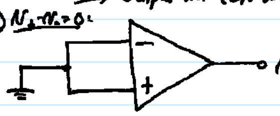
Device Mismatch Effects in Diff. Amplifiers

⇒ up to this point, we've assumed that Q_1 & Q_2 are perfectly matched
⇒ in actual ckt., get device mismatches due to processing variations

The Result:

① $v_+ - v_- = 0$: → Output not zero when Input is zero → $v_{od} \neq 0$ when $v_{id} = 0$!



Ideal Case: $v_o = 0$

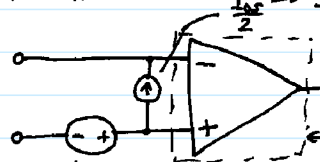
Reality: $v_o \neq 0$, even w/ $(v_+ - v_-) = 0$!

② Input $I_{B1} \neq I_{B2}$ if Q_1 & Q_2 not matched. (for BJT & JFET only.)

To model those effects, introduce:

① Input Offset Voltage, V_{os} → Typ. 1-5mV for BJT

② Input Offset Current, I_{os} → Typ. I_{os} = 10nA for BJT



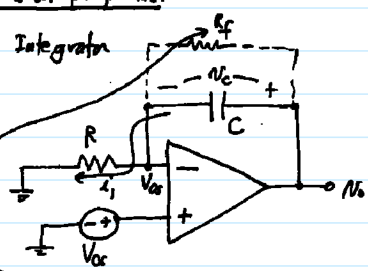
$\frac{I_{os}}{2}$ → $I_{os} \triangleq I_{B1} - I_{B2}$ (difference between input currents)

← Ideal Op Amp → well, it could have finite A_v, R_i, & R_o!

V_{os} → voltage needed to set $v_o = 0$ (negating the effect of mismatch) in this ckt.

Effect of V_{os} on Op Amp Ckts. –

Example. Integrator



$$v_o = V_{os} + \frac{1}{C}\int_0^t i_1 dt$$
$$= V_{os} + \frac{1}{C}\int_0^t \frac{V_{os}}{R} dt$$
$$= V_{os}\left(1 + \frac{t}{RC}\right) + v_c\big|_{t=0}$$

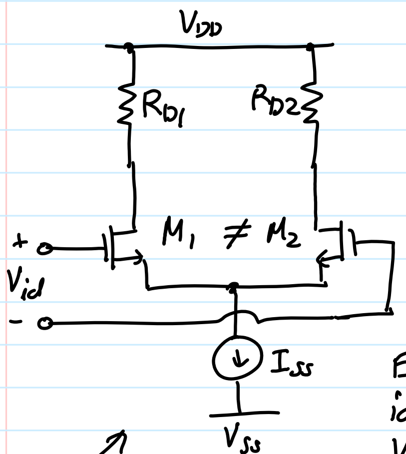
→ will continue to increase until op amp hits the voltage rails

Fix: Place an R_f in shunt w/ the C
→ then $v_o = V_{os}\left(1 + \frac{R_f}{R}\right)$, and railing doesn't happen
⇒ but, usually R_f = large to allow the C to dominate the integrator Xfer Function ∴ $v_o = V_{os}\left(1 + \frac{R_f}{R}\right)$ can be quite large → still want V_{os} = small

V_{os} is even more important in setting the resolution of A/D converters and other precision ckts.

V_{OS} of a Mismatched SCP

Objective: Derive an expression for V_{OS}.



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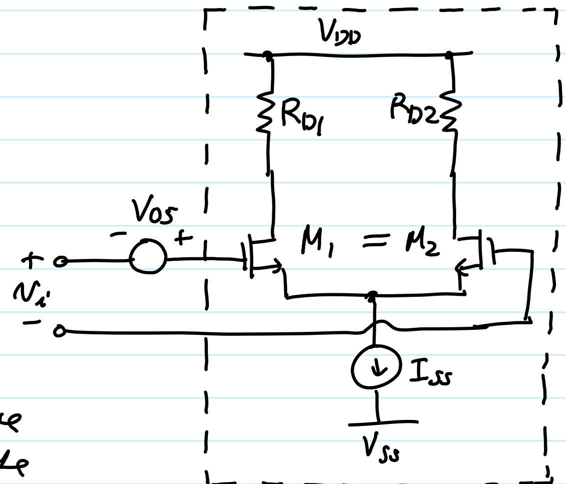


Actual SCP w/ Mismatched Xsistors & R's

Equate to an ideal SCP & use V_{OS} to model the effect of mismatches

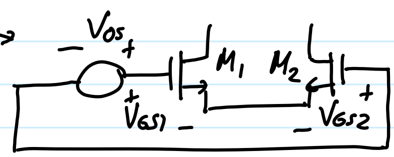
Ideal SCP w/ Matched M_1 & M_2 and $R_{D1} = R_{D2}$

Input offset voltage V_{OS} arises due to variations in:

① Xsistors, M_1 & M_2 $\longrightarrow$ $\frac{W}{L}$ and V_t vary

② $R_{D1} \neq R_{D2}$ $\longrightarrow$ causes gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$ in this ckt.



KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \qquad (1)$$

Define difference and average quantities:

$\Delta I_D = I_{D1} - I_{D2}$	$\Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$	$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$	$V_t = \frac{V_{t1} + V_{t2}}{2}$	$R_D = \frac{R_{D1} + R_{D2}}{2}$

EE 140/240A — V_{OS} of a Mismatched SCP — CTN — 3

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$V_{os} = \Delta V_t + \sqrt{\frac{\overbrace{2(I_D + \Delta I_D/2)}^{2I_D\left(1+\frac{\Delta I_D}{2I_D}\right)}}{\mu_n C_{ox}\underbrace{\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}_{\frac{W}{L}\left[1+\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}}} - \sqrt{\frac{\overbrace{2(I_D - \Delta I_D/2)}^{2I_D\left(1-\frac{\Delta I_D}{2I_D}\right)}}{\mu_n C_{ox}\underbrace{\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}_{\frac{W}{L}\left[1-\frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]}}}$$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1+nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{os} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore\ V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{os} \rightarrow V_{od} = 0 \;\therefore\; I_{D1}R_{D1} = I_{D2}R_{D2} \longrightarrow$ mismatch in I_D must be opposite that of R_D

$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$\boxed{V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right\}}$$

Threshold Mismatch → bias independent

Geometric (i.e., Layout) Variation

↳ scale w/ overdrive