Lecture 10: Supply & Temperature Indep. Biasing

- **Announcements**:
  - HW#4 due tomorrow at 8 a.m.
  - HW#5 online soon
  - Lab#1 reports are due the week of Oct. 8
    - Turn them in to Yang in your lab section
  - Lab#2 is online
    - This is a hardware lab
    - You must show up to lab for Lab#2
  - Office Hour Change: Yang's Thursday office hours moved to M 2:30-3:30
- **Lecture Topics**:
  - Supply & Temperature Independent Biasing
  - Output Swing
  - Dynamic Range
- ------------------------------------
- **Last Time**: Started supply independent biasing

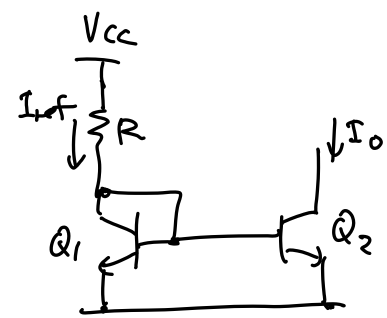
over

Define. Sensitivity of Y to X

$$S_X^Y = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{X}{Y}\frac{\Delta Y}{\Delta X} = \frac{X}{Y}\frac{\partial Y}{\partial X}$$

For supply dependence, we want $S_{V_{CC}}^{I_O} = 0$.

$\boxed{\text{Simple Current Source}}$



Neglecting base currents:

$$I_O = I_{REF} = \frac{V_{CC}-V_{BE(on)}}{R}$$

$$I_O \simeq \frac{V_{CC}}{R} \quad [V_{CC} \gg V_{BE(on)}]$$

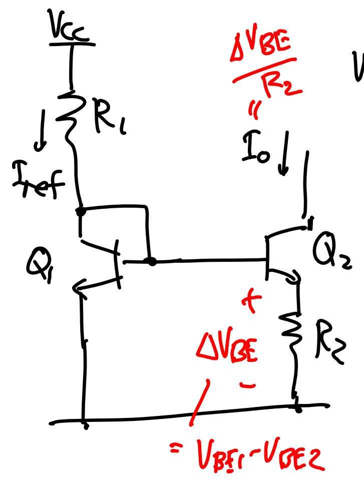
Thus:

$$S_R^{I_O} = \frac{R}{I_O}\frac{\partial I_O}{\partial R} = \frac{R^2}{V_{CC}}\left(-\frac{V_{CC}}{R^2}\right) \Rightarrow S_R^{I_O} = -1$$

$$S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{\partial I_O}{\partial V_{CC}} = R\left(\frac{1}{R}\right) \Rightarrow S_{V_{CC}}^{I_O} = 1$$

∴ a 10% change in V_{CC} leads to 10% change in I_O! terrible!

Widlar Current Source (Any better?)



$$V_T \ln \frac{I_{ref}}{I_o} = I_o R_2 \qquad (\ln I_{ref} - \ln I_o)$$

↓

Differentiate w/r to V_{cc}!

$$V_T\left(\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}} - \frac{1}{I_o}\frac{\partial I_o}{\partial V_{cc}}\right) = R_2 \frac{\partial I_o}{\partial V_{cc}}$$

↓ math

$$\frac{\partial I_o}{\partial V_{cc}} = \frac{\frac{V_T}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}}}{\left(R_2 + \frac{V_T}{I_o}\right)}$$

$$\therefore S^{I_o}_{V_{cc}} = \frac{V_{cc}}{I_o}\frac{\partial I_o}{\partial V_{cc}} = \frac{V_T\left(\frac{V_{cc}}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{cc}}\right)}{I_o R_2 + V_T}$$

$S^{I_{ref}}_{V_{cc}} = 1$

$$\Rightarrow S^{I_o}_{V_{cc}} = \left(\frac{1}{1+\frac{I_o R_2}{V_T}}\right) S^{I_{ref}}_{V_{cc}}$$

Since $I_{ref} = \frac{V_{cc} - V_{BE(on)}}{R_1} \simeq \frac{V_{cc}}{R_1} \Rightarrow S^{I_{ref}}_{V_{cc}} = 1$

$$\therefore S^{I_o}_{V_{cc}} = \frac{1}{1+\frac{I_o R_2}{V_T}}$$

For $I_{ref} = 1mA$, $I_o = 10\mu A$, $R_2 = 11.9k\Omega$, then

10% Δ in V_{cc} → 1.3% Δ in I_o

(better than the simple current source!)

