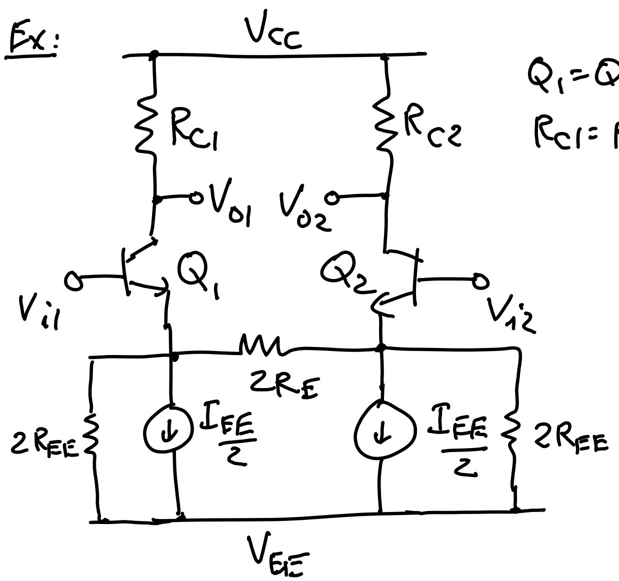
Lecture 14: SCP & Current Mirror Load

- Announcements:
  - Pre-Lecture materials online
  - HW#1A online for 240A folks; due Friday, Nov. 8 (more than 3 weeks from now)
  - HW#7 online soon
  - Midterm will be on the date specified in your syllabus: Thursday, Oct. 31, 9:00-11 a.m. in this room
    ??? → now uncertain
- Lecture Topics:
  - Emitter Coupled Pair Example
  - Source Coupled Pair
  - Current Mirror Load
- ----------------------------------
- Last Time:
- Going through op amp handout; continue this

Ex:

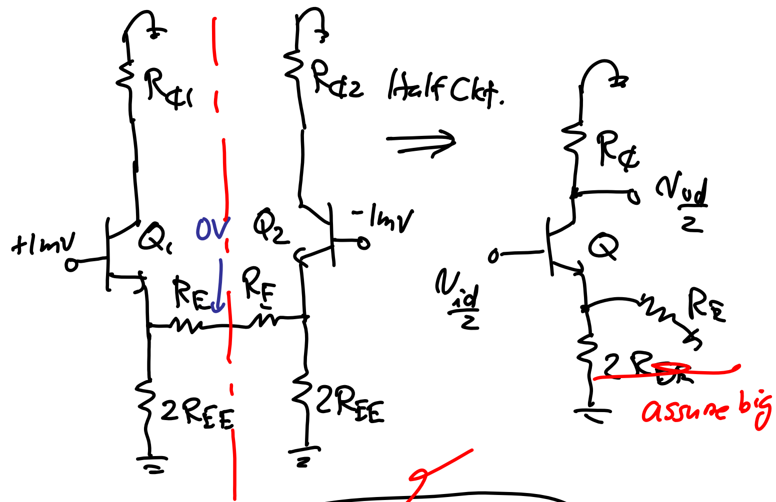


$Q_1 = Q_2$

$R_{C1} = R_{C2} = R_C$

(1) Find the differential gain (A_{dm})

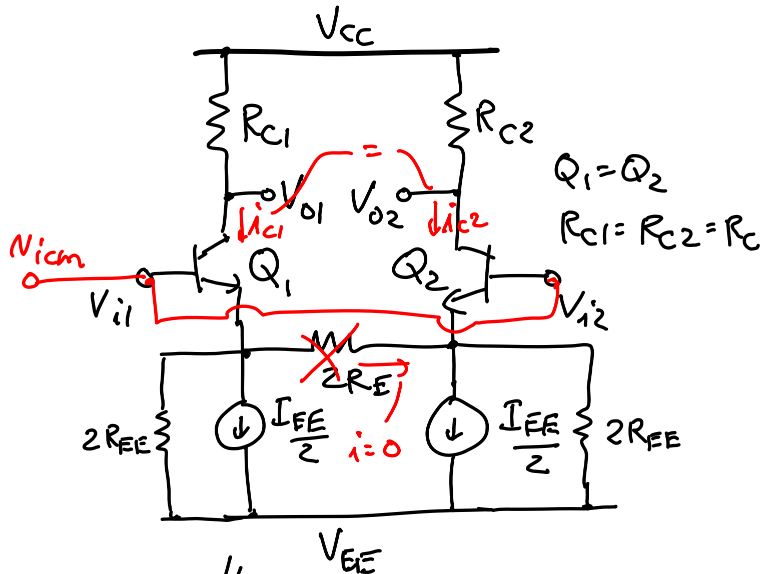
S.S. Ckt:



