Lecture #12

OUTLINE

– pn Junctions
  • narrow-base diode
  • charge-control model

Reading: Finish Chapter 6

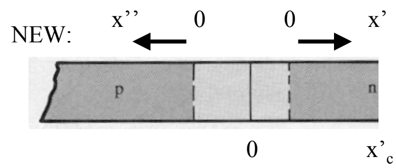
Carrier Concentration Profiles

Narrow or Short-Base Diode

- We have the following boundary conditions:

$$\Delta p_n(x_n) = p_{no}(e^{qV_A/kT} - 1) \qquad \Delta p_n(x' = x_c') \to 0$$

- With the following coordinate system:



- Then, the solution is of the form:

$$\Delta p(x) = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p}$$

Applying the boundary conditions, we have:

$$\Delta p_n(0) = A_1 + A_2$$
$$0 = A_1 e^{x_c'/L_p} + A_2 e^{-x_c'/L_p}$$

- So, we have:

$$\Delta p_n(x') = p_{n0}(e^{qV_A/kT} - 1)\left(\frac{e^{(x_c' - x')/L_P} - e^{-(x_c' - x')/L_P}}{e^{x_c'/L_P} - e^{-x_c'/L_P}}\right), \ 0 < x' < x_c'$$

- Note: $\sinh(\xi) = \frac{e^{\xi} - e^{-\xi}}{2}$

- So:

$$\Delta p_n(x') = p_{n0}(e^{qV_A/kT} - 1)\left(\frac{\sinh\left[\left(x_c' - x'\right)/L_P\right]}{\sinh\left[x_c'/L_P\right]}\right),\ 0 < x' < x_c'$$

- If we write:

$$I = I_0'(e^{qV_A/kT} - 1)$$

- Then

$$I_0' = qA\frac{D_P}{L_P}\frac{n_i^2}{N_D}\frac{\cosh\left(x_c'/L_P\right)}{\sinh\left(x_c'/L_P\right)}$$

where $\cosh(\xi) = \frac{e^{\xi}+e^{-\xi}}{2}$

Note: $\sinh(\xi) \to \xi$ as $\xi \to 0$ **and** $\cosh(\xi) \to 1+\xi^2$ as $\xi \to 0$

- **If $x_c' << L_P$:**

Narrow Base Diode: *I-V* Equation

Let $W_N \equiv \text{width of n-type region}$

$W_P \equiv \text{width of p-type region}$

and $W_N' \equiv W_N - x_n << L_P$

$W_P' \equiv W_P - x_p << L_N$

Then, $$J = qn_i^2\left[\frac{D_P}{W_N' N_D} + \frac{D_N}{W_P' N_A}\right]\left(e^{qV_A/kT} - 1\right)$$

$$I = qAn_i^2\left[\frac{D_P}{W_N' N_D} + \frac{D_N}{W_P' N_A}\right]\left(e^{qV_A/kT} - 1\right) = I_0\left(e^{qV_A/kT} - 1\right)$$

Current Flow in a One-Sided pn Junction

- Note that the diode current is dominated by the term associated with the more lightly doped side:

p^+n diode: $$I_0 \cong I_P(x_n) = \begin{cases} qAn_i^2\left(\dfrac{D_P}{L_P N_D}\right) & \text{long n-side} \\ qAn_i^2\left(\dfrac{D_P}{W_N' N_D}\right) & \text{short n-side} \end{cases}$$

pn^+ diode: $$I_0 \cong I_N(-x_p) = \begin{cases} qAn_i^2\left(\dfrac{D_N}{L_N N_A}\right) & \text{long p-side} \\ qAn_i^2\left(\dfrac{D_N}{W_P' N_A}\right) & \text{short p-side} \end{cases}$$

i.e. **current flowing across junction is dominated by carriers injected from the more heavily doped side**

Excess Carrier Profiles: Limiting Cases

Long base ($x_c' \rightarrow \infty$):

$$\Delta p_n(x') = p_{n0}(e^{qV_A/kT} - 1)\left(\frac{e^{(x_c' - x')/L_P} - e^{-(x_c' - x')/L_P}}{e^{x_c'/L_P} - e^{-x_c'/L_P}}\right)$$

$$= p_{n0}(e^{qV_A/kT} - 1)\left(\frac{e^{x_c'/L_P}e^{-x'/L_p} - e^{-x_c'/L_P}e^{x'/L_p}}{e^{x_c'/L_P} - e^{-x_c'/L_P}}\right)$$

$$\cong p_{n0}(e^{qV_A/kT} - 1)e^{-x'/L_p}$$

2. Short base ($x_c' \rightarrow 0$):

$$\Delta p_n(x') = p_{n0}(e^{qV_A/kT} - 1)\left(\frac{\sinh\left[(x_c' - x')/L_P\right]}{\sinh\left[x_c'/L_P\right]}\right)$$

$$= p_{n0}(e^{qV_A/kT} - 1)\left(\frac{(x_c' - x')/L_P}{x_c'/L_P}\right) = p_{n0}(e^{qV_A/kT} - 1)\left(1 - \frac{x'}{x_c'}\right)$$

