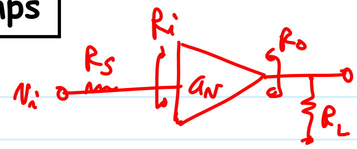
EE 140 | Ideal Op Amps | CTN | 1



Gain: $\frac{R_i}{R_i+R_s} a_v \frac{R_L}{R_L+R_o}$ — $R_i \to \infty$, $R_o \to 0$ — Gives max gain

Ideal Voltage Amplifier

→ ideal when $\frac{v_o}{v_s} = A_v$; i.e., when source and load R's do not influence the gain of the amplifier.

For this to occur, the voltage division at the input & output must be eliminated. This happens when:

$R_i = \infty$, $R_o = 0$ } These resistance values define an ideal voltage amplifier.

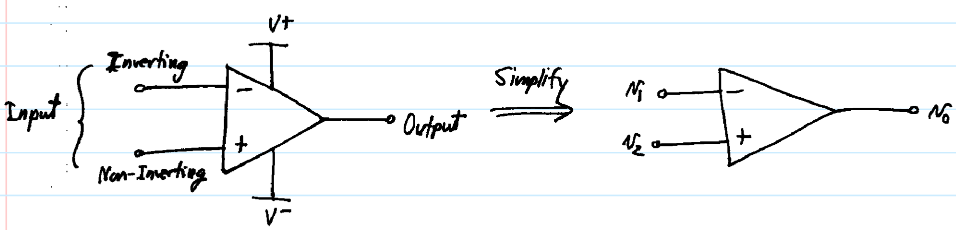
We'll look at other amplifier types later.

→ This, then, naturally leads us to:

Ideal Operational Amplifiers (Op Amps)

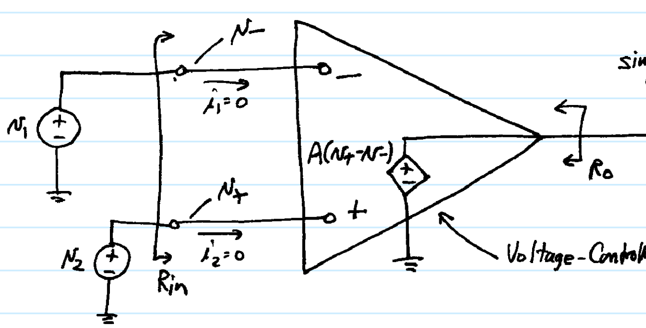
→ The work horse of analog electronics → combination of op amps w/ feedback components allow the implementation of analog computers, sampled-data systems, analog filters, A/D Converters, DAC's, instrumentation amplifiers

In general, have a minimum of 5 terminals:



Perhaps the best way to define an op amp is thru its equivalent ckt:

Equivalent Ckt. of an Ideal Op Amp:



single-ended output ↓ differential input ↓

$v_o = A(v_+ - v_-)$

$= A(v_2 - v_1)$

Voltage-Controlled Voltage Source (VCVS)

EE 140 **Ideal Op Amps** CTN (2)

Properties of Ideal Op Amps:

(1) $R_{in} = \infty$ $\xrightarrow{\text{leads to}}$ (4) $i_+ = i_- = 0$

(2) $R_o = 0$

(3) $A = \infty$ $\xrightarrow[\text{leads to}]{}$ (5) $v_+ = v_-$, assuming v_o = finite

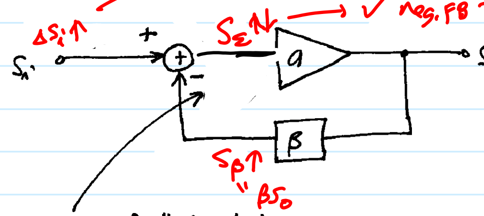
↳ Why? Because for $\infty(v_+ - v_-) = v_o$ = finite

$\therefore \underbrace{v_+ - v_-}_{= \frac{v_o}{\infty}} = 0 \rightarrow v_+ = v_-$

⇒ virtual short ckt. (virtual ground)

Big assumption! (v_o = finite)
How can we assume this? ⟹ only when there is an appropriate negative feedback path!

$\boxed{\text{Negative Feedback}}$ perturbation analysis to determine (−) or (+) FB



where S could be a current, voltage, displacement, etc., . . .