How can we do better?

→ Use another voltage reference: $\{R$ V_{ref} (+ −)

✓ ① V_{BE} → base emitter junction voltage

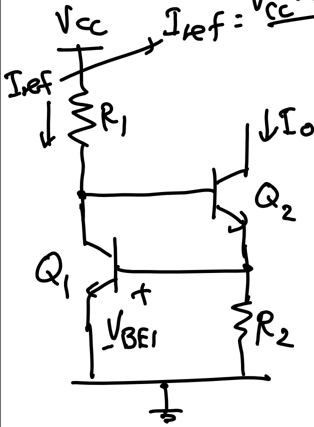
② V_Z → Zener diode

③ V_t → threshold voltage (MOS)

✓ ④ $V_T = \frac{kT}{q}$ → thermal voltage

✓ ⑤ E_g → bandgap

V_{BE}-Referenced Biasing



$I_{ref} = \frac{V_{CC} - 2V_{BE(on)}}{R_1}$

$I_0 = \frac{V_{BE1}}{R_2} = \frac{V_T}{R_2} \ln \frac{I_{ref}}{I_{S1}}$

$\frac{\partial I_0}{\partial V_{CC}} = \frac{V_T}{R_2}\left(\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}} - \frac{1}{I_{S1}}\frac{\partial I_{S1}}{\partial V_{CC}}\right)$

$S_{V_{CC}}^{I_0} = \frac{V_{CC}}{I_0}\frac{\partial I_0}{\partial V_{CC}}$

$= \frac{V_{CC}}{\ln(I_{ref}/I_{S1})}\left(\underbrace{\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}}}_{= \frac{1}{V_{CC}}} - \frac{1}{I_{S1}}\frac{\partial I_{S1}}{\partial V_{CC}}\right)$

Problem: Iref still depends on Vcc

$S_{V_{CC}}^{I_{ref}} \approx 1 = \frac{V_{CC}}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}}$

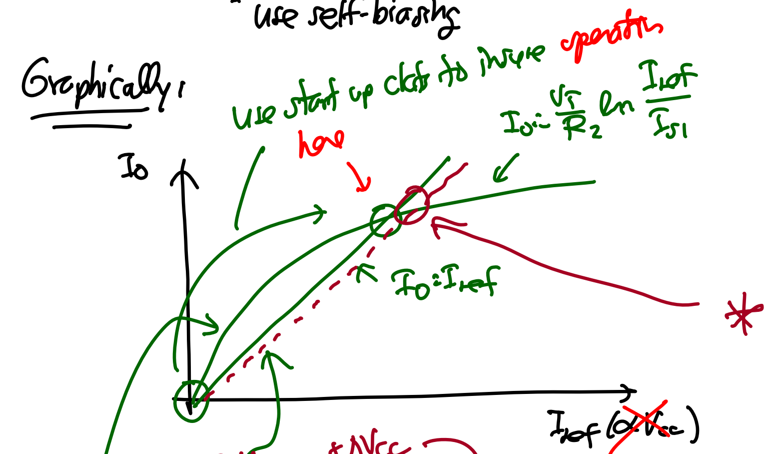
$S_{V_{CC}}^{I_0} = \frac{1}{\ln(I_{ref}/I_{S1})}\left[1 - \underbrace{\frac{V_{CC}}{I_{S1}}\frac{\partial I_{S1}}{\partial V_{CC}}}_{= 0}\right] = \frac{1}{\ln(I_{ref}/I_{S1})}$

