Professor Fearing   EEC128/MEC134   Problem Set 6   v1.01   Fall 2020

Due at 1800, Fri. Oct. 9 in gradescope.
Note: up to 2 students may turn in a single writeup. Reading Nise 8,9
Midterm: Thurs. Oct. 15. 1410 Berkeley time.

1. (25 pts) Root locus (Nise 8.7)
Given the unity gain feedback system in Fig. 1, where

$$G(s) = \frac{K(8000)(s+10)}{(s^2+20s+200)(s+20)^3}$$

`[15 pts]` a) Find and approximately hand sketch the root locus using RL rules 1-8 for $k > 0$.
`[4 pts]` b) Find the range of K which makes the system stable.
`[4 pts]` c) Using the second order approximation (from data tip in Matlab `'rlocus'` which assumes dominant 2nd order poles) find the value of K and the dominant closed-loop pole locations that gives $\zeta \approx 0.25$ for the system's dominant closed-loop poles.
`[2 pts]` d) Use MATLAB to plot the actual step response for the full closed-loop system with the K found above, and compare to the 2nd order poles approximation estimate. (Normalize the second order approximation to make comparison easier.) How far off is the approximation?

2. (18 pts) Generalized Root locus (Nise 8.8)
Given the unity gain feedback system in Fig. 1, where

$$G(s) = \frac{20}{(s+4)(s+1)(s+\alpha)}$$

`[2]` a) Determine the characteristic equation for the closed loop system.
`[14]` b) Find and approximately hand sketch the root locus using RL rules 1-6,8 with respect to positive values of α ($0 \le \alpha < \infty$), showing direction in which α increases on the locus.
`[2]` c) Find the pole location α such that closed-loop system has one pair of poles near $-2.33 \pm 0.62j$ (Matlab ok).

3. (10 pts) Positive Feedback Root locus (Nise 8.9)
The open loop transfer function for a system in unity feedback is given by:

$$G(s) = \frac{k(s+1)(s+5)}{(s+8)(s+4)(s+2)}$$

.
`[4 pts]` a) Sketch by hand the root locus with respect to **positive** values of k.
`[5 pts]` b) Sketch by hand the root locus with respect to **negative** values of k.
`[1 pts]` c) Determine the range of k for stability (Matlab ok).

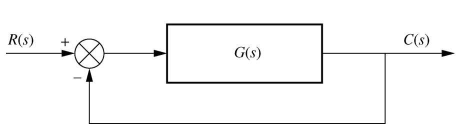

Fig. 1. Unity Gain Feedback.

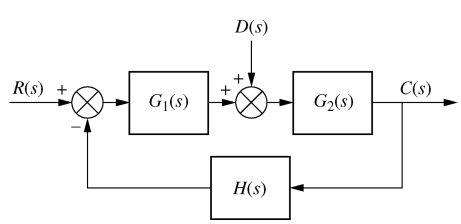

Fig. 2. Control System

4. (22 pts) PI compensation (Nise 9.2)
Consider open loop plant

$$G(s) = \frac{250K}{(s+10)(s^2+8s+25)}$$

and unity feedback.
`[3 pts]` a) find K such that damping factor $\zeta = 0.2$ (Matlab ok).
`[11 pts]` b) Design a PI controller with the same damping factor such that steady state error is 0, with $T_s \leq 2.8$ sec.
`[4 pts]` c) Using Matlab, plot the root locus for the original system and the system with a **PI** compensator, and mark the closed loop pole locations for each case.
`[2 pts]` d) Use Matlab to compare the step response for the closed-loop compensated and uncompensated systems, transient and steady state response. Is there pole-zero cancellation for the PI compensator? (Hint: use Matlab `stepinfo()`.)
`[2 pts]` e) Find the steady state error for a step for both systems.

5. (25 pts) PID Compensation (Nise 9.4)
Consider open loop plant

$$G(s) = \frac{K}{(s+4)^2(s+2)}$$

with unity feedback.
`[3`pts] a. Find the gain K for the uncompensated system to have percent overshoot of $\approx 27\%$ (Matlab ok). What is the settling time T_s? What is steady state error for a unit step input with this gain?

By using a proportional+ integral + derivative compensator, as shown in the next part, the system reponse can be made twice as fast, and the steady state error can be made zero.
`[20`pts] b. Design a PID controller such that percent overshoot of $25\% < O.S. < 30\%$ and settling time $T_{s1} < .52T_s$ sec, with zero steady state error for a step. Specify open and closed-loop poles, zero locations and gains (hint Matlab `tfdata()` for CLP, and `stepinfo()`). Note that the zero location for the PI part will have an effect on transient response will likely need to be adjusted).
`[2`pts] c. Show before and after compensation step response using Matlab on same plot.