Negative feedback acts to oppose or subtract from input.

$$\left.\begin{aligned} S_o &= a S_\varepsilon \\ S_\varepsilon &= S_i - \beta S_o \end{aligned}\right\} \Longrightarrow \begin{aligned} S_o &= a(S_i - \beta S_o) \\ S_o(1+a\beta) &= a S_i \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1+a\beta}} \end{aligned}$$

overall transfer function

$$[a \to \infty] \Longrightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$$

$$\therefore S_o = \frac{1}{\beta} S_i = \text{finite} \checkmark$$

(when there is neg. FB around the amplifier)

In Summary:

(1) Neg. FB can insure S_o = finite even with $a = \infty$.

(2) Overall Gain dependent (or overall T.F.) dependent only on external components. (e.g., β)

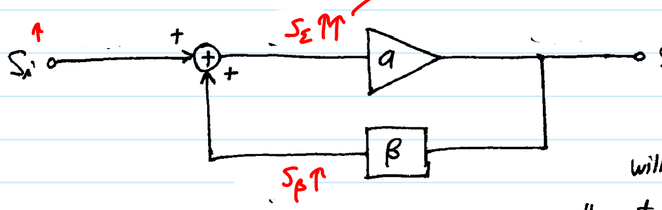
(3) Overall (closed-loop) gain $\frac{S_o}{S_i}$ is independent of amplifier gain a.

↖ very important! ⇒ as you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain. i.e., if you're shooting for a = 50,000, you might get 47,000 or 60,000 instead.

EE 140 | Op Amp Circuits | CTN | 3

Contrast w/ Positive Feedback

$\infty \rightarrow$ pos FB $\rightarrow$ can't use ideal op amp rules!



$\Rightarrow$ output blows up! (for $a\beta > 1$)

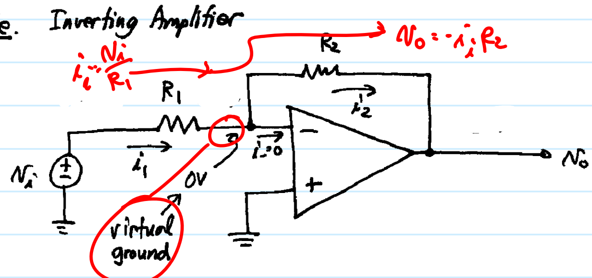
will be the case for $a = \infty$

$\Rightarrow$ usually get oscillation!

Thus, for a bounded, controllable function, need negative FB around an op amp.

Op Amp Ckts.

Example. Inverting Amplifier



① Verify that there is negative FB. ✓

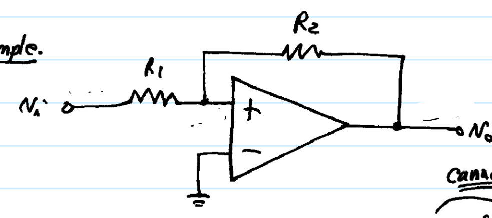
② $\therefore v_o =$ finite $\rightarrow v_+ = v_- \rightarrow$ node attached to (-) terminal is virtual ground

③ $i_- = 0 \;\therefore\; i_1 = i_2$

$$\left.\begin{array}{l} i_1 = \frac{v_i - 0}{R_1} = \frac{v_i}{R_1} = i_2 \\ v_o = 0 - i_2 R_2 = -i_2 R_2 \end{array}\right\} \Rightarrow v_o = -\left(\frac{v_i}{R_1}\right) R_2 = -\frac{R_2}{R_1} v_i \quad \therefore \boxed{\frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$

Note: Gain dependent only on R_1 & R_2 (external components), not on the op amp gain.

Example.



$v_o = L^+$ or L^- depending on initial conds.

$v_+ = (+) \rightarrow L^+$

$v_+ = (-) \rightarrow L^-$.

① Verify that there is neg. FB ✗

Cannot analyze using ideal op amp method!

