Lecture #19

ANNOUNCEMENT

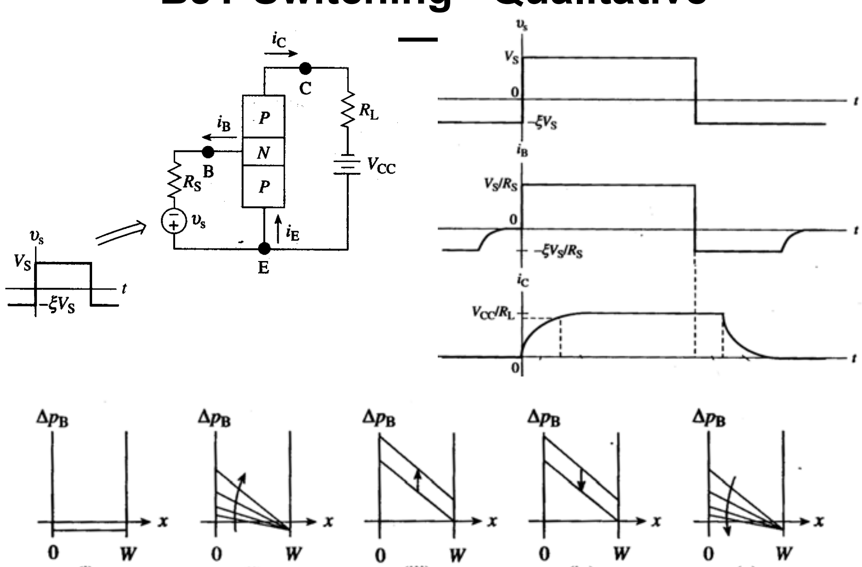
- Quiz #4 (Thursday 4/3) to cover Chapters 10 & 11

OUTLINE

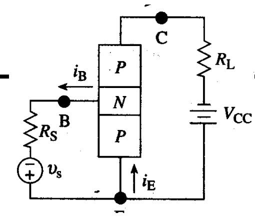
- BJT transient response
- BJT small-signal model, f_T

Reading: Chapter 12

BJT Switching - Qualitative

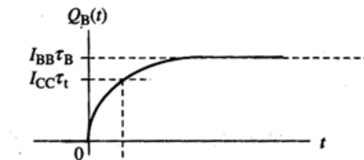


Turn-on transient



- We know: $\frac{dQ_B}{dt} = I_{BB} - \frac{Q_B}{\tau_B}$ where $I_{BB}=V_S/R_S$
- The general solution is:

$$Q_B(t) = I_{BB}\tau_B + Ae^{-t/\tau_B}$$

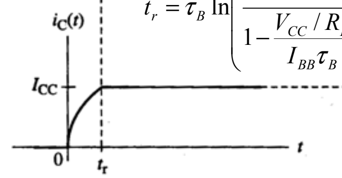


- Initial condition: $Q_B(0)=0$. since transistor is in cutoff

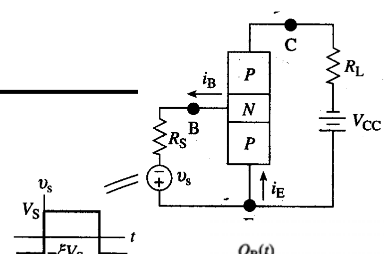
$$Q_B(t) = I_{BB}\tau_B(1 - e^{-t/\tau_B})$$

$$t_r = \tau_B \ln\left(\frac{1}{1 - \frac{V_{CC}/R_L}{I_{BB}\tau_B}}\right)$$

$$i_C(t) = \begin{cases} \frac{Q_B(t)}{\tau_t} = \frac{I_{BB}\tau_B + Ae^{-t/\tau_B}}{\tau_t} \cdots 0 \le t \le t_r \\ \frac{V_{CC}}{R_L} \cdots t \ge t_r \end{cases}$$

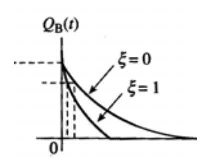


Turn-off transient



- We know: $\frac{dQ_B}{dt} = -\xi I_{BB} - \frac{Q_B}{\tau_B}$
- The general solution is:

$$Q_B(t) = -\xi I_{BB}\tau_B + Ae^{-t/\tau_B}$$

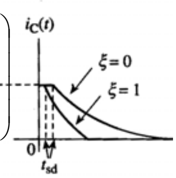


- Initial condition: $Q_B(0)=I_{BB}\tau_B$

$$Q_B(t) = I_{BB}\tau_B\left[(1+\xi)e^{-t/\tau_B} - \xi\right]$$

$$i_C(t) = \begin{cases} I_{CC} \cdots 0 \le t \le t_{sd} \\ \frac{Q_B(t)}{\tau_t} = \frac{I_{BB}\tau_{B\left[(1+\xi)e^{-t/\tau_B} - \xi\right]}}{\tau_t} \cdots t \ge t_{sd} \end{cases}$$

