Problem Set 6: Solutions

Problem 1

a. We will list the results from the rules:

1. 2. Open loop poles: 3 poles at -20. Complex conjugate poles at $-10 \pm 10j$. 5 poles implies 5 branches.
   Open loop zero: -10.

3. Since we know the root locus starts at the open loop poles, we observe that at -10 we have two poles ($-10 \pm 10j$) and 1 zero (three in total), so we should expect a real axis segment to the left of -10. Note that to the left of -20 we have 5 poles + 1 zero, that is an even number, so we should not expect a real axis segment to the left of -20. Hence, we should expect a real axis segment only between -10 to -20.

4. We only have 1 zero at -10, with a real-axis segment between -20 to -10, so we expect one pole at -20 to move to -10. The other 4 poles must go to an infinite zero.

5. Let us compute the angle asymptotes:

$$\theta_a = \frac{-(2l+1)\pi}{n-m}$$

on which $n = 5$ and $m = 1$, then:

$$\begin{aligned}
k = 0 &: \theta_a = \frac{-\pi}{4} \\
k = 1 &: \theta_a = \frac{-3\pi}{4} \equiv= \frac{5\pi}{4} \\
k = 2 &: \theta_a = \frac{-5\pi}{4} \equiv \frac{3\pi}{4} \\
k = 3 &: \theta_a = \frac{-7\pi}{4} \equiv \frac{\pi}{4} \\
k = 4 &: \theta_a = \frac{-9\pi}{4} \equiv \frac{-\pi}{4} \text{ repeats}
\end{aligned}$$

Real axis intercept:

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \frac{(-20-20-20-10-10)-(-10)}{4} = -17.5$$

From the first 5 rules we can create a reasonable sketch of the root-locus:

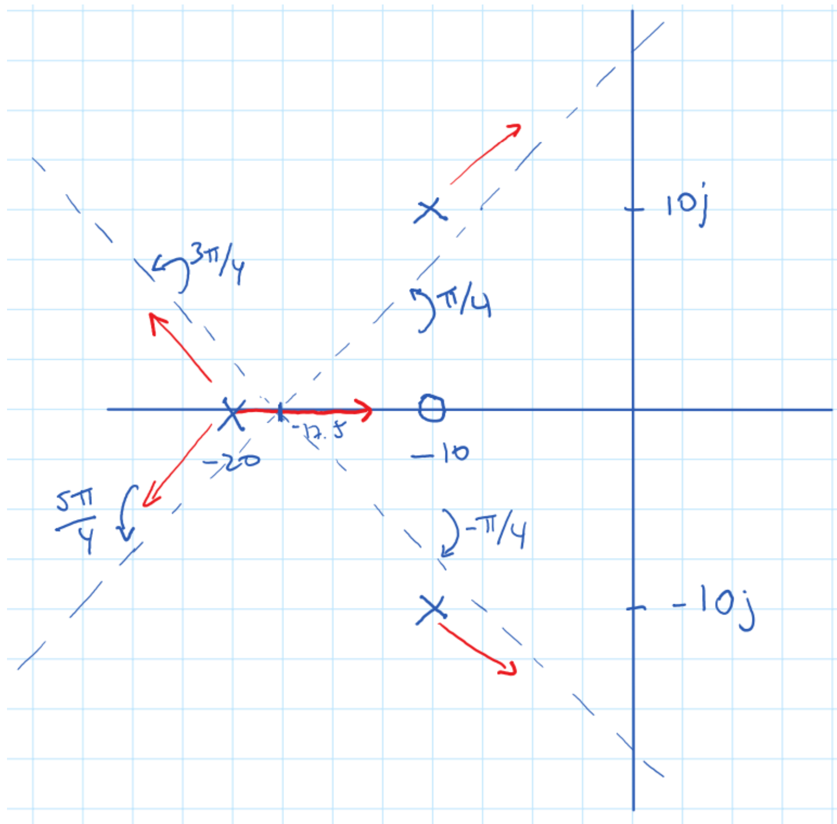

Figure 1: Sketch of the RL using the first 5 rules.

We then can refine our RL using rules 6 and 7 and 8.

6. Based on our sketch we expect a break away for the of the poles of -20 on the real axis. Since we have complex conjugates poles we can clear K to obtain:

$$K = \frac{(s^2 + 20s + 200)(s + 20)^3}{8000s + 80000}$$

Taking the derivative we obtain:

$$K' = \frac{(s + 20)^2(2s^3 + 65s^2 + 800s + 3000)}{4000(s + 10)^2}$$

The real roots of K' are on -20 (two times) and ≈ -6.4856. Based on this, we note the two poles at -20 will breakaway at -20. The other point is not in the locus, so no breakaway or break-in occurs at -6.4856.

Remark: Usually, based on rules 1-5 you can suspect the breakaway points. Since we know, that the poles at $-10 \pm 10j$ will go in the asymptotes of $\pm\pi/4$ we don't expect a break-in of such poles. On the other hand, we know that other two poles at -20 will go in the direction of $\pm 3\pi/4$, so it is expected to assume that they will breakaway exactly at -20.

7. The $j\omega$ axis crossing is usually obtained via the Routh-Hurwitz test, but in this case we can

solve numerically the closed-loop roots:

$$T = \frac{KG}{1+KG} = \frac{K(8000)(s+10)}{(s^2+20s+200)(s+20)^3+8000K(s+10)}$$

Numerically we find the roots of the denominator until the real part of at least one-root is equal to zero. The following code can be used for such purpose:

Listing 1: MATLAB code that numerically find the $j\omega$ axis cross.

```
[Ng, Dg]= zp2tf([-10]', [-20 -20 -20 -10+10j -10-10j]',8000)
G=tf(Ng,Dg)
K=(0:1000)/10;
[roots, gains]= rlocus(G,K);
idx = find(max(real(roots)) ≥ 0, 1, 'first')
Kcrit = gains(idx)
```

The critical K occurs at approximately $K = 49.4$ at a cross of approximately $\pm 18.816j$. Alternatively, the angle can be computed by finding a value at the $j\omega$-axis such that the sum of angles of open loop poles and zeros add to π. That is, let d be the value at the $j\omega$-axis, then we want to find d such that:

$$\begin{aligned}
\pi &= -\sum_{i=1}^{n} \angle(p_i - dj) + \sum_{i=1}^{m}(z_i - dj) \\
&= -\angle(-20+dj) - \angle(-20+dj) - \angle(-20+dj) \\
&\quad - \angle(-10+10j+dj) - \angle(-10-10j+dj) + \angle(-10+dj) \\
&= -3\arctan(d/20) - \arctan((10+d)/10) - \arctan((d-10)/10) + \arctan(d/10)
\end{aligned}$$

that can be solved numerically to obtain $d = -18.815$, as expected. The following code allows you to solve that equation:

