Lecture 10: Supply & Temperature Indep. Biasing

• Announcements:
  - HW#4 due tomorrow at 8 a.m.
  - HW#5 online soon
  - Lab#1 reports are due the week of Oct. 8
    - Turn them in to Yang in your lab section
  - Lab#2 is online
    - This is a hardware lab
    - You must show up to lab for Lab#2
  - Office Hour Change: Yang's Thursday office hours moved to M 2:30-3:30

• Lecture Topics:
  - Supply & Temperature Independent Biasing
  - Output Swing
  - Dynamic Range

• Last Time: Started supply independent biasing

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Define: Sensitivity of $Y$ to $X$

$$S_Y = \frac{\Delta Y}{\Delta X} = \frac{X}{Y} \frac{\Delta Y}{\Delta X} = \frac{X}{Y} \frac{\partial Y}{\partial X}$$

For supply dependence, we want $S_{V_{CE}} = 0$.

**Simple Current Source**

$$V_{CC}$$

Neglect base currents:

$I_0 = I_{\text{REF}} = \frac{V_{CC} - V_{BEQ}}{R}$

$I_0 \approx \frac{V_{CC}}{R} \left[ V_{CC} > V_{BEQ} \right]$}

Thus:

$$S_{I_0} = \frac{\partial I_0}{\partial V_{CC}} : V_{CC} \left( \frac{R}{R^2} \right) \Rightarrow S_{I_0} = -1$$

$$S_{V_{CC}} = \frac{\partial I_0}{\partial V_{CC}} : R \left( \frac{1}{R} \right) \Rightarrow S_{V_{CC}} = 1$$

...a 10% change in $V_{CC}$ leads to 10% change in $I_0$... **Terrible!**
Since $I_{\text{ref}} = \frac{V_{cc} - V_{B\text{Eoff}}}{R_1} = \frac{V_{cc}}{R_1} \Rightarrow S_{V_{cc}} = 1$

$$S_{V_{cc}} = \frac{1}{1 + \frac{I_0 R_2}{V_T}}$$

For $I_{\text{ref}} = 1mA$, $I_0 = 10\mu A$, $R_2 = 11.9k\Omega$, then

10% $\Delta$ in $V_{cc} \rightarrow 1.3\% \Delta$ in $I_0$
(better than the simple current source!)

How can we do better?

→ Use an offset voltage reference:

1. $V_{BE}$ → base emitter junction voltage
2. $V_T$ → thermal voltage
3. $V_T$ → threshold voltage (MOS)
4. $V_T = \frac{kT}{I}$ → thermal voltage
5. $E_g$ → bandgap
**V_{BE} - Reference Biasing**

\[ I_{ref} = \frac{V_{cc} - V_{BE}\text{lin}}{R_1} \]

\[ I_0 = \frac{V_{BE1}}{R_2} \left( \frac{1}{I_{ref}} \ln \left( \frac{I_{ref}}{I_{sat}} \right) - 1 \right) \frac{\partial I_{ref}}{\partial V_{cc}} \]

\[ I_0 = \frac{V_{cc}}{R_2} \frac{\partial I_{ref}}{\partial V_{cc}} \]

Problems:
- \( I_{ref} \) depends on \( V_{cc} \)
- \( S_{V_{cc}} = \frac{1}{\ln(I_{ref}/I_{sat})} \left( 1 - \frac{V_{cc} \partial I_{ref}}{I_{sat} \partial V_{cc}} \right) \)

**Solutions: Desired \( I_{ref} \) independent of \( V_{cc} \)**

Use self-biasing

Graphically:

- Use shunt of VBE to move operation
- Use \( I_0 = I_{ref} \)

**Derivation**

\[ V_{CE} = V_{cc} - V_{BE} + V_{BEQ} \]

\[ I_0 = I_{ref} \]

\[ S_{V_{cc}} = 0 \]

Pretty damn good!
Determine Temperature Dependence of $I_E$

VBE Reference

Define Fractional Temperature Coefficient

$$T_C = \frac{1}{I_o} \frac{dT}{dT}$$

For $I_o = V_{BE}/R$:

$$T_C = \frac{1}{I_o} \frac{dT_{V_{BE}}}{dT} - \frac{1}{R} \frac{dR}{dT}$$

$$T_C = \frac{1}{I_o} \frac{dV_{BE}}{dT} - \frac{1}{R} \frac{dR}{dT}$$

Some Typical $T_C$'s:

