Professor Fearing EECS128/MEC134/Problem Set 4 v1.01 Fall 2020

Due at 1700, Fri. Sep 25 on gradescope.

Note: up to 2 students may turn in a single writeup. Reading Nise 4, 5-5.3, 6-6.1, 7-7.4

1. (20 pts) Pole-Zero cancellation (Nise 4.8)

Find the partial fraction expansion for the following transfer functions $C_1(s), C_2(s)$. Determine if pole-zero cancellation is a reasonable approximation for the step response by comparing coefficients of the terms of the partial fraction expansion. Using MATLAB, plot the step response for the complete system and the approximate system assuming pole-zero cancellation. (Note that the `step()` command should be on the systems $H(s)$ and $\tilde{H}(s)$). For C_1, C_2, briefly discuss if the step response approximation ($\tilde{c}_i(t)$) is reasonably close to the step response of the original system, $c_i(t)$.

a. $C_1(s) = \frac{1}{s} \cdot H_1(s) = \frac{8(s+2.5)}{s(s+2)(s^2+3s+10)} \qquad \tilde{C}_1(s) = \frac{1}{s} \cdot \tilde{H}_1(s) \approx \frac{10}{s(s^2+3s+10)}$

b. $C_2(s) = \frac{1}{s} \cdot H_2(s) = \frac{10(s+2.01)}{s(s+2)(s^2+3s+10)} \qquad \tilde{C}_2(s) = \frac{1}{s} \cdot \tilde{H}_2(s) \approx \frac{10}{s(s^2+3s+10)}$

2. (25 pts) Time Domain Solution - Convolution (Nise 4.11)

For the system below,

`[ 5 pts]` a. Find e^{At}.

`[15 pts]` b. Find $\mathbf{x}(t)$ and $y(t)$ using convolution (4.109) for the system below with unit step input $u(t)$:

`[ 5 pts]` c. Show that the $\mathbf{x}(t)$ found above is a solution to $\dot{\mathbf{x}} = A\mathbf{x} + Bu$ (by direct substitution).

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \ , \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \ y = [1 \quad 1]\mathbf{x}$$

3. (15 pts) Time Domain Solution - Laplace Transform (Nise 4.10)

For the system $\{A, B, C, D\}$ above, find $\mathbf{x}(t)$ and $y(t)$ using the Laplace transform method with unit step input $u(t)$.

4. (20 pts) Block Diagram Equivalence (Nise 5.2)

a) Find and draw the unity feedback system that is equivalent to the system in Fig. 1 below. Assume delay can be neglected.

b) Find the transfer function from input $R(s)$ to error $E(s)$ where $E(s) = R(s) - Y(s)$, neglecting delay.

5. (20 pts)Steady State Error (Section 7-7.4)

For the system in Fig. 2 below, let $G_1(s) = 1$, $G_2(s) = \frac{10(s+10)}{s(s+2)}$, $D(s) = 0$, and $H(s) = s + 4$.

`[ 10 pts]` a) What is the system type and appropriate static error constant?

`[ 5 pts]` b) What input waveform $r(t)$ would yield a constant error? (e.g. step, ramp, parabola, or ?)

`[ 5 pts]` c) What is the steady state error ($e = r - c$) for a unit input of the $r(t)$ found from b)?

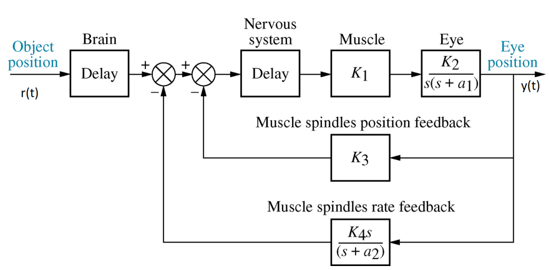

Fig. 1. Block Diagram.

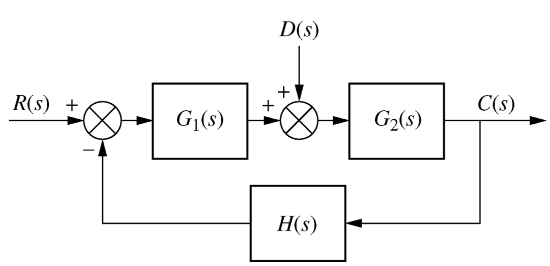

Fig. 2. Control System