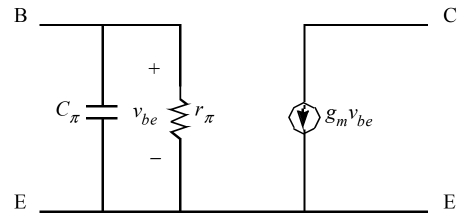
$$t_{sd} \cong \tau_B \ln\left(\frac{1+\xi}{\frac{I_{CC}\tau_t}{I_{BB}\tau_B} + \xi}\right)$$



Small-Signal Model

Forward-active mode,
Common-emitter configuration:

$$I_C = \alpha_F I_F e^{qV_{BE}/kT}$$



transconductance:

$$g_m \equiv \frac{dI_C}{dV_{BE}} = \frac{d}{dV_{BE}}\left(\alpha_F I_F e^{qV_{BE}/kT}\right)$$

$$= \frac{q}{kT}\alpha_F I_F e^{qV_{BE}/kT} = I_C/(kT/q)$$

At 300 K, for example, $g_m = I_C/26\text{mV}$.

$$\boxed{g_m = I_C/(kT/q)}$$

Small-Signal Model (cont.)

$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{BE}} = \frac{1}{\beta_{dc}}\frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta_{dc}}$$

$$r_\pi = \frac{\beta_{dc}}{g_m} r_\pi = \frac{\beta_{dc}}{g_m}$$

$$C_\pi = \frac{dQ_F}{dV_{BE}} = \frac{d(\tau_F I_C)}{dV_{BE}} = \tau_F g_m$$

This is the minority-carrier charge-storage capacitance, better known as the ***diffusion capacitance***.

Add the depletion-layer capacitance, C_{JBE} :

$$\boxed{C_\pi = \tau_F g_m + C_{dBE}}$$

Forward Transit Time τ_F

$$\tau_F = \tau_E + \tau_{BE} + \tau_t + \tau_{BC} = \frac{Q_F}{I_C}$$

where

$\tau_E =$ emitter delay time

$\tau_{BE} =$ emitter-base depletion region transit time

$\tau_t =$ base transit time

$\tau_{BC} =$ base-collector depletion-region transit time

- To reduce the forward transit time, the emitter as well as the depletion layers must be kept thin.

Example: Small-Signal Model Parameters

A BJT is biased at I_C = 1 mA and V_{CE} = 3 V. β_{dc}=90, τ_F=5 ps, and T = 300 K. Find (a) g_m , (b) r_π, (c) C_π.

Solution:

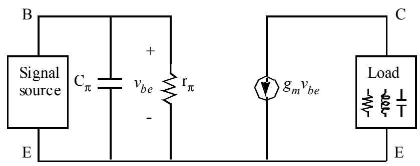
(a) $g_m = I_C/(kT/q) = \dfrac{1\,\text{mA}}{26\,\text{mV}} = 39\,\dfrac{\text{mA}}{\text{V}} = 39\,\text{mS}$ *(milli siemens)*

(b) $r_\pi = \beta_{dc}/g_m =$ 90/0.039 = 2.3 kΩ

c) $C_\pi = \tau_F g_m = 5\times10^{-12}\times0.039 \approx 1.9\times10^{-14}\,\text{F} = 19\,\text{fF}$ *(femto farad)*

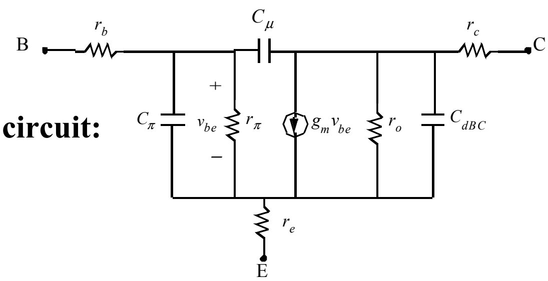
Application of Small-Signal Model

Once the model parameters have been determined, one can analyze circuits with arbitrary source and load impedance.

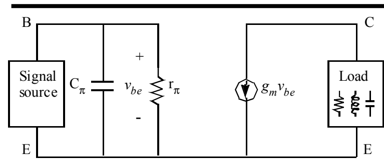


The parameters are routinely determined through comprehensive measurement of the BJT AC and DC characteristics.