$\therefore v_o \neq$ finite, $v_+ \neq v_-$

$\Rightarrow$ this ckt. will "rail out"

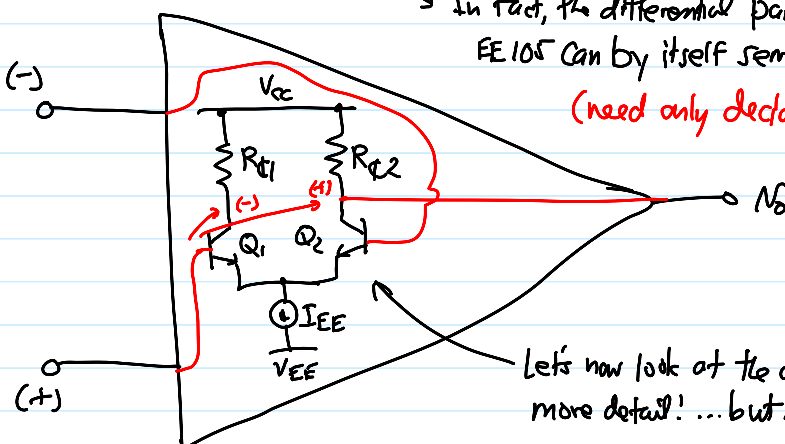
EE 140 | Basic Op Amp Design | CTN | 4

How does one make an op amp? (It turns out, you already know!)

⇒ Basic Needed Attributes:

1. Gain (voltage gain).
2. Two inputs, (+) and (−).
3. One output equal to the difference of the inputs multiplied by some gain.

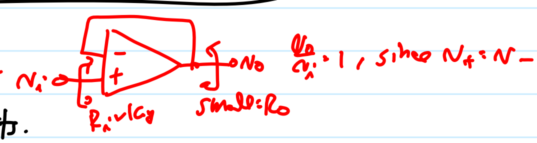
In fact, the differential pair you studied in EE105 can by itself serve as an op amp! (need only decide which of the inputs is the (+) and (−) terminals)



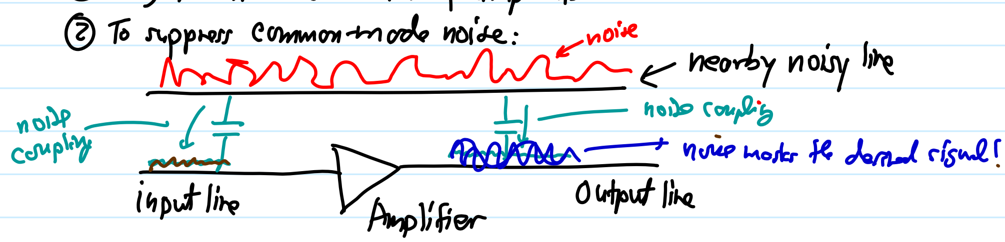
Let's now look at the diff. pair in more detail! ... but first

Why have 2 inputs?

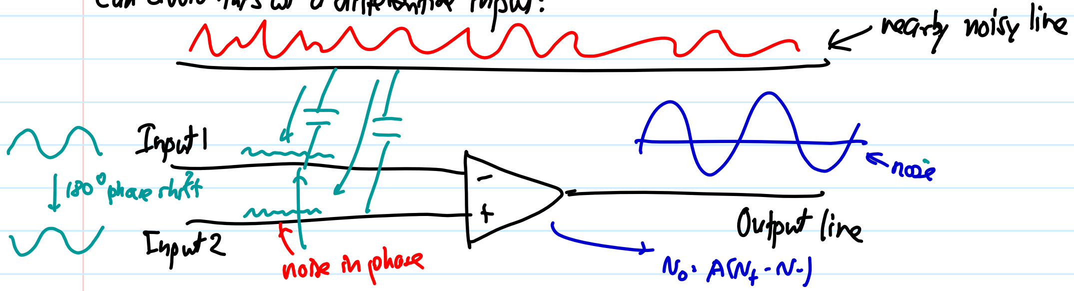
1. To get a virtual short for op amp ckts.



2. To suppress common-mode noise:

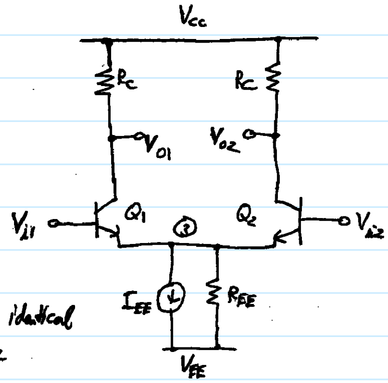


Can avoid this w/ a differential input:



EE 140 — Differential Pair (Bipolar) — CTN — 5

Differential Pair (Emitter-Coupled Pair)