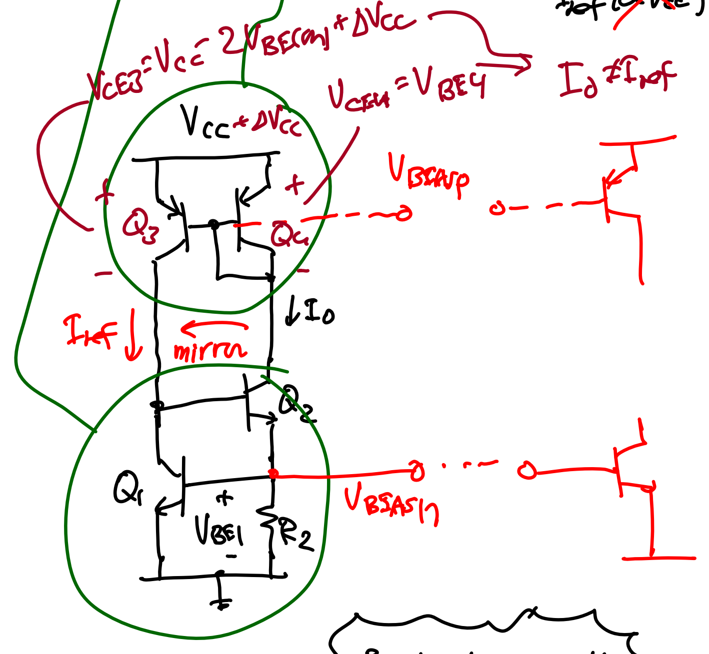
If we can eliminate this "1", then $S_{V_{CC}}^{I_0} = 0$

→ need to eliminate this dependence of Iref on Vcc!

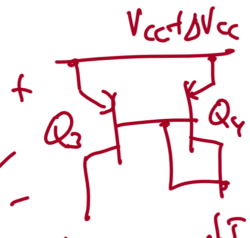
Solution: Derive Iref independently of Vcc.
→ use self-biasing

Graphically:





$S_{V_{CC}}^{I_{ref}} \to S_{V_{CC}}^{I_0} \cong 0$ Pretty darn good!



$|V_{CE4}| = |V_{BE(on)}|$ ← stays still

← $\Delta V_{CE} \approx 0$

$\downarrow I_o = I_s \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$

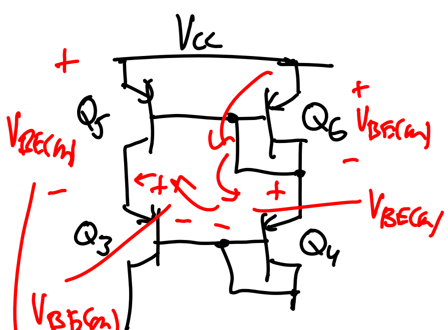
$\downarrow I_{ref}$

$|V_{CE3}| = V_{CC} - 2V_{BE(on)} + \delta V_{CC} \rightarrow \delta V_{CE3} \approx \Delta V_{CC}$

$I_{ref} = I_s \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE3}}{V_A}\right)$

* ← $I_{ref} \neq I_o$ ∴ $S^{I_o}_{V_{CC}}$ no longer 0 (but close)

But what if we want better?



clamped @ $V_{BE(on)}$ → δV_{cc} won't change it

$S^{I_o}_{V_{cc}} \approx 0$!

Determine Temperature Dependence of the V_{BE}-Reference

Define: Fractional Temperature Coefficient

$$TC_F = \frac{1}{I_o}\frac{\partial I_o}{\partial T} \leftarrow \text{temperature}$$

For $I_o = \frac{V_{BE}}{R}$:

$$TC_F = \frac{1}{I_o}\frac{\partial I_o}{\partial T} = \frac{R}{V_{BE}}\left(\frac{1}{R}\frac{\partial V_{BE}}{\partial T} - \frac{V_{BE}}{R^2}\frac{\partial R}{\partial T}\right)$$

$$\therefore \boxed{TC_F = \underbrace{\frac{1}{V_{BE}}\frac{\partial V_{BE}}{\partial T}}_{TC_F|_{V_{BE}}} - \underbrace{\frac{1}{R}\frac{\partial R}{\partial T}}_{TC_F|_R}}$$

Some Typical TC_F's:

parts per million
" 10^{-6}

Diffused R's ~ 1000 – 1500 ppm/°C

poly Si R's ~ 500 ppm/°C

V_{BE} ~ −3300 ppm/°C

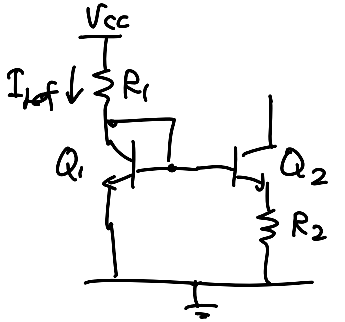
∴ TC_F ~ −3300 − 1000 = −4300 ppm/°C ~ 0.43%/°C