Full BJT equivalent circuit:



Cutoff Frequency f_T



$$\beta_{ac} = 1 \text{ at } f_T = \frac{1}{2\pi\left(\tau_F + C_{JBE}kT/qI_C\right)}$$

The load is a short circuit, and the signal source is a current source, i_b, at frequency, f. At what frequency does the a.c. current gain fall to unity?

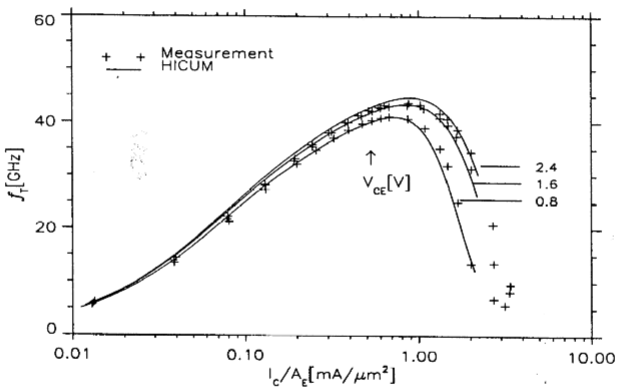
$$v_{be} = \frac{i_b}{\text{input admittance}} = \frac{i_b}{1/r_\pi + j\omega C_\pi}$$

$$i_c = g_m v_{be}$$

$$\beta(\omega) = \left|\frac{i_c}{i_b}\right| = \frac{g_m}{\left|1/r_\pi + j\omega C_\pi\right|} = \frac{1}{\left|1/\beta_F + j\omega\tau_F + j\omega C_{dBE}kT/qI_C\right|}$$

For the full BJT equivalent circuit:

$$f_T = \frac{1}{2\pi\left(\tau_F + \left(C_{dBE} + C_{dBC}\right)kT/\left(qI_C\right) + C_{dBC}\left(r_e + r_c\right)\right)}$$



SiGe HBT by IBM

f_T is commonly used as a metric for the speed of a transistor.

Cutoff Frequency f_T

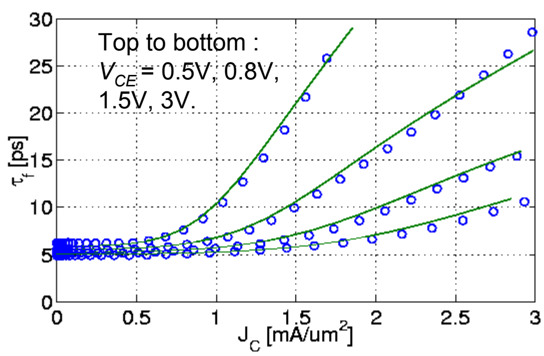
$$f_T = \frac{1}{2\pi\left(\tau_F + \left(C_{dBE} + C_{dBC}\right)kT/\left(qI_C\right) + C_{dBC}\left(r_e + r_c\right)\right)}$$

- To maximize f_t:
  - Increase I_C
  - Minimize C_{dBE}, C_{dBC}
  - Minimize r_e, r_c
  - Minimize τ_F

Base Widening at High I_C: the Kirk Effect

- At very high current densities (> 0.5 mA/μm²), base widening occurs*, so Q_B increases.

  $\rightarrow t_t$ and τ_{BC} increase, so τ_F increases and $\mathbf{f_T}$ **decreases**.



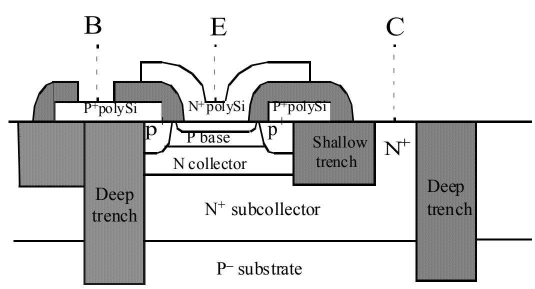
*For an NPN BJT, the electron density in the collector ($n = N_C$) becomes insufficient to support the collector current even if the electrons move at the saturation velocity.

$$I_C = qAnv_{sat}$$

$$\rho_{dep,C} = qN_C - qn = qN_C - \frac{I_C}{Av_{sat}}$$

Eventually, ρ changes sign as I_C increases (for fixed V_{BC}), and the base width is effectively widened.

BJT Structure for High Speed



- Narrow base
- n+ poly-Si emitter
- Self-aligned p+ poly-Si base contacts
- Lightly-doped collector
- Heavily-doped epitaxial subcollector
- Shallow trenches and deep trenches filled with SiO_2 for electrical **isolation**