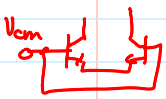
Assume: Q_1 & Q_2 identical, $R_{c1} = R_{c2}$

Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

Definition: $V_{id} = V_{i1} - V_{i2}$ (differential input)

$V_{icm} = \dfrac{V_{i1} + V_{i2}}{2}$ (common-mode input)

$$\Rightarrow \begin{cases} V_{i1} = V_{icm} + \dfrac{V_{id}}{2} \\ V_{i2} = V_{icm} - \dfrac{V_{id}}{2} \end{cases}$$



Differential Gain $= A_d = \dfrac{V_{o1} - V_{o2}}{V_{id}} = \dfrac{V_{od}}{V_{id}}$ (want this to be large for this differential amplifier)

Common-Mode Gain $= A_{cm} = \dfrac{V_{o1}}{V_{cm}}$ or $\dfrac{V_{o2}}{V_{cm}}$ (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio $=$ CMRR $= \dfrac{A_{dm}}{A_{cm}}$ (should be very high to favor the differential-mode and reject the common-mode)

⇒ we also want a high Common-Mode Input Range to reject DC input offsets

⇒ Note: No need for bypass capacitors (large) to the inputs or outputs → can just use direct coupling!

Biasing & Large Signal Common-Mode Behavior

Case: $R_{EE} = \infty$ → ideal current source biasing → $I_{E1} = I_{E2} = \dfrac{I_{EE}}{2}$ → $V_{o1} = V_{o2} \Rightarrow V_{od} = 0$

If $V_{cm}\uparrow$ → $V_3\uparrow$, but current draw from I_{EE} stays constant ∴ I_{c1} & I_{c2} stay constant → bias pt. doesn't change

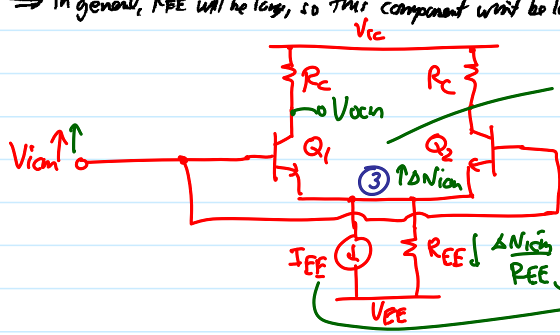
$g_{m1} = \dfrac{1}{2}\dfrac{I_{EE}}{V_T}$

Case: R_{EE} = finite

$V_3 = V_{i1} - V_{BE(on)}$

If $V_{icm}\uparrow$ → $V_3\uparrow$ → $I_{E1} = I_{E2}\uparrow$ (current draw $= I_{EE} + \dfrac{V_3}{R_{EE}}$)

⇒ in general, R_{EE} will be large, so this component won't be large, and the bias pt. won't Δ much



$\dfrac{\Delta V_{icm}}{R_{EE}}$ ⊕ = total current I'_{EE}

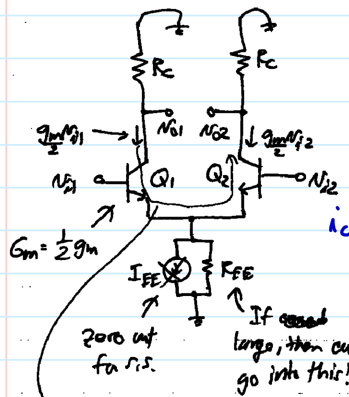
$V_{icm}\uparrow$ → $V_3\uparrow$ → $I'_{EE} > I_{EE}$

$I_{E1} = I_{E2} = \dfrac{I'_{EE}}{2}\uparrow$ → $I_{c1} = I_{c2}\uparrow$

$V_{ocm} = V_{DD} - I_{c1}R_c \downarrow$

EE 140 | Differential Mode Analysis | CTN | 6

Small-Signal Analysis of Diff. Pair



$$v_{o1} = -\frac{1}{2} g_m v_{i1} R_c + \frac{1}{2} g_m v_{i2} R_c$$

$$v_{o2} = -\frac{1}{2} g_m v_{i2} R_c + \frac{1}{2} g_m v_{i1} R_c$$

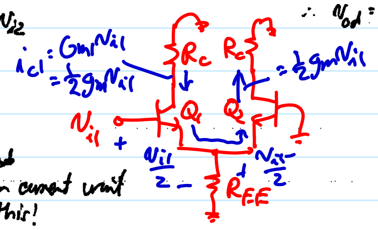
~~v_{o1} & v_{o2}~~ →

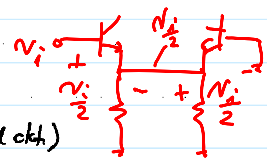
$$v_{o1} = -\frac{1}{2} g_m R_c (v_{i1} - v_{i2})$$

