In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

### Tables for reference

#### Trigonometry:

\[
\begin{array}{cccccc}
\tan^{-1} \frac{1}{7} & \approx & 5.7^\circ & \tan^{-1} \frac{1}{2} & \approx & 7.1^\circ \\
\tan^{-1} \frac{1}{3} & \approx & 11.3^\circ & \tan^{-1} \frac{1}{4} & \approx & 14^\circ \\
\tan^{-1} \frac{1}{5} & \approx & 18.4^\circ \\
\tan^{-1} \frac{1}{7} & \approx & 26.6^\circ & \tan^{-1} \frac{1}{\sqrt{3}} & \approx & 30^\circ \\
\tan^{-1} \frac{1}{6} & \approx & 33.7^\circ & \tan^{-1} \frac{1}{8} & \approx & 36.9^\circ \\
\tan^{-1} \frac{1}{9} & = & 45^\circ & \tan^{-1} \frac{1}{10} & = & 56^\circ \\
\sin 30^\circ & = & \frac{1}{2} & \cos 30^\circ & = & \frac{\sqrt{3}}{2} \\
\cos 45^\circ & = & \frac{\sqrt{2}}{2} & \sin 45^\circ & = & \frac{\sqrt{2}}{2}
\end{array}
\]

#### Constants

\[
\begin{array}{ccc}
\pi & \approx & 3.14 \\
\pi/2 & \approx & 1.57 \\
\pi/4 & \approx & 0.79 \\
1/e & \approx & 0.37 \\
1/e^2 & \approx & 0.14 \\
1/e^3 & \approx & 0.05
\end{array}
\]

#### Logarithm:

\[
\begin{array}{ccc}
\log_{10} 2 & = & 0.30 \\
\log_{10} 3 & = & 0.48 \\
\log_{10} 5 & \approx & 0.7 \\
\log_{10} 6 & \approx & 0.78 \\
20 \log_{10} 2 & = & -6dB \\
20 \log_{10} 1 & = & 0dB \\
20 \log_{10} \sqrt{2} & = & 3dB \\
20 \log_{10} \sqrt[3]{2} & = & 6dB \\
20 \log_{10} \sqrt{10} & = & 10dB \\
20 \log_{10} 5 & = & 20db - 6dB = 14dB
\end{array}
\]

#### Square roots

\[
\begin{array}{ccc}
1/\sqrt{3} & \approx & 0.58 \\
1/\sqrt{2} & \approx & 0.71 \\
\sqrt{3}/2 & = & 0.87 \\
\sqrt{2} & \approx & 1.41 \\
\sqrt{3} & \approx & 1.73 \\
\sqrt{3.5} & \approx & 1.87 \\
\sqrt{5} & \approx & 2.24 \\
\sqrt{7} & \approx & 2.65 \\
\sqrt{10} & \approx & 3.16 \\
\sqrt{13} & \approx & 3.61 \\
\sqrt{17} & \approx & 4.12 \\
\sqrt{20} & \approx & 4.47
\end{array}
\]
Problem 1 (25 pts)
Each part is independent.

[7 pts] a) Consider a single-input single-output system with input \( u(t) \) and output \( y(t) \) described by the block diagram below with coefficients as given. (Note that this diagram is a modified version of the Lec#3 handout.)

\[
\begin{align*}
a_0 &= 1 & a_1 &= 0 & a_2 &= 3 & a_3 &= 0 & b_4 &= 4 \\
b_0 &= 2 & b_1 &= 0 & b_2 &= 0 & b_3 &= 0
\end{align*}
\]

[3 pts] i) Write the transfer function for the system:

\[
\frac{Y(s)}{U(s)} =
\]

[4 pts] ii) For the output equation

\[
y = Cx + Du = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + b_4u(t), \text{ find } C.
\]

\[
C = [\quad \quad \quad \quad ]
\]

[5 pts] b) A nonlinear system with output \( x(t) \) and input \( f(t) \) is described by the differential equation

\[
\ddot{x} + \dot{x} + kx^2 = f(t)
\]

The system input is a constant offset with small variations such that \( f(t) = 1 + \delta f(t) \). The output \( x(t) \approx x_0 + \delta x(t) \). Find the transfer function relating output variation \( \delta x(t) \) to input variation \( \delta f(t) \).

\[
\frac{\Delta X(s)}{\Delta F(s)} =
\]
c) Consider a system with a step input which has output transfer function:

\[ Y(s) = \frac{2(s - 2)}{s(s + 1)(s + 4)} = \frac{A}{s} + \frac{B}{s + 1} + \frac{C}{s + 4} \]

[3 pts] i) Find the partial fraction expansion coefficients for \( Y(s) \):

\[ A = \ldots \]
\[ B = \ldots \]
\[ C = \ldots \]