- Diffused $R_s \sim 1000 - 1500$ ppm/$^\circ$C
- Poly $R_s \sim 500$ ppm/$^\circ$C
- $V_{BE} \sim 3300$ ppm/$^\circ$C

- $T_C \sim -3300 - 1600$
- $-4200$ ppm/$^\circ$C $\sim 0.4870^\circ$C

- $0 \to 70^\circ$C: $\sim 25\%$ $I_o$ variation
- $55 \to 125^\circ$C: $\sim 60 - 70\%$ $I_o$ variation
\[ I_{q2} - \text{Reference Bias Clip} \]

Based on the Widlar current source:

\[ I_{ref} = \frac{V_{cc}}{R_1} \]

Here, \( I_{ref} \) is \( I_o \).

\[ I_0 \] so sensitive to

\[ V_{cc} \] variations

But can fix this by

\[ \text{setting} \ I_{ref} = I_0. \]

Use self-biasing:

\[ V_{cc} \]

Assuming \( I_{ref} = N I_o \):

KVL:

\[ I_o R = V_{BE1} - V_{BE2} = V_T \ln N \]

\[ I_o = \frac{V_T}{R} \ln N \]

\[ I_{ref} = \frac{k T}{q} \text{(const)} \]

\[ I_0 = I_{ref} \text{ if } \]

Two possible operating pts.

\[ \text{we startup clip to guarantee} \]

\[ \text{this one} \]

(\( \text{why we have very small} \ I_o \))

Just use

\[ \text{more emitters to make } N \text{X device!} \]

\[ \frac{1}{2} \text{- Reference Temperature Dependence} \]

\[ TC_f = \frac{\frac{\partial I_o}{\partial T}}{I_o} = \ln N \left( \frac{1}{R} \frac{\partial V_T}{\partial T} - \frac{V_T}{R} \frac{\partial R}{\partial T} \right) \frac{R}{V_T \ln N} \]

\[ TC_f = \frac{1}{V_T} \frac{\partial V_T}{\partial T} - \frac{1}{R} \frac{\partial R}{\partial T} \]

\[ = TC_f V_T - TC_f R \sim 1500 \text{ ppm/°C} \]

\[ \frac{3300 \text{ ppm/°C}}{1800 \text{ ppm/°C}} \]

Better than \( V_{BE}-\text{Ref} \), but still not zero...

How can get \[ \text{?} \]
Bandgap Reference

Basic Idea:

- $V_{BE} \rightarrow \text{neg. } TCF$
- $\frac{kT}{q} \rightarrow \text{pos. } TCF$
- Add together to cancel $\rightarrow$ get $\textit{not } TCF = 0$!

0th Order Picture:

$$
\begin{align*}
V_{BE} + k \left( \frac{kT}{q} \right) & \\
& \approx k
\end{align*}
$$

Choose $k$ to get $TCF = 0$.

What should $k$ be? (0th Order Picture)

$\Rightarrow$ get ballpark estimate

\[ \Rightarrow \text{ over} \]

Bandgap voltage obtained via a linear extrapolation to 0K. (Not the same as what one usually sees in physics)

\[ V_{BE} \approx 1.205V \]

\[ V_{GO} \approx 0.105V \text{ at } T = 800K \]

\[ \frac{kT}{q} \approx 0.26mV \]

\[ k \approx \frac{1.205V}{300K} \]

\[ \frac{kT}{q} \approx 0.70mV \]

\[ V_{GO} \approx 0.105V \text{ at } T = 800K \]

\[ k \approx \frac{1.205V}{300K} \]

\[ \frac{kT}{q} \approx 0.26mV \]

\[ k \approx \frac{1.205V}{300K} \]

\[ \frac{kT}{q} \approx 0.70mV \]
240A folks: read Gray & Meyer

- Sections 4.4.2 through 4.4.3
- These cover supply and temperature independent biasing, including bandgap references
- Can also read Razavi, Chpt. 11, on bandgap references

Output Supply (Headroom)

$I_D$ vs. $V_{DS}$ Characteristic:

- Skew: $\frac{1}{r_o}$ = large
- $r_o$ = small

Linear

Saturation

Overlap Voltage ($V_{OL}$)

Gain ~ $g_m r_o$ → $g_m r_o < g_m r_o$