$$v_{o2} = +\frac{1}{2} g_m R_c (v_{i1} - v_{i2})$$

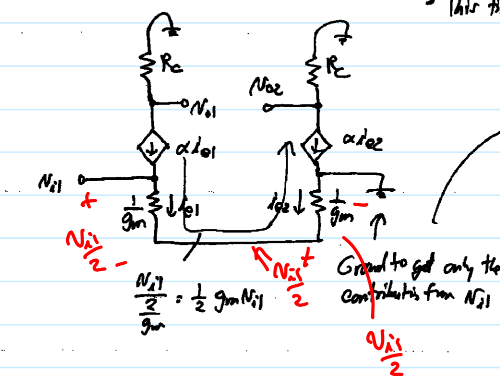
$$\therefore\ v_{od} = v_{o1} - v_{o2} = -g_m R_c (v_{i1} - v_{i2})$$

$$\therefore\ \boxed{\frac{v_{od}}{v_{id}} = A_{dm} = -g_m R_c}$$





⇒ Easiest to see this happening using the T-model: (for those who must see the model ckt.)
↳ This time also get the $\frac{v_{o1} - v_{o2}}{v_{i1}}$ gain!



$$\left.\begin{aligned} v_{o1} &= -\frac{1}{2} g_m R_c v_{i1} \\ v_{o2} &= +\frac{1}{2} g_m R_c v_{i1} \end{aligned}\right\} v_{o1} - v_{o2} = -g_m R_c v_{i1}$$

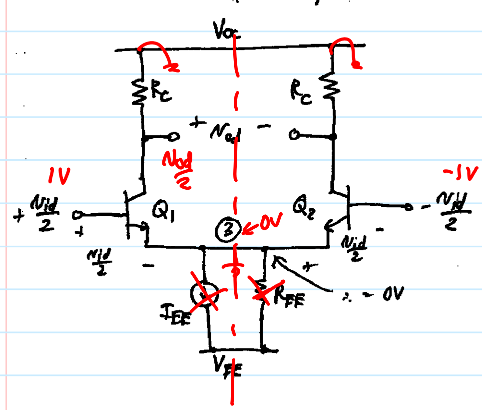
$$\therefore\ \boxed{\frac{v_{od}}{v_{i1}} = -g_m R_c}$$

$$\boxed{\frac{v_{o2}}{v_{i1}} = \frac{1}{2} g_m R_c}$$

$$\boxed{\frac{v_{o1}}{v_{i1}} = -\frac{1}{2} g_m R_c}$$

Diff. Mode Analysis

Assume a ckt. w/ only diff. input:



Total current thru I_{EE} = const.

→ V_E = const. as input changes

→ ③ acts as an incremental ground! → $V_{③} = 0V$ (always!)

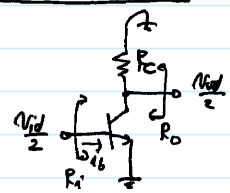
∴ we can ground ③, and thus, have a **Differential Half Ckt.**

Note: Can really only make this for a purely symmetrical ckt.!

EE 140 | Common-Mode Analysis

CTN (7)

Differential Half Ckt.



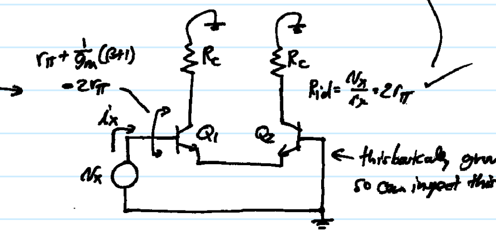
By inspection: $\frac{v_{od}/2}{v_{id}/2} = \frac{v_{od}}{v_{id}} = \boxed{A_{dm} = -g_m R_C}$

S.S. params. determined w/ $I_C = \frac{I_{EE}}{2}$

$\frac{v_{id}/2}{i_b} = r_\pi \longrightarrow R_{id} = \frac{v_{id}}{i_b} = \boxed{2r_\pi = R_{id}}$