∴ 0 → 70°C: ~ 25% I_o variation

−55 → 125°C: ~60–70% I_o variation

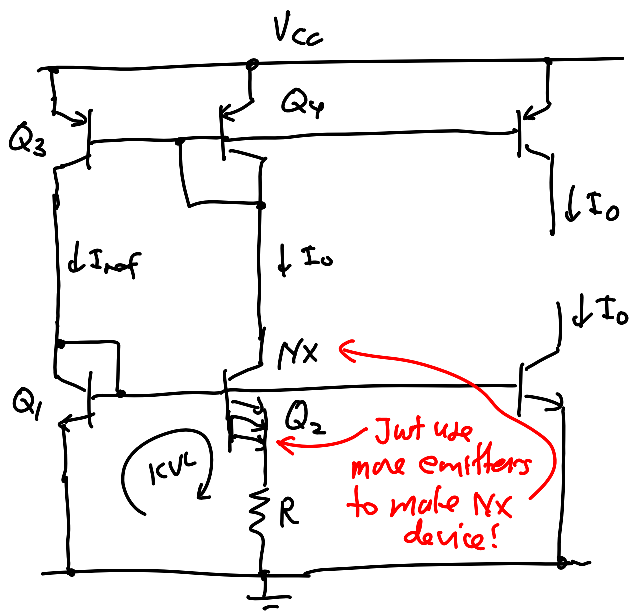
$\frac{kT}{q}$-Referenced Bias Ckt.

Based on the Widlar current source:



Here, $I_{ref} \neq I_o$ & so sensitive to V_{cc} variations ↓ But can fix this by setting $I_{ref} = I_o$.

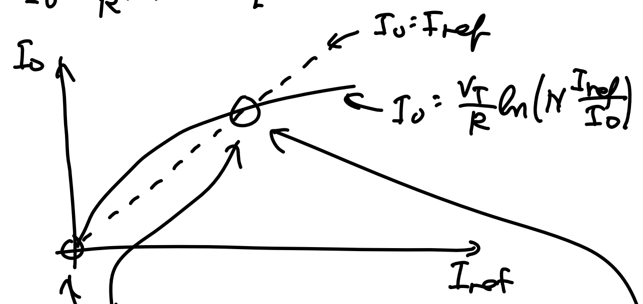
→ Use self-biasing:



Assuming $I_{S2} = N I_{S1}$:

KVL: $I_o R = V_{BE1} - V_{BE2} = V_T \ln N$

$I_o = \frac{V_T}{R}\ln N = \frac{kT}{q}(\text{const.})$ ← at least w/r to V_{cc}



Two possible operating pts.
⇒ use startup ckt to guarantee this one
(where we have very small $S_{V_{cc}}^{I_o}$)

$\frac{kT}{q}$-Reference Temperature Dependence

$$TC_F = \frac{1}{I_o}\frac{\partial I_o}{\partial T} = \cancel{\ln N}\left(\frac{1}{R}\frac{\partial V_T}{\partial T} - \frac{V_T}{R^2}\frac{\partial R}{\partial T}\right)\frac{R}{V_T \cancel{\ln N}}$$

$$\therefore TC_F = \frac{1}{V_T}\frac{\partial V_T}{\partial T} - \frac{1}{R}\frac{\partial R}{\partial T}$$

$$= \underbrace{TC_F|_{V_T}}_{3300\,ppm/^\circ C} - \underbrace{TC_F|_R}_{1800\,ppm/^\circ C} \sim 1500\,ppm/^\circ C$$

→ Better than V_{BE}-Ref, but still not zero...

How can get ↗ ?

Bandgap Reference

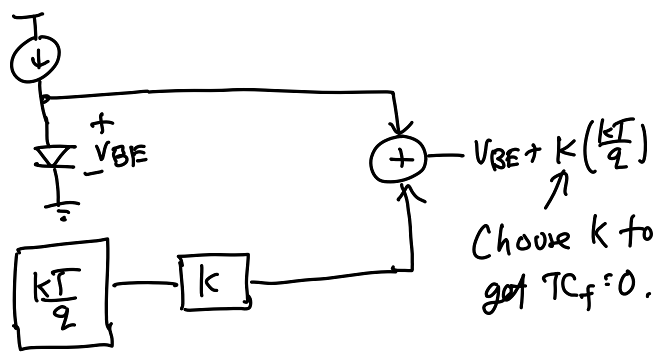
Basic Idea:

$V_{BE} \longrightarrow$ neg. TC_f

$\frac{kT}{q} \longrightarrow$ pos. TC_f

Add together to cancel ⇒ get net $TC_f = 0$!