Listing 2: MATLAB code that numerically find the $j\omega$ axis cross using the angle equation.

```
syms d
aux = -3*atan(d/20) - atan((10+d)/10) - atan((d-10)/10) + atan(d/10)
double(solve(aux == pi, d))
```

8. We have two complex conjugate poles at $-10 \pm 10j$. Recall that the rule for angle of departure of such poles is given by:

$$\sum_{i=1}^{m} \angle(s - z_i) - \sum_{i=1}^{n} \angle(s - p_i) = \pm(2\ell+1)\pi$$

The trick here is to pick a value of s around the specific pole and obtain the angles directly. Then the rule for a pole j is simply:

$$\angle(s - p_j) = \pi + \sum_{i=1}^{m} \angle(p_j - z_i) - \sum_{i=1, i\neq j}^{n} \angle(p_j - p_i)$$

In such case, for $-10+10j$ we have:

$$\theta_1 = \pi + \theta_3 - \theta_2 - \theta_4 - \theta_5 - \theta_6$$

as depicted in the following figure:

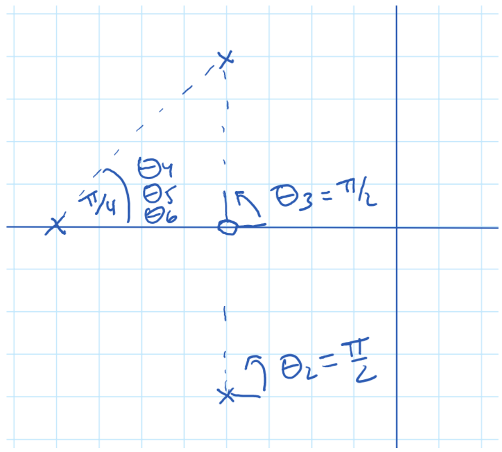

Figure 2: Sketch for the angle of departure of $-10+10j$.

Based on this:

$$\theta_1 = \pi + \pi/2 - \pi/2 - \pi/4 - \pi/4 - \pi/4 = \pi/4$$

Similarly for $-10-10j$ we obtain $\theta_3 = -\pi/4$.

With this, the sketch of the root-locus is:

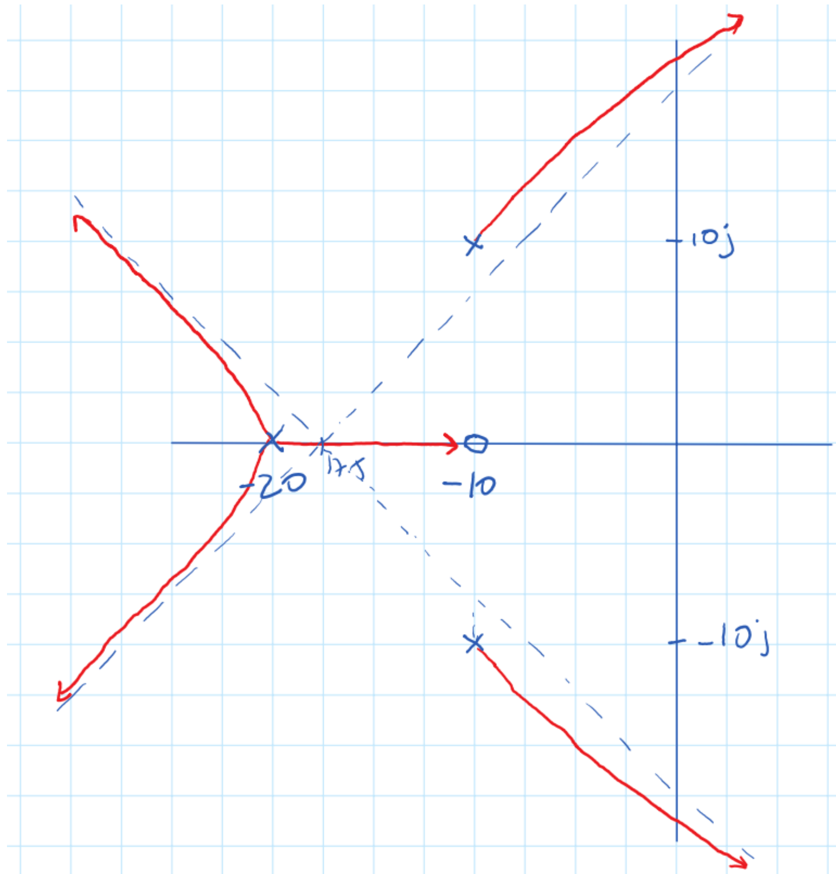

Figure 3: Final sketch of the RL.

b. From the $j\omega$ crossing and the RL sketch, the range of K on which the closed loop system stable is approximately given by $K \in [0, 49.4]$.

c. As mentioned before, the denominator of the closed loop system is given by:

$$\text{den}(T) = (s^2 + 20s + 200)(s + 20)^3 + 8000K(s + 10)$$

We know that the dominant poles are the ones closer to the $j\omega$ axis, that are the poles that start from $-10 \pm 10j$ based on the root locus. First recall that the damping ratio ζ satisfies that: $\cos(\zeta) = \theta$ on which θ is the angle of the complex conjugate poles, i.e.:

$$\tan(\theta) = \frac{|\text{Im}(p)|}{|\text{Re}(p)|}$$

With that note:

$$\tan(\arccos(\zeta)) = \frac{|\text{Im}(p)|}{|\text{Re}(p)|} \rightarrow \zeta = \cos\left(\arctan\left(\frac{|\text{Im}(p)|}{|\text{Re}(p)|}\right)\right)$$

Or equivalently the damping can be computed as:

$$\zeta = \frac{|\text{Im}(p)|}{|p|} = \frac{|\text{Re}(p)|}{\sqrt{\text{Re}(p)^2 + \text{Im}(p)^2}}$$

We will find numerically the gain such that the complex poles with the larger real part satisfies the previous condition. This can be done with the following code:

Listing 3: MATLAB code to find the K that satisfies $\zeta \approx 0.25$ for the two dominant poles.

```
[Ng, Dg]= zp2tf([-10]', [-20 -20 -20 -10+10j -10-10j]',8000)
G=tf(Ng,Dg)
K=(0:1000)/10;
[roots, gains]= rlocus(G,K);
zeta = abs(real(roots(1,:)))./abs(roots(1,:));
idx = find(zeta ≤ 0.25, 1, 'first')
gains(idx)
```