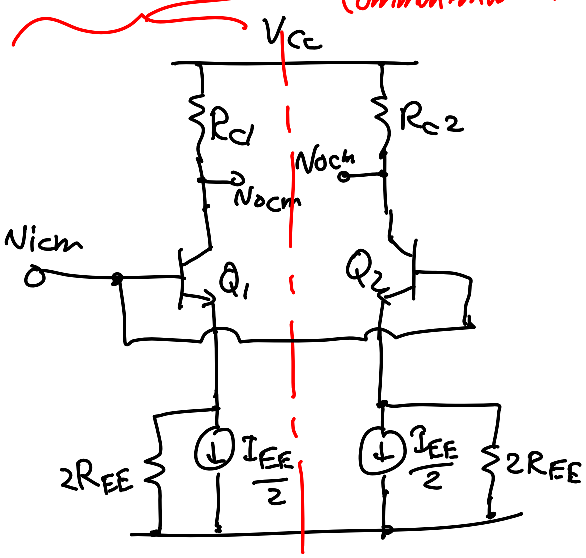
$$\frac{(v_{od}/2)}{(v_{id}/2)} = -\frac{g_m(r_o(1+g_mR_E) \| R_c)}{1+g_mR_E}$$

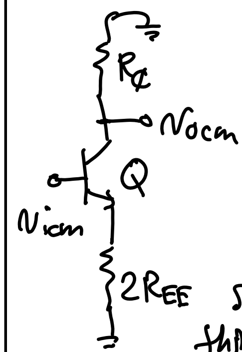
$$A_{dm} = \frac{v_{o1}-v_{o2}}{v_{i1}-v_{i2}} = \frac{v_{od}}{v_{id}} =$$

(2) Find the common-mode gain: (A_{cm})



common-mode half-ckt





$$A_{cm} = \frac{v_{ocm}}{v_{icm}} = \frac{-g_m R_C}{1 + g_m(2R_{EE})} \approx -\frac{R_C}{2R_{EE}}$$

$2R_{EE}$ same thing as "want a good current source" → want $2R_{EE} \gg R_C$

$A_{cm} \downarrow$ → reject noise better!

If there is a mismatch in the ckt., e.g., $R_{C1} \neq R_{C2}$, then we can also define:

$A_{cm\text{-}dm} \triangleq$ common-mode input to differential-mode output gain

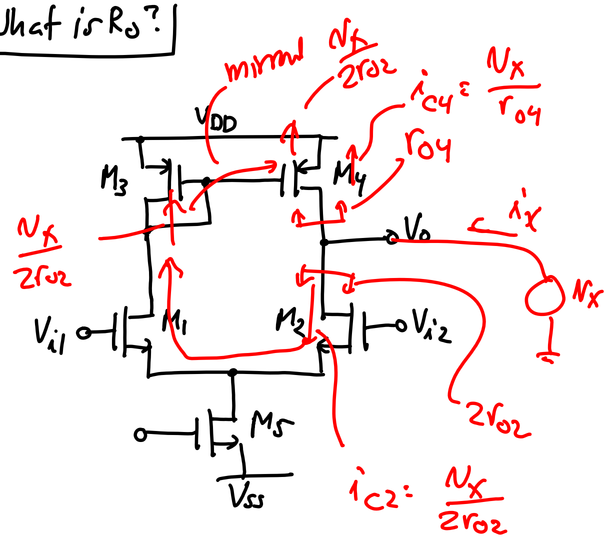
$$= \frac{v_{od}}{v_{ic}} = \frac{v_{o1} - v_{o2}}{v_{i1}} = \frac{v_{o1} - v_{o2}}{v_{i2}} \quad (w/\ v_{i1} = v_{i2})$$

$A_{dm\text{-}cm} \triangleq$ differential-mode input to common-mode output gain

$$= \frac{v_{oc}}{v_{id}} = \frac{v_{oc}}{v_{i1} - v_{i2}} \quad \left[w/\ v_{oc} = \tfrac{1}{2}(v_{o1} + v_{o2})\right]$$

You will be experiencing these in a future HW.

What is R_o?



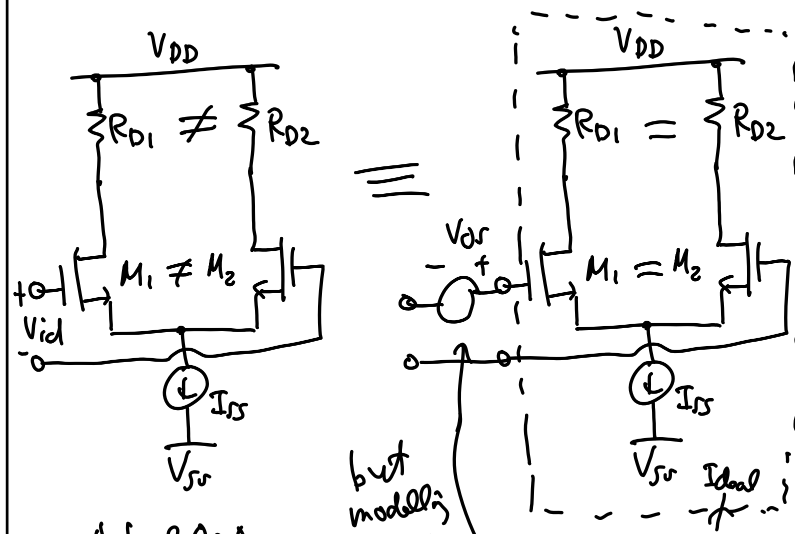
$$i_x = \frac{v_x}{r_{o4}} + \frac{v_x}{2r_{o2}} + \frac{v_x}{2r_{o2}} = \left(\frac{1}{r_{o4}} + \frac{1}{r_{o2}}\right) v_x$$

