Due at 17:00, Fri. Feb. 3 in homework box under stairs, first floor Cory .

Note: up to 2 students may turn in a single writeup. Reading Nise 2, 3, 4-4.5.

1. (15 pts) Equivalent Circuit (Nise 2.9)

Draw the equivalent electrical circuit for the system shown in in Fig. 1.

2. (25 pts) Rotary Mechanical System (Nise 2.6, 2.7, 3.4, 3.5)

[10pts] a. Find the transfer function for the system shown in Fig. 2, $\frac{\theta_2(s)}{T(s)}$.

[10pts] b. Write the state space equations for this system in phase-variable form and find A, B, C, D.

[5pts] c. Draw the equivalent block diagram of the system in phase variable form using integrator, scale, and summing blocks.

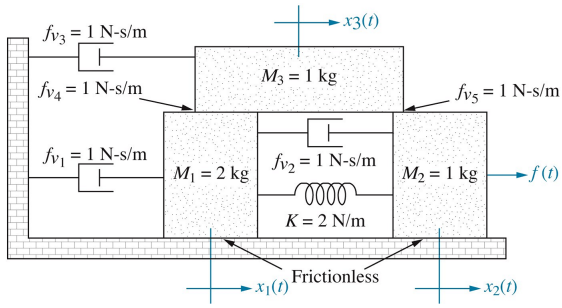

Fig. 1

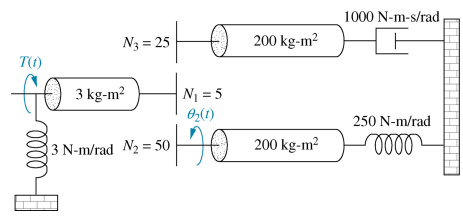

Fig. 2

3. (10 pts) Linearization (Nise 2.11)

A system is described by $f(t) = m\ddot{x}(t) + b\dot{x}(t) + f_s(x,t)$, where f_s is the force from a non-linear spring. The spring force is defined by $f_s(x,t) = e^{100x} - 1$ where $x(t)$ is the spring displacement. Find the transfer function $X(s)/F(s)$ for small excursions around $x = 0$.

4. (15 pts) State Space (Nise 3.4, 3.5)

Given $\frac{Y(s)}{U(s)} = \frac{4s+5}{s^4+5s^3+s^2+5s+13}$, write the state space equations for this system in phase-variable form and find A, B, C, D. (Hint: long division).

5. (15 pts) Transfer function from state space (Nise 3.6)

Find the transfer function $Y(s)/U(s)$ for the following systems:

[5pts] a.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u(t) \qquad \text{and} \quad y = [1 \quad 0 \quad 0]\mathbf{x}$$

[10pts] b.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -8 & 2 & -3 \\ 0 & 5 & 3 \\ -3 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ 3 \end{bmatrix} u(t) \qquad \text{and} \quad y = [1 \quad 3 \quad 2]\mathbf{x}$$

6. (20 pts) Linearization (Nise 3.7)

For the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} u \log_e x_2 \\ \cos(\pi(u + x_4)) \\ \sin(\pi(u + x_2)) \\ x_3^2 + x_1^2 \end{bmatrix} \tag{1}$$

Linearize the system about $x_1 = x_2 = 1, x_3 = 2, x_4 = \frac{-1}{2}$, $u = 1$, and express in state space form:

$\dot{\delta}\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u$.