The value obtained is $K = 17.3$.
Using Matlab **rlocus** we can interactively found this K as showcased in the following figure:

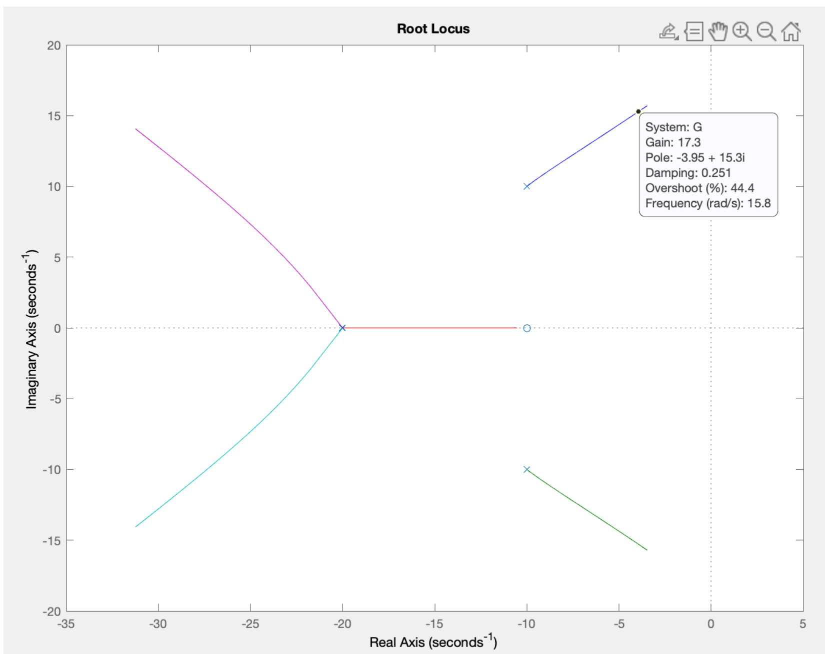

Figure 4: Value of K with $\zeta = 0.25$ via **rlocus** in Matlab.

d. The closed-loop poles with $K = 17.3$ are given by: $p_{12} = -3.9444 \pm 15.2819j$ and hence the denominator is given by:

$$\text{den} = (s - (-3.9444 - 15.2819j))((s - (-3.9444 + 15.2819j)) = s^2 + 7.889s + 249.1$$

Normalizing our approximated transfer function we obtain:

$$H_{\text{app}} = \frac{249.1}{s^2 + 7.889 + 249.1}$$

In addition we normalized our original transfer function and we compare step responses. The following code can be used for that:

Listing 4: MATLAB code to plot the two step responses.

```
[Ng, Dg]= zp2tf([-10]', [-20 -20 -20 -10+10j -10-10j]',8000)
G=tf(Ng,Dg)
K=(0:200)/10;
[roots, gains]= rlocus(G,K);

%Obtain feedback TF
H = feedback(17.3*G,1)
numconst = get(H).Numerator{1}(6)
denconst = get(H).Denominator{1}(6)
H_normalized = H * denconst/numconst;
step(H_normalized);

%Construct TF using roots at K = 17.3
idx = 174
ansroots=roots(1:5, idx)
denapp=conv([1 -ansroots(1)],[1 -ansroots(2)]);
numapp=denapp(3); % get constant term for normalizing
Gapp=tf(numapp,denapp)

hold on;
step(Gapp)
```

The resulting plot is:

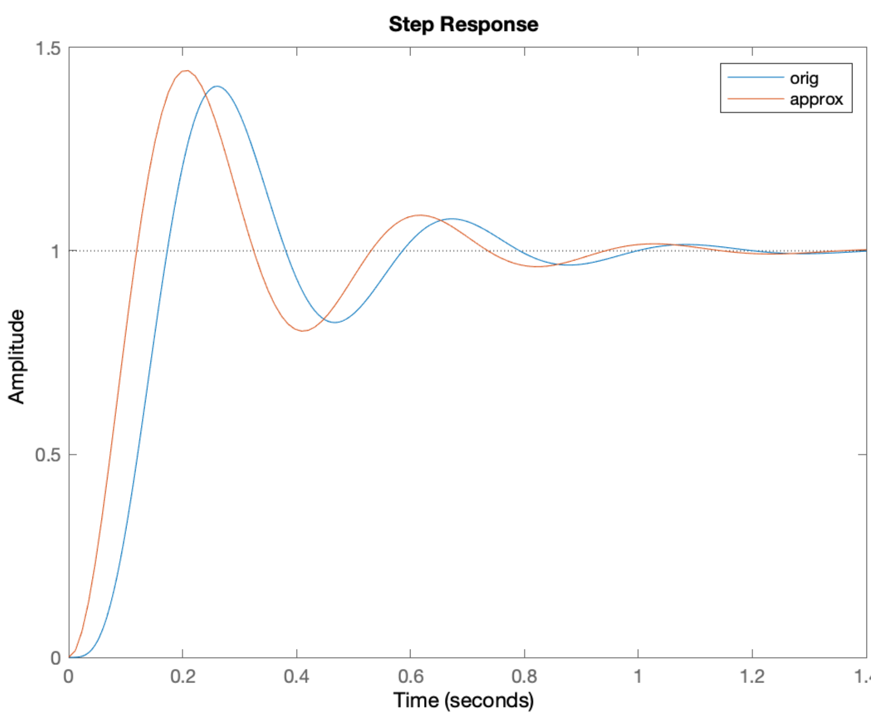

Figure 5: Step response of both transfer functions.

We observe that the overshoot and settling time are similar, but the rise time is faster in the approximated TF, that showcase that the approximation is not good. Why is this? The closed loop poles are found from Matlab:

$$
\begin{aligned}
&-3.9444 \pm 15.2819i \\
&-10.6019 + 0.0000i \\
&-30.7546 \pm 13.5672i
\end{aligned}
$$

the poles at -30 are more than $5\times$ away from dominant poles and have little effect. However, pole at -10.6 with the zero at -10 slows down the response compared to the pure 2nd order approximation, that produces this difference.