mirroring (F3)

$$\frac{v_x}{i_x} = R_o = (r_{o2} \| r_{o4})$$

V_{os} of a Mismatched SCP

Objective: Determine an expression for V_{os}.



Actual Op Amp w/ mismatches

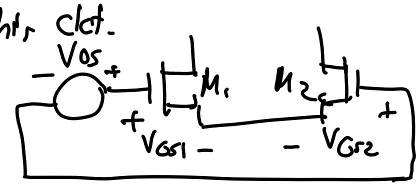
but modelling mismatch via V_{os}

Ideal → No mismatches

V_{os} arises due to variations in:

1. Xristors M_1 & M_2 → $\frac{W}{L}$ & V_t vary
2. $R_{D1} \neq R_{D2}$ → cause variations in gain

Definition. V_{os} = V_{id} needed to get $V_{od} = 0V$ in this ckt.



KVL: $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS} - V_{GS2}$$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}(W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} \quad (1)$$

Define difference and average quantities:

$$\Delta I_D = I_{D1} - I_{D2} \qquad \Delta\left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$$

$$I_D = \frac{I_{D1} + I_{D2}}{2} \qquad \left(\frac{W}{L}\right) = \frac{1}{2}\left[\left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2\right]$$

$$\Delta V_t = V_{t1} - V_{t2} \qquad \Delta R_D = R_{D1} - R_{D2}$$

$$V_t = \frac{1}{2}(V_{t1} + V_{t2}) \qquad R_D = \frac{1}{2}(R_{D1} + R_{D2})$$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \qquad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \qquad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \qquad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1): $2I_D\left(1 + \frac{\Delta I_D}{2I_D}\right)$

$$V_{OS} = \Delta V_t + \sqrt{\frac{2(I_D + \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) + \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}} - \sqrt{\frac{2(I_D - \Delta I_D/2)}{\mu_n C_{ox}\left[\left(\frac{W}{L}\right) - \frac{1}{2}\Delta\left(\frac{W}{L}\right)\right]}}$$

$\frac{W}{L}\left[1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right]$

$$\left[V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\right]$$

$$= \Delta V_t + (V_{GS} - V_t)\left\{\sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}}}\right\}$$

Binomial Theorem:

$$(1 + nx)^m \xrightarrow[n = \text{small}]{} 1 + mnx$$

$$V_{OS} = \Delta V_t + (V_{GS} - V_t)\left\{\left(1 + \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4}\frac{\Delta I_D}{I_D}\right)\left(1 + \frac{1}{4}\frac{\Delta(W/L)}{(W/L)}\right)\right\}$$

$$\cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} - \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}} - \cancel{1} + \frac{1}{4}\frac{\Delta I_D}{I_D} - \frac{1}{4}\frac{\Delta(W/L)}{(W/L)} + \cancel{\frac{1}{16}\frac{\Delta I_D}{I_D}\frac{\Delta(W/L)}{(W/L)}}$$

$$= \Delta V_t + (V_{GS} - V_t)\left(\frac{1}{2}\frac{\Delta I_D}{I_D} - \frac{1}{2}\frac{\Delta(W/L)}{(W/L)}\right)$$

$$\therefore V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{\frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

When $V_{id} = V_{OS} \rightarrow V_{od} = 0 \quad \therefore I_{D1}R_{D1} = I_{D2}R_{D2}$

→ mismatch in I_D must be opposite that of R_D

$$\therefore \frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

$$V_{OS} = \Delta V_t + \frac{1}{2}(V_{GS} - V_t)\left\{-\frac{\Delta R_D}{R_D} - \frac{\Delta(W/L)}{(W/L)}\right\}$$

Threshold Mismatch ↑ Bias Indep.

Geometric Variations (i.e., layout) ↓ scale as overdrive

Again, signs mean nothing as could have

$$\frac{\Delta R_D}{R_D} = (-) \;\; \& \;\; \frac{\Delta(W/L)}{(W/L)} = (-)$$

Take the worst case by convention → add everything