Δp_n is a linear function of x

→ J_p is constant (no recombination)

Minority-Carrier Charge Storage

- When V_A>0, excess minority carriers are stored in the quasi-neutral regions of a pn junction:

$$Q_N = -qA\int_{-x_p}^{-\infty} \Delta n_p(x)dx \qquad\qquad Q_P = qA\int_{x_n}^{\infty} \Delta p_n(x)dx$$

$$= -qA\Delta n_p(-x_p)L_N \qquad\qquad = qA\Delta p_n(x_n)L_P$$

Derivation of Charge Control Model

- Consider a forward-biased pn junction. The total excess hole charge in the n quasi-neutral region is:

$$Q_P = qA\int_{x_n}^{\infty} \Delta p_n(x,t)dx$$

- The minority carrier diffusion equation is (without G_L):

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

- Since the electric field is very small,

$$J_P = -qD_P \frac{\partial \Delta p_n}{\partial x}$$

- Therefore: $\frac{\partial (q\Delta p_n)}{\partial t} = -\frac{\partial J_P}{\partial x} - \frac{q\Delta p_n}{\tau_p}$

(Long Base Diode)

- Integrating over the n quasi-neutral region:

$$\frac{\partial}{\partial t}\left[qA\int_{x_n}^{\infty}\Delta p_n dx\right] = -A\int_{J_p(x_n)}^{J_P(\infty)} dJ_P - \frac{1}{\tau_p}\left[qA\int_{x_n}^{\infty}\Delta p_n dx\right]$$

- Furthermore, in a p⁺n junction:

$$-A\int_{J_p(x_n)}^{J_P(\infty)} dJ_P = -AJ_P(\infty) + AJ_P(x_n) = AJ_P(x_n) \cong i_{DIFF}$$

- So: $\dfrac{dQ_P}{dt} = i_{DIFF} - \dfrac{Q_P}{\tau_p}$

Charge Control Model

We can calculate pn-junction current in 2 ways:

1. From slopes of $\Delta n_p(-x_p)$ and $\Delta p_n(x_n)$
2. From steady-state charges Q_N, Q_P stored in each excess-minority-charge distribution:

$$\frac{dQ_P}{dt} = AJ_P(x_n) - \frac{Q_P}{\tau_p} = 0$$

$$\Rightarrow AJ_P(x_n) = I_P(x_n) = \frac{Q_P}{\tau_p}$$

$$\text{Similarly, } I_N(-x_p) = \frac{-Q_N}{\tau_n}$$

Charge Control Model for Narrow Base

- For a narrow-base diode, replace τ_p and/or τ_n by the ***minority-carrier transit time*** τ_{tr}
  - time required for minority carrier to travel across the quasi-neutral region
  - For holes on narrow n-side:

$$Q_P = qA\int_{x_n}^{W_N} \Delta p_n(x)dx = qA\frac{1}{2}\Delta p_n(x_n)W_N'$$

$$I_P = AJ_P = -qAD_P\frac{d\Delta p_n}{dx} = qAD_P\frac{\Delta p_n(x_n)}{W_N'}$$

$$\Rightarrow \quad \tau_{tr} = \frac{Q_P}{I_P} = \frac{(W_N')^2}{2D_P}$$

  - Similarly, for electrons on narrow p-side: $\tau_{tr} = \dfrac{(W_P')^2}{2D_N}$

Summary: Narrow (Short) Base Diode

- If the width of the quasi-neutral region (*e.g.* x_c') is **not** much larger than the minority-carrier diffusion length (*e.g.* L_P), then the solution to the minority-carrier diffusion equation is

$$\Delta p_n(x') = \Delta p_n(x_n)\frac{\sinh\left(\dfrac{x_c' - x}{L_P}\right)}{\sinh\left(\dfrac{x_c'}{L_P}\right)}$$

- If $x_c' < 0.1L_P$:

$$\Delta p_n(x') \cong \Delta p_n(x_n)\left(1 - \frac{x'}{x_c'}\right)$$

- If L_P >> x_c', negligible recombination occurs
  $\Rightarrow$ hole current is constant throughout n-type region

$$J_p = -qD_P \frac{d\Delta p_n}{dx} = qD_P \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)\left(\frac{1}{x_c'}\right)$$

- Compare this to the hole current contribution in the long-base diode:

$$J_p = qD_P \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)\left(\frac{1}{L_P}\right)$$