Problem 2

a. The closed loop system is given by:

$$
\begin{aligned}
T(s) &= \frac{G}{1+G} = \frac{\frac{20}{(s+4)(s+1)(s+\alpha)}}{1+\frac{20}{(s+4)(s+1)(s+\alpha)}} \\
&= \frac{20}{(s+4)(s+1)(s+\alpha)+20} \\
&= \frac{20}{\alpha(s+1)(s+4)+s(s+1)(s+4)+20} \\
&= \frac{\frac{20}{s(s+1)(s+4)+20}}{\frac{\alpha(s+1)(s+4)}{s(s+1)(s+4)+20}+1}
\end{aligned}
$$

With that, the characteristic equation for the traditional root locus is:

$$
G_1(s) = \frac{\alpha(s+1)(s+4)}{s(s+1)(s+4)+20} = \frac{\alpha(s^2+5s+4)}{s^3+5s^2+4s+20} = \frac{\alpha(s^2+5s+4)}{(s+5)(s^2+4)} = \frac{\alpha(s+1)(s+4)}{(s+5)(s+2j)(s-2j)}
$$

b. 1.2. We have three poles, at -5, $-2j$ and $2j$, with 2 zeros at -1 and -4. Then we have three branches.

3. We should expect real axis segments, to the left of -1 up to -4 and to the left of -5.

4. We have two zeros at -1 and -4, and we know that there is a real axis segment to the left of the -5 pole. Then we expect that -5 moves to the $-\infty$ in the real axis, while the other two poles at $\pm 2j$ moves to the finite zeros.

5. We skip the asymptotes since we know how the root locus will end.

6. We compute the break-in point as:

$$
\alpha = \frac{s^3+5s^2+4s+20}{s^2+5s+4}
$$

and hence:

$$
\alpha' = \frac{s^4+10s^3+33s^2-84}{(s^2+5s+4)}
$$

equating the numerator to zero we find that the real roots are -2.3752 and 1.3231. The first one is in the locus, so the break-in of the two complex conjugates poles occur at -2.3752.

8. We compute angle of departure of both complex conjugates poles as:

$$
\begin{aligned}
\theta_1 &= \pi - \theta_2(p=-2j) + \theta_3(z=-1) + \theta_4(z=-4) - \theta_5(p=-5) \\
&= \pi - \frac{\pi}{2} + \arctan(2/1) + \arctan(2/4) - \arctan(2/5) \\
&\approx 2.761 = 158.19^\circ
\end{aligned}
$$

and

$$
\theta_1 = \pi - \theta_1(p=-2j) + \theta_3(z=-1) + \theta_4(z=-4) - \theta_5(p=-5)
$$

$$
\begin{aligned}
&= \pi - \frac{3\pi}{2} + \arctan(-2/1) + \arctan(-2/4) - \arctan(-2/5) \\
&\approx -2.761 = -158.19^\circ
\end{aligned}
$$

Then the sketch of the root locus is given by:

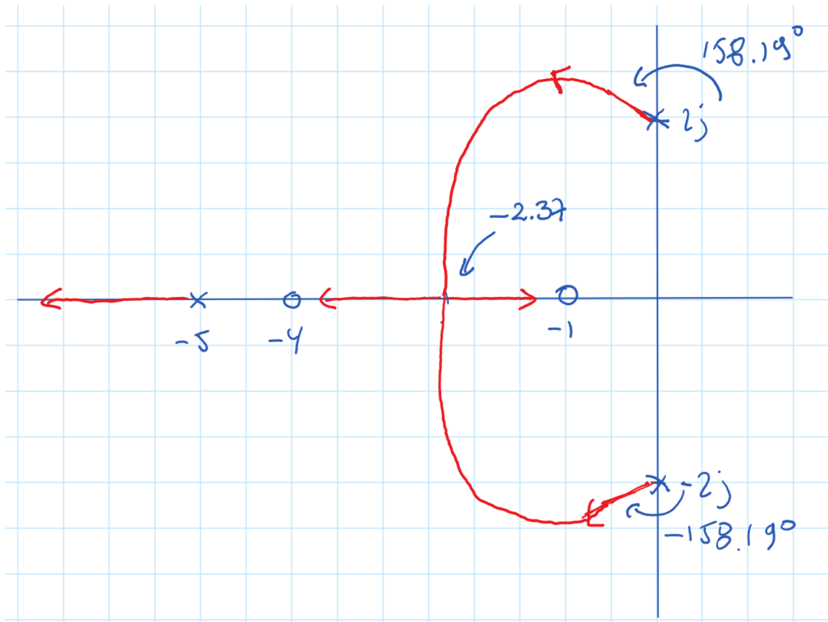

Figure 6: Final sketch of the RL.

c. We can do this numerically by finding α such that the closed loop denominator has such poles:

$$\mathrm{den}(T) = (s+4)(s+1)(s+\alpha) + 20 = 0$$

The following code can be used for such purpose:

Listing 5: MATLAB code to find the α that the poles around $-2.33 \pm 0.63j$.

```
syms s;

sol = zeros(3,200);
for i = 1:200
     a = i/10;
     den = (s+4)*(s+1)*(s+a) + 20;
     aux = double(solve(den ==0, s));
     sol(:, i) = aux;
end
```

Observing the results, for $\alpha \approx 9.9$, the closed loop poles are: $-2.3269 \pm 0.6344j, -10.24$. Alternatively, you can use the code `[roots, gains] = rlocus(G, K)`, as in problem 1, to find the case when the roots are in the requested locations. Similarly for $\alpha = 9.9$ we obtain such result for the complex conjugate poles. You can also use `controlSystemDesigner('rlocus',G)` or `sisotool(G)` to vary α in an interactive way.

Problem 3

a. Repeating the process of the previous problem we obtain the following sketch (that is quite easy given the real axis segments):

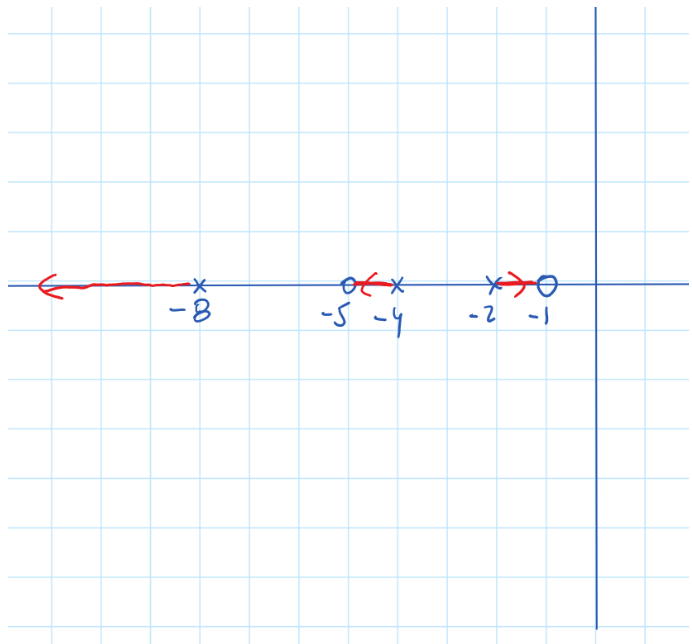

Figure 7: Final sketch of the RL for negative feedback.

b. In the case of the positive feedback root-locus (negative K) we will go through the rules again:

1. 2. Branches are the same, so we will have three branches with the same poles and zeros.