[2 pts] ii) Find \( y(t) \), the inverse Laplace transform of \( Y(s) \), using the partial fraction expansion above.

\[ y(t) = \ldots \]

[8 pts] d) Draw the equivalent mechanical circuit for this electrical system, with voltage corresponding to force and current to velocity. Input force is \( r(t) \), output force is \( y(t) \).
Problem 2 Steady State Error (16 pts)

For parts a) and b), use:

\[ G_1(s) = k \]
\[ G_2(s) = \frac{s+2}{(s+3)(s+4)} \]
\[ H(s) = \frac{1}{s} \]

[4 pts] a) Let \( R(s) = 0 \). Find the response to a general disturbance input \( D(s) \) in terms of \( G_1(s), G_2(s), H(s) \).

\[
\frac{C(s)}{D(s)} = 
\]

[4 pts] b) For a disturbance input \( d(t) = u(t) \), a unit step, (with \( r(t) = 0 \)) show that \( \lim_{t \to \infty} c(t) = 0 \).

[4 pts] c) Let \( D(s) = 0 \). Let \( c(t) = r(t) - c(t) \). Find \( \frac{E(s)}{R(s)} \) in terms of \( G_1(s), G_2(s), H(s) \).

\[
\frac{E(s)}{R(s)} = 
\]

[4 pts] d) For \( r(t) = u(t) \), a unit step, (with \( d(t) = 0 \)), show that \( \lim_{t \to \infty} c(t) \neq 0 \).
Problem 3. Root Locus Plotting (27 pts)
For the root locus \((1 + kG(s) = 0)\) with \(k > 0\), and given open loop transfer function \(G(s)\):

\[
G(s) = \frac{s(s + 1)}{(s^2 - 2s + 5)(s^2 + 6s + 13)}
\]

[1 pts] a) Determine the number of branches of the root locus =

[2 pts] b) Determine the locus of poles on the real axis

[3 pts] c) Determine the angles for each asymptote:

[4 pts] d) determine the real axis intercept for the asymptotes \(\sigma =

[6 pts] e) Use the angle criteria for poles and zeros to show that \(p \approx 0 + 2.1j\) is on the root locus.
zeros: \(\angle p = \) \(\angle (p + 1) =

poles:
\[
\angle (p - 1 + 2j)^{-1} = \quad \angle (p + 3 + 2j)^{-1} =
\]
\[
\angle (p - 1 - 2j)^{-1} = \quad \angle (p + 3 - 2j)^{-1} =
\]

[6 pts] f) Estimate the value of \(k\) for which \(p \approx +2.1j\) is a closed-loop pole. (Show work for full credit).
\(k =

[5 pts] g) Sketch a root locus below which is consistent with the information found above. Draw arrows on branches showing increasing gain. Draw asymptotes.
Problem 4. Root Locus Compensation (18 pts)

Given open loop transfer function \( G(s) \), where \( G_1(s) \) is the open-loop plant:

\[
G(s) = G_c(s)G_1(s) = G_c(s) \frac{64}{(s^2 + 4s + 8)^2}
\]

and \( G_c(s) \) is a PI compensation of the form \( G_c(s) = k_p \frac{s + z_c}{s} \). The closed loop system, using unity gain feedback and the PI controller, should have a pair of poles at \( p \approx -1 + j \) and \( p^* \approx -1 - j \).

[7 pts] a. To obtain the closed loop pole at \( p \), estimate the angle contribution (in degrees) for each of the open loop poles, and the total angle contribution for all open-loop poles:

<table>
<thead>
<tr>
<th>pole</th>
<th>angle</th>
<th>pole</th>
<th>angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s = 0 )</td>
<td></td>
<td>( s = -2+2j )</td>
<td></td>
</tr>
<tr>
<td>( s = -2+2j )</td>
<td></td>
<td>( s = -2-2j )</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[2 pts] b. What is the necessary angle contribution of the zero \( z_c \) for the closed loop pole \( p \) to be on the root locus?

[9 pts] c. Find \( z_c \) to within \( \pm 0.5 \) such that \( p \) is approximately on the root locus, within \( \pm 2 \) degrees. (Show work.)

\[
z_c = \text{__________}
\]

(Pole-Zero plot below for scratch work. It will not be graded).
Problem 5. Routh-Hurwitz (14 pts)

Given system with closed loop transfer function (assuming unity feedback)

\[ T(s) = \frac{k}{s^4 + 10s^3 + 33s^2 + 40s + (16 + k)} \]

[10 pts] a. Using the Routh-Hurwitz table, show that the maximum positive \( k \) for which the closed loop system is stable is approximately 100.

[4 pts] b. For \( k \approx 100 \), approximately find the pair of closed loop poles on the imaginary axis. (Show work).

\[ s = \pm j\omega = \pm j \text{______} \]
page for scratch work