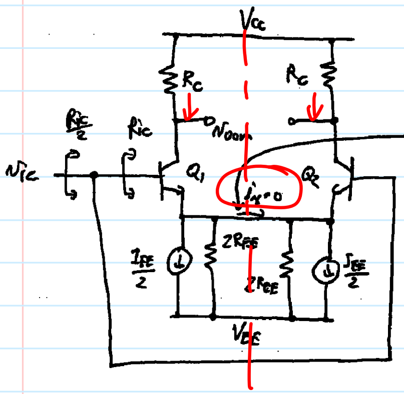
$\frac{v_{od}/2}{i_o} = r_o \| R_C \longrightarrow R_{od} = \frac{v_{od}}{i_o} = 2(r_o \| R_C) \approx \boxed{2R_C = R_{od}}$

First define



Common-Mode Analysis

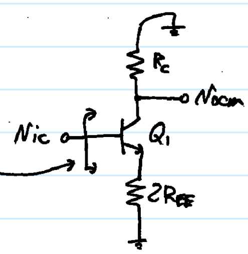
Assume a pure CM input → tie inputs together



By symmetry, $i_x = 0$. ⇒ thus, really have the equivalent of an open ckt. here

∴ ⇒ can split the ckt. into CM half-ckts.!

S.S. CM Half-Ckt.



$$\boxed{A_{cm} = \frac{v_{ocm}}{v_{ic}} = -\frac{g_m R_C}{1 + g_m(2R_{EE})} \approx -\frac{R_C}{2R_{EE}}}$$

Want small for large CMRR ∴ want R_{EE} = large!

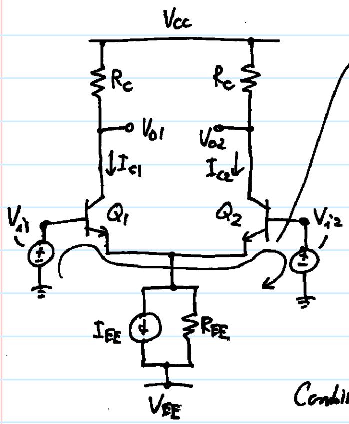
$R_{ic} = r_\pi + (\beta+1)(2R_{EE})$
@ each input

Common-Mode Rejection Ratio = $CMRR = \frac{A_{dm}}{A_{cm}} = \frac{-g_m R_C}{-\frac{g_m R_C}{1+g_m(2R_{EE})}} \Rightarrow \boxed{CMRR = 1 + 2g_m R_{EE}}$

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the ECP.

EE 140 | Large Signal ECP Performance | CTN | 8

Large Signal ECP performance



Find I_{C1} & I_{C2}:

KVL: $V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$

$I_{C1} = I_{S1}\exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right)$, $V_{be2} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right)$

$V_{i1} - V_T \ln\left(\frac{I_{C1}}{I_{C2}}\frac{\cancel{I_{S2}}^{1}}{\cancel{I_{S1}}}\right) - V_{i2} = 0 \rightarrow \ln\frac{I_{C1}}{I_{C2}} = \frac{V_{i1}-V_{i2}}{V_T} = \frac{V_{id}}{V_T}$

$$\frac{I_{C1}}{I_{C2}} = \exp\left(\frac{V_{id}}{V_T}\right) \qquad (1)$$

$$I_{EE} = I_{E1} + I_{E2} = \frac{1}{\alpha}(I_{C1} + I_{C2}) \qquad (2)$$

Combine (1) & (2) to get:

$$I_{C1} = \frac{\alpha I_{EE}}{1+\exp\left(-\frac{V_{id}}{V_T}\right)}, \quad I_{C2} = \frac{\alpha I_{EE}}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \qquad (3)$$

