the dimensions (width w and length l) of a $t = 1\mu m$ thick Si layer which gives the desired time to peak and displacement.

4. (15 pts) Second order poles (Nise 4.6)

For each pair of second-order system step response specifications, find the location of the second order pair of poles (approximations ok).

a. $T_p = 1$ second; $T_s = 3$ seconds.

b. $\xi = 0.2$; $T_r = 0.2$ seconds (hint: approximate using Fig. 4.16).

c. $\xi = 0.1$; $T_p = 0.2$ seconds.

5. (20 pts) Dominant poles (Nise 4.7)

A system has transfer function $H(s) = \frac{40a}{(s^2+2s+40)(s+a)}$. Consider the step reponse of the system to be $y(t)$. The claim is made that for some range of a, (with $\omega_d = \omega_n\sqrt{1-\xi^2}$), that

$$y(t) \approx 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

i.e. the pole at $s = -a$ can be neglected. If so, find the range of a (to within 0.01) for which the approximate step response has peak c_{max} within 2% of the c_{max} of the true step response for $H(s)$. Also provide a plot using Matlab for the true and approximate step response with the a found above.

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