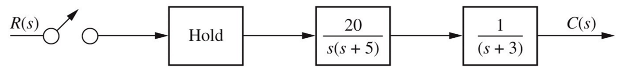
Professor Fearing   EECS128/MEC134   Problem Set 12 v1.0   Spring 2017

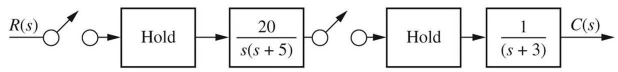
Due at 1700, Fri. April 28 in homework box under stairs, first floor Cory . Note: up to 2 students may turn in a single writeup. Reading Nise 13.

1. (15 pts) Discrete Time Conversion (Nise 13.4)

For each of the block diagrams below (b,c), find $\frac{C(z)}{R(z)}$ using $T = 0.5$ sec. If the transfer functions are different, explain why.



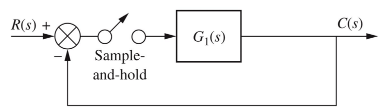
(b)



(c)

2. (15 pts) Stability of DT (Nise 13.6)

Find the range of gain K for which the system below with $G_1(s) = \frac{K}{s(s+4)}$ with $T = 0.25$ is stable.



3. (20 pts) SS to TF (Nise 3.6, 13.3, 13.4, DT handout)

Given the following discrete time (DT) system, with sample period $T = 1$:

$$\mathbf{x}(k+1) = G\mathbf{x}(k) + Hu(k) = \begin{bmatrix} -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{2} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u \tag{1}$$

[6pts] a. Find the transfer function $\frac{X(z)}{U(z)}$.

[2pts] b. Is the system BIBO stable?

[12pts] c. Find $x[n]$ for a unit step input and zero initial conditions using partial fraction expansion.

4. (30 pts) Continuous vs Discrete Time Control (Handout and Matlab)

For each part, hand in relevant Matlab code as well as plots. Use `hold on` *to superimpose plots.*

Given the following continuous time (CT) system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -25 & -35 & -11 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t), \quad y = [1\ 1\ 0]\ \mathbf{x}$$

[4pts] a) Find the corresponding discrete time (DT) system $x[n+1] = Gx[n] + Hu[n]$, $y[n] = Cx[n]$ which can be found using the Matlab function `c2d(ctsys,T,'zoh')`, with sampling period $T_s = 0.25$ sec. (Note `ctsys` can be found from `ss(A,B,C,D)`.) Compare eigenvalues for CT A and DT G; are both systems stable?

[8pts] b) The continuous time system has state feedback such that $u(t) = r(t) - K\mathbf{x}(t)$. Find K such that closed loop poles are at $-10, -3 \pm 6j$. Plot the closed-loop step response using Matlab.

[6pts] c) Consider the CT system having state feedback applied in discrete time using a D/A converter such that $u[n] = u(nT) = r[n] - K\mathbf{x}[n]$ using the K found above. Thus the closed loop DT system has state equation $x[n+1] = (G - HK)x[n] + Hr[n]$, $y[n] = Cx[n]$. Plot the step response for the closed loop step response on the same axes as the CT step response of part b).

[6pts] d) Explain why the state feedback for the DT system does not look like the DT version of $\dot{\mathbf{x}} = (A - BK)\mathbf{x} + Br$.

[6pts] e) Use Matlab (iteratively if necessary) to find a sampling period T_s which gives a closed-loop step response for DT that is "reasonably close" (say within 5%) to the CT closed-loop step response. Determine DT closed-loop pole locations, and plot the DT step response on same axes as part c.

5. (20 pts) Transient performance using gain compensation (Nise 13.8,13.9)
Given a CT plant $G(s) = \frac{K}{(s+5)(s+1)}$.
[6pts] a. With sample period $T = 0.25$, find $G(z)$, the Z transform of $G(s)$.
[4pts] b. Sketch the root locus for $G(z)$ in unity gain feedback, and find the range of K for stability.
[4pts] c. With unity gain feedback, find the value of K for damping factor $\zeta = 0.6$, and note the K in root locus for CT and DT.
[6pts] d. Plot step response for the closed-loop CT and DT system in Matlab, and compare.