Find V_{od}:

$$\left.\begin{aligned} V_{o1} &= V_{CC} - I_{C1}R_C \\ V_{o2} &= V_{CC} - I_{C2}R_C \end{aligned}\right\} \quad V_{od} = V_{o1} - V_{o2} = (I_{C2} - I_{C1})R_C$$

using (3)

$$= \alpha_F I_{EE} R_C \left\{ \frac{1}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)} - \frac{1}{1+\exp\left(-\frac{V_{id}}{V_T}\right)} \times \frac{\exp\left(+\frac{V_{id}}{2V_T}\right)}{\exp\left(+\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right) + \exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha_F I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_C \frac{\sinh\left(-\frac{V_{id}}{2V_T}\right)}{\cosh\left(-\frac{V_{id}}{2V_T}\right)}$$

$$\left[\left.\begin{aligned} \sinh u &= \tfrac{1}{2}(e^{u} - e^{-u}) \\ \cosh u &= \tfrac{1}{2}(e^{u} + e^{-u}) \end{aligned}\right\} u = -\frac{V_{id}}{2V_T} \right]$$

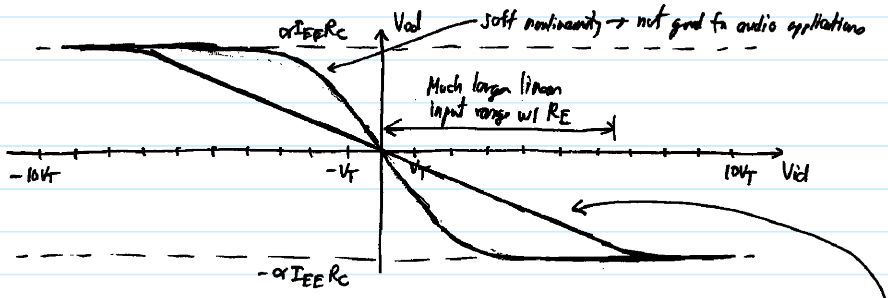
$$\therefore \boxed{V_{od} = \alpha_F I_{EE} R_C \tanh\left(-\frac{V_{id}}{2V_T}\right)}$$

From our knowledge of the Taylor series for

$$\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \ldots$$

this is fairly linear for small V_{id}, but gets nonlinear abruptly when V_{id} approaches a threshold value!

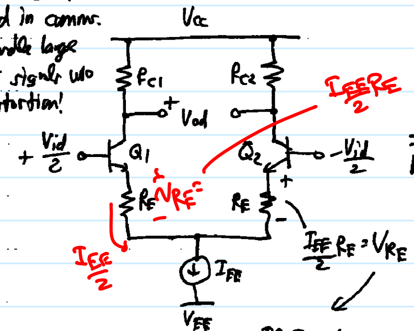
EE 140 | Large Signal ECP Performance | CTN | 9



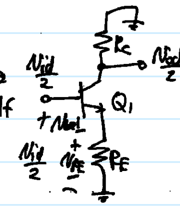
In the above curve, the $\frac{V_{od}}{V_{id}}$ Xfer function is really only linear for $V_{id} \ll V_T$ → beyond V_T, start to enter the nonlinear realm of curve → causes signal distortion: e.g., phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xistor amplifiers)

Needed in comms. to handle large input signals w/o distortion!



⟹ Diff Half Ckt.



$$A_{dm} = -\frac{g_m R_C}{1+g_m R_E}$$

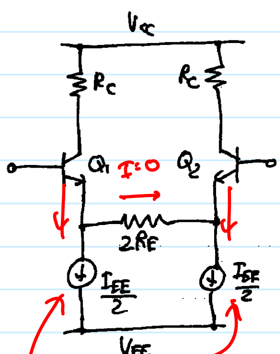
⟹ s.s. gain reduced, but the linear range is increased

If I_{EE} = large, then this can force large supply voltages

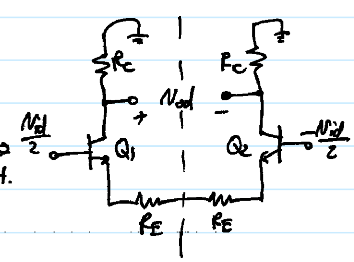
$\frac{v_{id}}{2} = v_{be1} + v_{RE}$

This can still be $v_{be1} \ll V_T$ if this absorbs some of the input voltage!

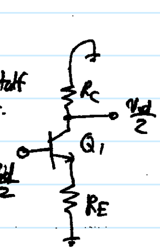
Alternative Biasing Technique If Need Larger DC Currents:–



⟹ s.s. ckt.



⟹ Diff. Half Ckt.



Current source matching important!

Same s.s. performance w/o the need to drop a DC voltage across R_E → ~~get better~~ Can use lower V_{cc} & V_{EE}.