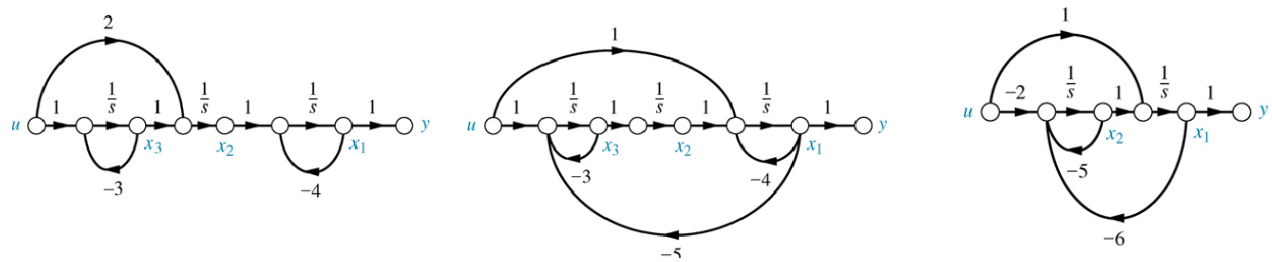
Due at 1800, Fri. Nov. 6 in gradescope.
Note: up to 2 students may turn in a single writeup. Reading Nise 12

1. (24 pts) State Feedback/Pole placement (Nise 12.2)
Consider the plant, where $G(s) = Y(s)/U(s)$:

$$G(s) = \frac{100}{s(s+3)^3}.$$

[6pts] a. Draw the signal graph in phase variable form and write the corresponding state equations.
[10pts] b. Find $K = [k_1\ k_2\ k_3\ k_4]$ such that feedback $u = r - K\mathbf{x}$ yields an equivalent second order step response with $\zeta = 0.5$ and $\omega_n = 10$. (Place third and fourth pole with real part 5 times further from $j\omega$ axis as the dominant pole pair).
[8pts] c. With zero initial conditions, use Matlab to plot the step response $y(t)$. Is the response 2nd order? Also plot $u(t)$, and each individual component $k_1x_1(t)$, $k_2x_2(t)$, $k_3x_3(t)$, $k_4x_4(t)$. Which state contributes most to $u(t)$?

2. (24 pts) Controllability and Observability (Nise 12.3, 12.6)
For each of the systems below, input is $u(t)$, output is $y(t)$, states x_1, x_2, x_3.
[6pts] a) Write state and output equations for the graph.
[1pts] b) Determine if the system is controllable.
[1pts] c) Determine if the system is observable.



i) ii) iii)

3. (10 pts) Controllability (Nise 12.3, 12.6)

Consider the system $\dot{\mathbf{x}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

[6pts] a) For what values of a, b, c, d is the system uncontrollable?
[4pts] b) Draw the signal flow graph with a, b, c, d chosen for the system to be uncontrollable, and briefly explain how one can see from the signal flow graph that the system is uncontrollable.

4. (20 pts) Control Form transformation (Nise 12.4, 5.8)

Given the following:

$$\dot{\mathbf{z}} = A\mathbf{z} + Bu = \begin{bmatrix} -13 & -3 \\ -2 & -12 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t), \quad y = [1 \quad 4]\ \mathbf{z}$$

[10pts] a) Find the transformation P such that $(\bar{A}, \bar{B})$ is in <u>diagonal</u> form, where $\bar{A} = P^{-1}AP$ and $\bar{B} = P^{-1}B$.
[10pts] b) Find $\bar{A}, \bar{B}, \bar{C}$ such that $\dot{\bar{\mathbf{x}}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{x}$.

5. (22 pts) Cayley-Hamilton. (Notes under <u>Resources</u> on web page)

Given: $A = \begin{bmatrix} -13 & -3 \\ -2 & -12 \end{bmatrix}$

[2pts] a) Find the eigenvalues of A and the characteristic polynomial $\Delta(\lambda) = 0$.
[4pts] b) Show that $\Delta(A) = \mathbf{0}$.
[10pts] c) Find e^{At} by Cayley-Hamilton; that is ~~show, that~~ <u>find</u> $e^{At} = \alpha_0(t)I + \alpha_1(t)A$, where $\alpha_i(t)$ are found by solving $e^{\lambda_i t} = \alpha_0(t) + \alpha_1(t)\lambda_i$.
[6pts] d) Verify e^{At} is the same as found by Laplace transform.