   3. The real axis segments are the opposite of the ones in the negative feedback locus. That is $[-8,-5], [-4,-2]$, $[-1,\infty]$.

   4. Same rule.

   5. We don't need to compute asymptotes, since we know given the real axis segments that the pole -8 will go to the -5 zero. And the other two poles from -4 and -2 must go to the zero at -1, and the one at $+\infty$, since we know that the segment is in the locus due to rule 3.

   6. Given rule 5, we know both poles on -4 and -2 must breakaway at some point. We can compute again such points by taking the derivative of K:

$$K = \frac{(s+8)(s+4)(s+2)}{(s+1)(s+5)} \rightarrow K' = \frac{s^4+12s^3+43s^2+12s-104}{(s+1)^2(s+5)^2}$$

Equating the derivative to zero we find that the real roots are:

$$-2.872, 1.2276$$

Then, based on the real axis segments, we expect that the poles from -4 and -2 breakaway at -2.872 and breakin in 1.2276.

7. The $j\omega$-axis cross can be found numerically as in problem 1. Approximately the cross occurs at $\pm 1.97j$ with $K = -8.7$.

With that, the sketch is given by:

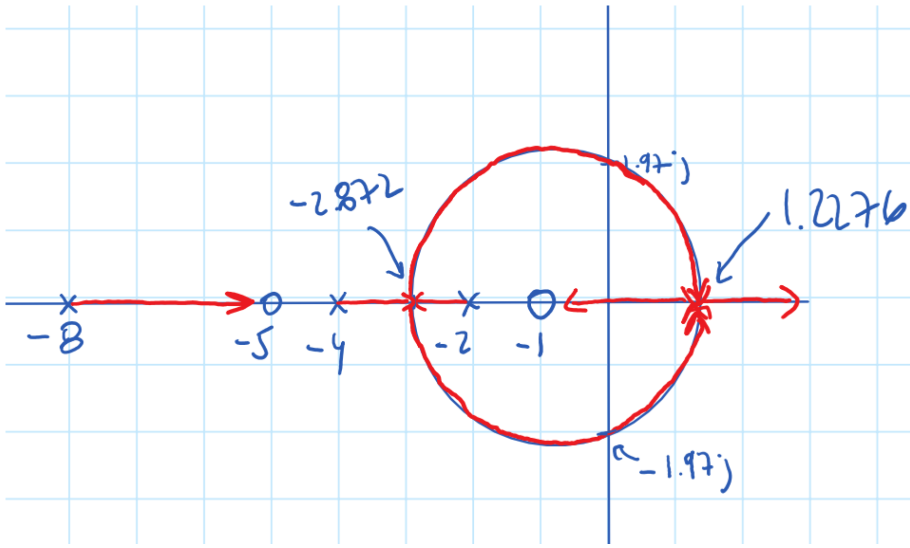

Figure 8: Final sketch of the RL for positive feedback.

c. From the negative feedback root locus (positive K) we observe that for any K the system is stable. While from the positive feedback the system becomes unstable at the $j\omega$-axis crossing, so at $K = -8.7$. With that, the range is $(-8.7, \infty)$.

Problem 4

a. Similar to problem 1.c) we can find numerically the poles via the equation:

$$\zeta = \frac{|\mathrm{Re}(p)|}{|p|} \quad \text{or} \quad \zeta = \cos\left(\arctan\left(\frac{|\mathrm{Im}(p)|}{|\mathrm{Re}(p)|}\right)\right)$$

for the dominant poles located at $-4 \pm 3j$. Using the following code we obtain the poles and gain:

Listing 6: MATLAB code to find the gain at which $\zeta = 0.2$.

```
num = 250;
den = [1 18 105 250];
G = tf(num,den)

K = (1:200)/100;
[roots, gains] = rlocus(G,K);
aux = abs(real(roots(2,:)))./abs(roots(2,:))
idx = find(aux ≤ 0.2, 1, 'first')
gains(idx)
```

Observing the results, for $K = 2.53$, the closed loop poles are: $-1.5368 \pm 7.534j$.

b. Now we will add a compensator of the form:

$$C(s) = \frac{s + \alpha}{s}$$

and hence the system with the compensator takes the form of:

$$G_1(s) = \frac{250K(s + \alpha)}{s(s + 10)(s^2 + 8s + 25)}$$

We try $\alpha = 2$, and repeating the process we obtain that $K = 1.87$, with dominant poles at $-1.3096 \pm 6.4208j$ (i.e. $\zeta = 0.2$). The settling time is given by:

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.2 \cdot \sqrt{1.3096^2 + 6.4208^2}} = 2.6029$$

that satisfies the requirement.

c. Using `controlsystemDesigner('rlocus',G)` we obtain (you can also use `rlocus(G)`):

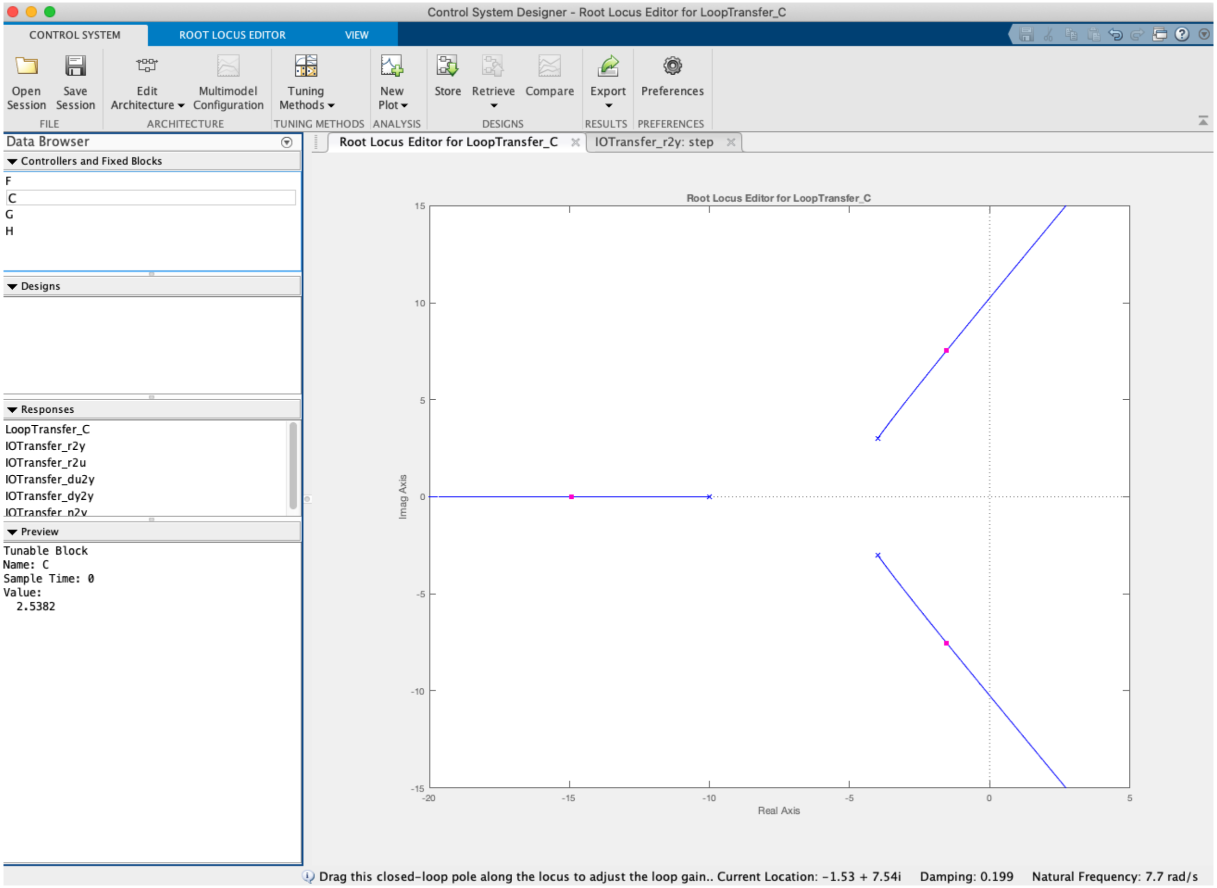

Figure 9: Root-locus for the uncompensated system. It is observed that the damping is 0.2 at $-1.53+7.54j$ with $K = 2.5382$.

For the compensated block (you can also use `rlocus(G)`):

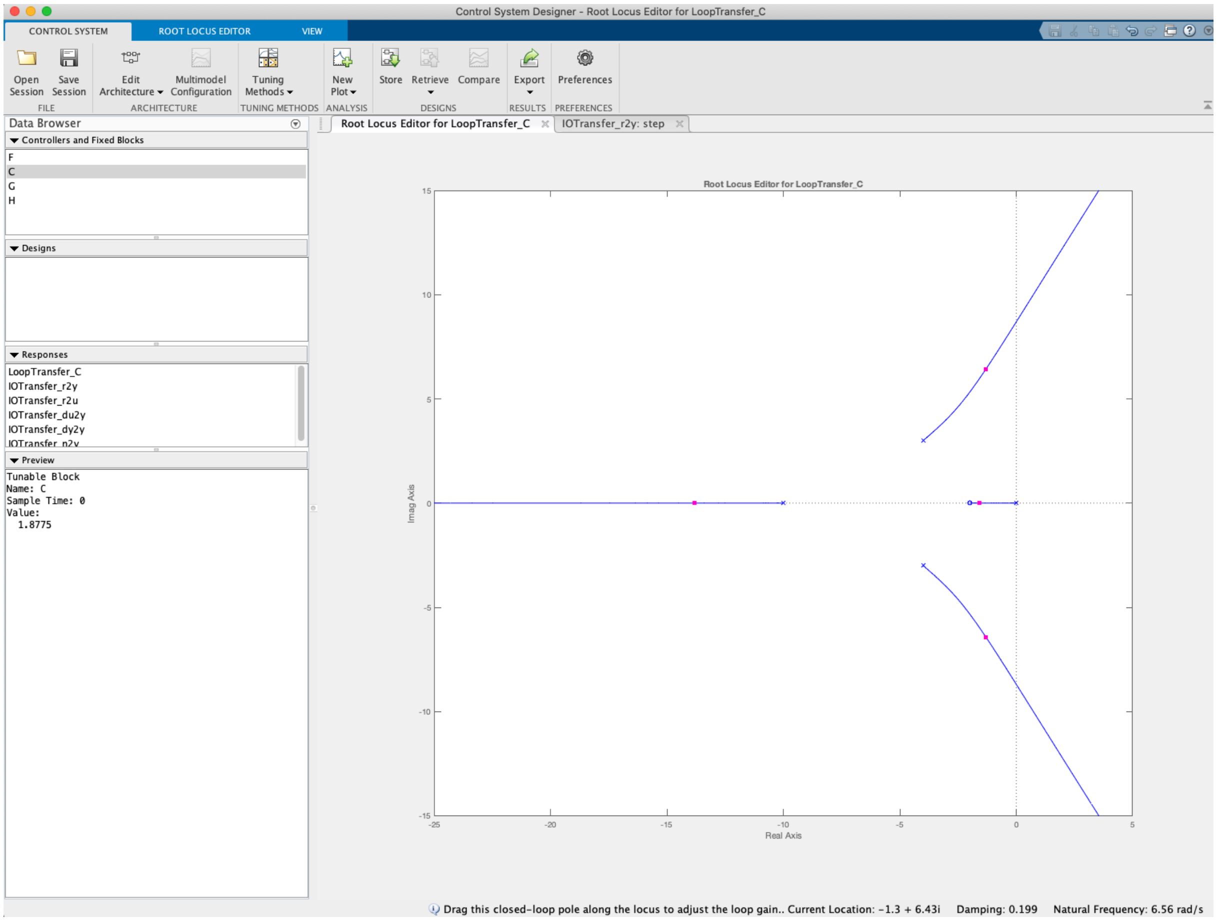

Figure 10: Root-locus for the compensated system. It is observed that the damping is 0.2 at $-1.3+6.43j$ with $K = 1.8775$.

d. The following code can be used to compare plots:

Listing 7: MATLAB code to plot both responses.

```
num = 250;
den = [1 18 105 250];
num1 = 250*[1 2];
den1 = [1 18 105 250 0];
G = tf(num,den)

K = (1:400)/100;
[roots, gains] = rlocus(G,K);
aux = abs(real(roots(2,:)))./abs(roots(2,:))
idx = find(aux ≤ 0.2, 1, 'first')
Kcrit = gains(idx)
zeta = abs(real(roots(2,idx)))./abs(roots(2,idx));
wn = abs(roots(2,idx));
roots(2,idx)
Ts = 4/(zeta*wn)
H = feedback(Kcrit * G,1)
step(H)

```

```
%Compensated case
Gcomp = tf(num1, den1)
[roots2, gains2] = rlocus(Gcomp,K);
aux2 = abs(real(roots2(2,:)))./abs(roots2(2,:));
idx = find(aux2 ≤ 0.2, 1, 'first');
Kcrit2 = gains2(idx)
zeta2 = abs(real(roots2(2,idx)))./abs(roots2(2,idx));
wn2 = abs(roots(2,idx));
roots2(2,idx)
Ts2 = 4/(zeta*wn)
Hcomp = feedback(Kcrit2 * Gcomp,1)
[num, den] = tfdata(Hcomp)
[r,p,k] = residue(num{1},den{1})
hold on
step(Hcomp)
legend('uncompensated','compensated')
```

The step plot is:

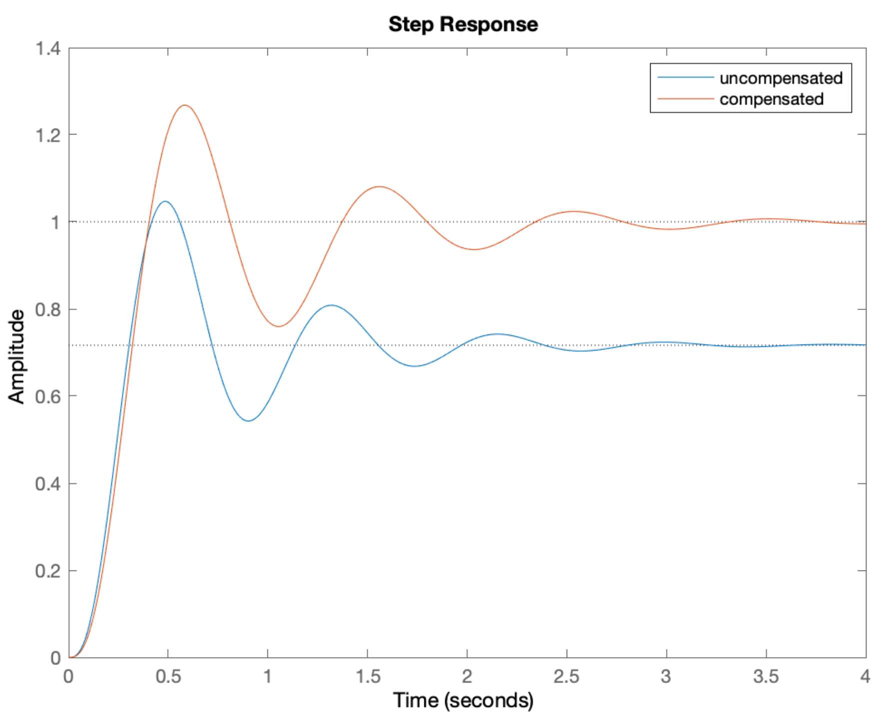

Figure 11: Step response of both transfer functions.

From Figure 10 we see that there is no pole cancellation by using $\alpha = 2$, since the other pole is at -1.577. However, from the partial fraction residues (using `[r,p,k] = residue(num,den)`) on the closed loop compensated transfer function we obtain the following expansion:

$$H_{\text{comp}} = \frac{2.287}{s+13.8} + \frac{-1.3393 - 2.3337j}{s+(1.3-6.43j)} + \frac{-1.3393 + 2.3337j}{s+(1.3+6.43j)} + \frac{0.3913}{s+1.577}$$

$$= \frac{2.287}{s+13.8} + \frac{-2.678s+25.17}{s^2+2.6192s+42.9411} + \frac{0.3913}{s+1.577}$$

Observe that the residue of the pole at 1.577 is around one order of magnitude smaller than the other residues, that showcase that our second order approximation using the poles at $-1.3 \pm 6.43j$ is an appropriate approximation (note that the other pole is at -13.8, around 10 times faster than our dominating second order poles).

e. Finally, from the step plots, we observe that the steady state error is zero for the compensated plot, while the error for the uncompensated is $1 - 0.717 \approx 0.283$.

Problem 5

a. Using `stepinfo(G)` we can obtain the step-response characteristics. By setting $K = 65.5$ we obtain an overshoot of 27%, with a settling time of 3.1357s. The steady error can obtained observing the step response, and computed as: $e(\infty) \approx 1 - 0.672 \approx 0.328$.

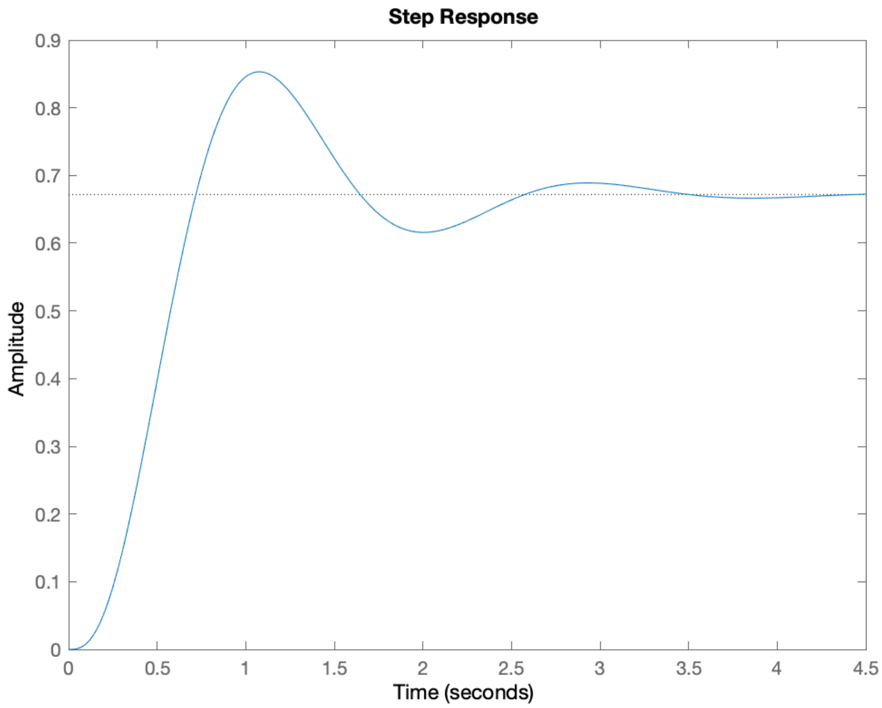

Figure 12: Step response for the closed-loop system with $K = 65.5$.

b. The PID controller will be of the form:

$$KC(s) = \frac{K(s+\alpha_1)(s+\alpha_2)}{s}$$

We will choose the same overshoot from part a), and attempt to compensate our plant first with our PD and then adding the PI. From the overshoot we can obtain our required damping:

$$\zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + \ln^2(OS)}} = \frac{-\ln(0.27)}{\sqrt{\pi^2 + \ln^2(0.27)}} \approx 0.385$$

The required settling time given the uncompensated plant $T_s = 3.13357$, is given by $T_{s1} < 0.52 \cdot 3.13357 \approx 1.629$. With this we can obtain a condition for our natural frequency since:

$$T_s = \frac{4}{\zeta\omega_n} \;\rightarrow\; \omega_n = \frac{4}{T_s\zeta} = \frac{4}{1.629 \cdot 0.385} = 6.338$$

Now, we are interested in adding our first PD term of the form $K(s+\alpha_1)$. Observe that after adding our first zero, the asymptotes angles are given by:

$$\theta_a = \frac{(2\ell+1)\pi}{3-1} = \frac{\pi}{2}, \frac{3\pi}{2} = 90°, 270°$$

Based on the RL, if we include a zero between $[-\infty, 0]$, we expect that the one pole will go to that zero, while the other poles will breakaway to $\pm j\infty$ with those asymptotes angles. Given this, we can assume that the real part of our complex conjugate poles (computed as $\sigma_d = -\zeta\omega_n = -2.44$) can be equated with the real-axis intercept via root-locus. Thus:

$$\text{real-axis-intercept} = \frac{\sum p - \sum z}{\#p - \#z} = \frac{-4-4-2+\alpha_1}{3-1} = \sigma_d = -2.44 \to \alpha_1 = 5.12$$

Then adding a zero at -5.12 provides such performance. Now, the K can be tuned now via traditional root-locus to ensure that the imaginary part of the complex poles satisfies that:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 5.8494$$

For example, $K = 33.4$ satisfies that, and that will be sufficient for a PD compensation with the required overshoot and settling time. However, we know that we require a pole at the origin to ensure zero steady-state error and hence we require to add an additional PI compensator:

$$\frac{(s+\alpha_2)}{s}$$

that will modify our requirements. The standard approach here will be to include our zero from the PI compensator close to the -2 pole to ensure an approximate zero-pole cancellation. A value of $\alpha_2 = -1.9$ is convenient for such purpose (we don't want an exact zero pole cancellation to avoid numerical issues altering our system behavior). However, this new zero modify our real-axis intercept that we are looking to keep intact from the previous PD compensation. Then, we update our previous zero α_1 as:

$$\text{real-axis-intercept} = \frac{\sum p - \sum z}{\#p - \#z} = \frac{-0-4-4-2+\alpha_1+1.9}{4-2} = \sigma_d \to \alpha_1 = 3.22$$

With this, we can use our root-locus techniques to update K to satisfy:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} > 5.8494$$

A value of $K = 40$ satisfies such purpose. We confirm our design using Matlab. From `stepinfo(H)` of the closed loop system we obtain a settling time of 1.317 seconds and an overshoot of 25.3%, that are consistent with the design requirements. With that our final compensated open-loop transfer function is:

$$G(s) = \frac{40(s+3.22)(s+1.9)}{s(s+4)^2(s+2)}$$

Using `[z,p,k] = zpkdata(G)`, the open-loop poles, zeros and gains given by the compensated system are:

$$\text{poles} = -4, -4, -2, 0$$
$$\text{zeros} = -3.22, -1.9$$
$$\text{gain} = 40$$

Now, the closed-loop system $KG/(1 + KG)$ has:

$$\text{poles} = -2.4745 + 5.909j, -2.4745 - 5.909j, -3.1697, -1.8813$$
$$\text{zeros} = -3.22, -1.9$$
$$\text{gain} = 40$$

We can observe that the other poles are close enough to the zeros of the system, to ensure a sufficiently good zero-pole cancellation, that confirms that our second order approximation is good. You can reassure yourself by computing the residues of the closed-loop compensated transfer function.

c. The code used is:

Listing 8: MATLAB code to plot both responses.

```
num = 1;
den = [1 10 32 32];
G = tf(num,den);
H = feedback(65.5*G, 1);
stepinfo(H)
fig1 = figure();
step(H)
numcomp = [1 5.12 6.118]
dencomp = [1 10 32 32 0];
Gcomp = tf(numcomp,dencomp);
%rlocus(Gcomp)
Hcomp = feedback(40*Gcomp, 1);
stepinfo(Hcomp)
hold on
step(Hcomp)
legend('uncompensated', 'compensated')
```

The step plot is:

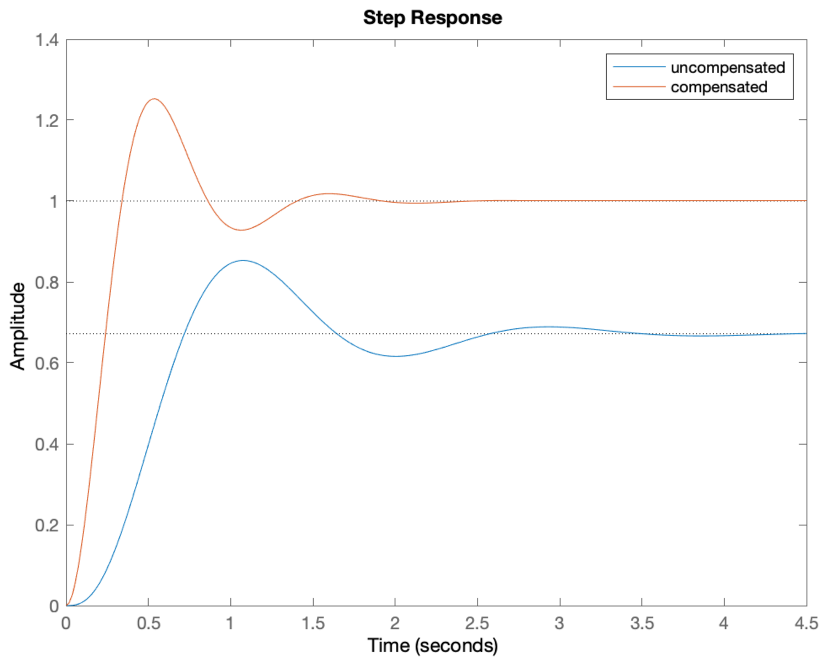

Figure 13: Step response of both transfer functions.