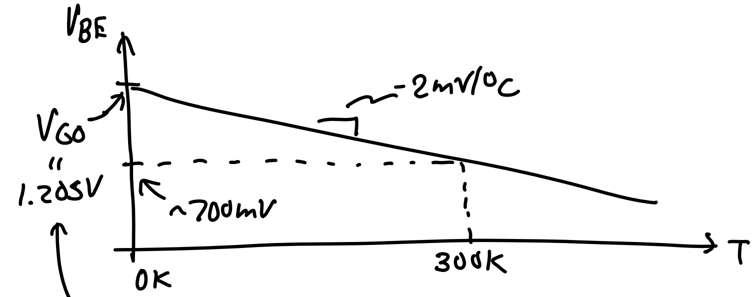
0th Order Picture:



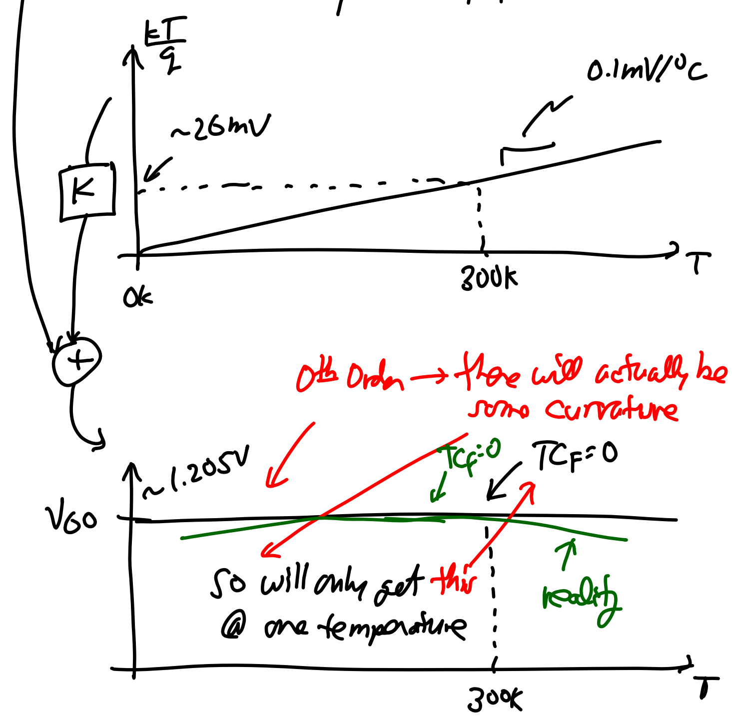
What should k be? (0th Order Picture)

⇒ get ballpark estimate

over

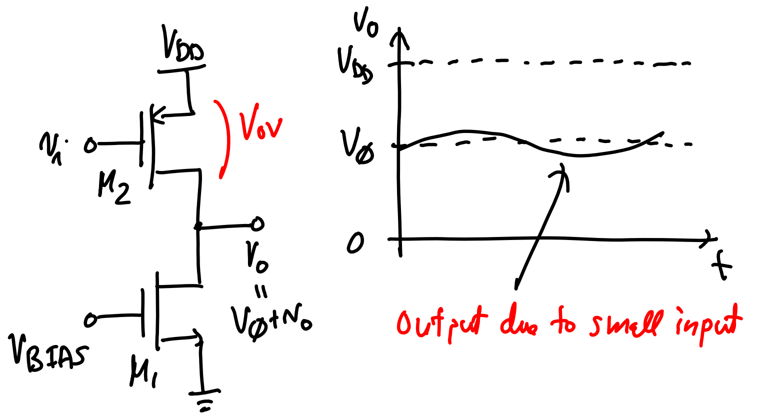


Bandgap voltage obtained via a linear extrapolation to 0K. (Not the same as what one usually sees in physics.)



- 240A folks: read Gray & Meyer
  - Sections 4.4.2 through 4.4.3
  - These cover supply and temperature independent biasing, including bandgap references
  - Can also read Razavi, Chpt. 11, on bandgap references

Output Swing (Headroom)



I_D vs. V_{DS} Characteristic: