

EE 140/240A

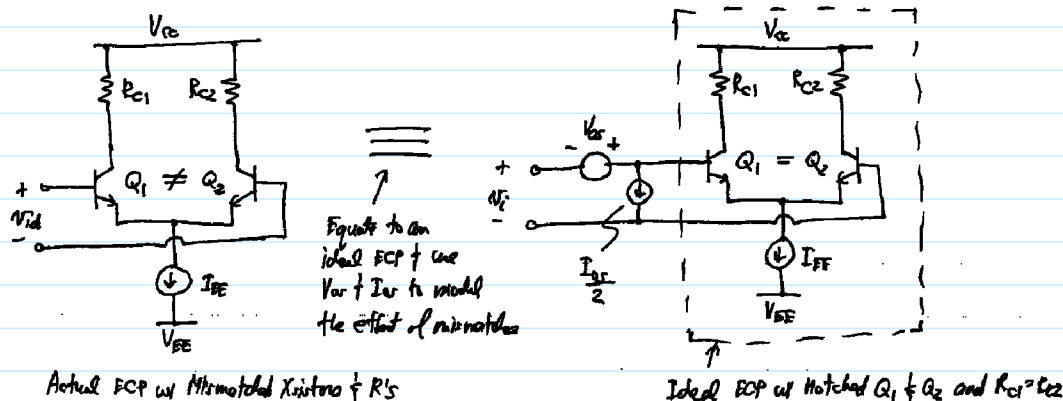
V_{OS} of a Mismatched ECP

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V_{OS} in a Mismatched ECP

Objective: Derive an expression for V_{OS} .



Input Offset Voltage V_{OS} arises due to variations in:

① x_{mis} , $Q_1 \neq Q_2 \rightarrow I_s \neq \beta$ vary: $I_s = \frac{q n_i^2 D_n A}{N_A W_B (V_{CB})}$

$I_{S1} \neq I_{S2}$ can be caused by:
(i) $A_1 \neq A_2$ (etching tolerance limits)
(ii) $N_{A1} \neq N_{A2}$ (doping variations of base)
(iii) $W_B = f(V_{CB})$ (with variations exacerbated by V_{CB} diff)

② $R_{c1} \neq R_{c2} \rightarrow$ cause gain variation

Definition. $V_{OS} = V_{id}$ to get $V_{od} = 0$, which occurs when:

$V_{OS} - V_{be1} + V_{be2} = 0$

$V_{OS} = V_{be1} - V_{be2} = V_T \ln \frac{I_{c1}}{I_{s1}} - V_T \ln \frac{I_{c2}}{I_{s2}} = V_T \ln \left(\frac{I_{c1}}{I_{c2}} \frac{I_{s2}}{I_{s1}} \right)$

Find $\frac{I_{c1}}{I_{c2}}$ in terms of design elements:

[When $V_{id} = V_{OS} \rightarrow V_{od} = 0V \rightarrow V_{od} = (V_{CC} - I_{c1}R_{c1}) - (V_{CC} - I_{c2}R_{c2}) = 0$

$I_{c1}R_{c1} = I_{c2}R_{c2} \rightarrow \frac{I_{c1}}{I_{c2}} = \frac{R_{c2}}{R_{c1}}$

$V_{OS} = V_T \ln \left(\frac{R_{c2}}{R_{c1}} \frac{I_{s2}}{I_{s1}} \right)$

This is an exact equation for V_{OS} . It's often more useful & intuitive to express this in terms of percent variations (and eventually standard deviations).

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Convert to Percent Variation Form -

Define $R_c = \frac{R_{c1} + R_{c2}}{2}$, $\Delta R_c = R_{c1} - R_{c2}$ } Objective: Express Var in terms of percent variations $\frac{\Delta R_c}{R_c} \neq \frac{\Delta I_s}{I_s}$.

$I_s = \frac{I_{s1} + I_{s2}}{2}$, $\Delta I_s = I_{s1} - I_{s2}$ }

In general: $\Delta X = X_1 - X_2$ } $X_1 = X + \frac{\Delta X}{2}$ } Thus: $R_{c1} = R_c + \frac{\Delta R_c}{2}$, $R_{c2} = R_c - \frac{\Delta R_c}{2}$

$X = \frac{X_1 + X_2}{2}$ } $X_2 = X - \frac{\Delta X}{2}$ } $I_{c1} = I_s + \frac{\Delta I_s}{2}$, $I_{c2} = I_s - \frac{\Delta I_s}{2}$

With these formulations:

$$V_{OS} = V_T \ln \left[\frac{R_{c2} I_{c2}}{R_{c1} I_{c1}} \right] = V_T \ln \left\{ \frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}} \right\} = V_T \ln \left\{ \frac{1 - \frac{\Delta R_c}{2R_c}}{1 + \frac{\Delta R_c}{2R_c}} \frac{1 - \frac{\Delta I_s}{2I_s}}{1 + \frac{\Delta I_s}{2I_s}} \right\}$$

$$\left[\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right] \Rightarrow V_{OS} \approx V_T \left\{ -\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s} \right\}$$

taking the first term assuming $\Delta R \ll R_c + \Delta I_s \ll I_s$

$$V_{OS} = V_T \left\{ -\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s} \right\}$$

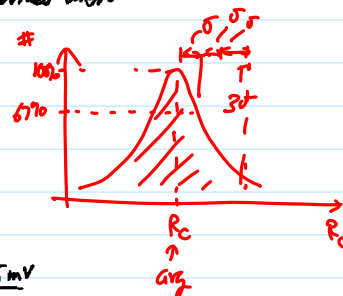
Since $\frac{\Delta R_c}{R_c}$ and $\frac{\Delta I_s}{I_s}$ are statistically ^{varying} parameters for a given process run & layout, one usually expresses terms in the form of variances when specifying V_{OS} :

→ since $\frac{\Delta R_c}{R_c} \neq \frac{\Delta I_s}{I_s}$ are uncorrelated, their variances add like power:

$$\sigma_{V_{OS}}^2 = V_T^2 \left(\sigma_{\Delta R_c/R_c}^2 + \sigma_{\Delta I_s/I_s}^2 \right)$$

Ex: Typ. $\sigma_{\Delta R_c/R_c} \sim 0.01$, $\sigma_{\Delta I_s/I_s} \sim 0.05$

$$\therefore \sigma_{V_{OS}} = (26m) \sqrt{(0.01)^2 + (0.05)^2} = 1.3mV \quad \text{Typ. Var for BJT} \sim 1-5mV$$



V_{OS} Drift w/ Temperature

$$\frac{dV_{OS}}{dT} = \frac{kT}{q} \left\{ \underbrace{-\frac{\Delta R_c}{R_c}}_{\text{indep. of T}} - \frac{\Delta I_s}{I_s} \right\} \frac{1}{T} = \frac{V_{OS}}{T}$$

Ex: $\frac{dV_{OS}}{dT} = \frac{13m}{300K} = 4.3 \mu V/K$ around $T = 300K$.

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I_{OS} of a Mismatched ECP

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I_{OS} in a Mismatched ECP

By Definition: $I_{OS} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2} = I_{OS}$

To express in percent variation:

$$\begin{cases} I_{C1} = I_C + \frac{\Delta I_C}{2} \\ I_{C2} = I_C - \frac{\Delta I_C}{2} \end{cases} \quad \begin{cases} \beta_1 = \beta + \frac{\Delta \beta}{2} \\ \beta_2 = \beta - \frac{\Delta \beta}{2} \end{cases}$$

$$\therefore I_{OS} = \frac{I_C + \frac{\Delta I_C}{2}}{\beta + \frac{\Delta \beta}{2}} - \frac{I_C - \frac{\Delta I_C}{2}}{\beta - \frac{\Delta \beta}{2}} = \frac{I_C}{\beta} \left\{ \frac{1 + \frac{\Delta I_C}{2I_C}}{1 + \frac{\Delta \beta}{2\beta}} - \frac{1 - \frac{\Delta I_C}{2I_C}}{1 - \frac{\Delta \beta}{2\beta}} \right\}$$

$$\left[\frac{1}{1+x} \approx 1 - x + x^2 - \dots \right] \rightarrow = \frac{I_C}{\beta} \left\{ \left(1 + \frac{\Delta I_C}{2I_C}\right) \left(1 - \frac{\Delta \beta}{2\beta}\right) - \left(1 - \frac{\Delta I_C}{2I_C}\right) \left(1 + \frac{\Delta \beta}{2\beta}\right) \right\}$$

$$= \frac{I_C}{\beta} \left\{ 1 + \frac{\Delta I_C}{2I_C} - \frac{\Delta \beta}{2\beta} - \frac{\Delta I_C \Delta \beta}{2I_C \beta} - 1 + \frac{\Delta I_C}{2I_C} - \frac{\Delta \beta}{2\beta} + \frac{\Delta I_C \Delta \beta}{2I_C \beta} \right\}$$

$$I_{OS} = \frac{I_C}{\beta} \left\{ \frac{\Delta I_C}{I_C} - \frac{\Delta \beta}{\beta} \right\}$$

But for $V_{od} = 0V \Rightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}} \rightarrow \frac{\Delta I_C}{I_C} = -\frac{\Delta R_C}{R_C}$

$$\therefore I_{OS} = -\frac{I_C}{\beta} \left(\frac{\Delta R_C}{R_C} + \frac{\Delta \beta}{\beta} \right)$$

Ex. Typ: $\sigma_{\Delta R_C} = 0.1$, $\sigma_{\Delta \beta} = 0.01$

$$\rightarrow I_{OS} = -\frac{I_C}{\beta} \left[\sigma_{\Delta R_C}^2 + \sigma_{\Delta \beta}^2 \right]^{1/2} \approx -0.1 \frac{I_C}{\beta} \approx -0.1 I_B = I_{OS}$$

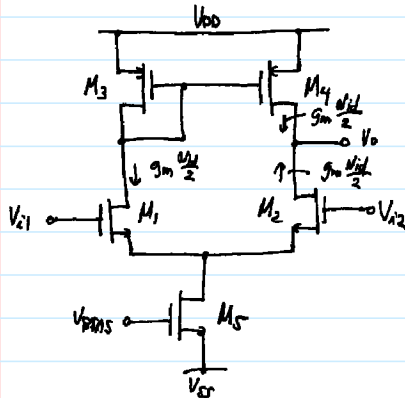
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Diff. Pair w/ I-Mirror Load V_{OS}

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MOS Differential Stage w/ Current Mirror Load



Small-Signal Gain: (similar to BJT)

$$\frac{V_o}{V_{id}} = \frac{g_{m3}(r_{o2} \parallel r_{o4})}{g_{m2} + g_{m4}} = \frac{\sqrt{2\mu_n C_{ox}} \left(\frac{W}{L}\right)_2 I_{D2}}{\lambda_2 I_{D2} + \lambda_4 I_{D4}}$$

$$= \frac{\sqrt{2\mu_n C_{ox}} \left(\frac{W}{L}\right)_2 I_{D2}}{\frac{I_{D2}}{2} (\lambda_2 + \lambda_4)} \Rightarrow \frac{V_o}{V_{id}} = \frac{2}{\lambda_2 + \lambda_4} \sqrt{\frac{\mu_n C_{ox} (W/L)_2}{I_{D2}}}$$

$$\left[\frac{\Delta(W/L)_{i2}}{(W/L)_{i2}} - \frac{\Delta(W/L)_{s,y}}{(W/L)_{s,y}} \right]$$

Offset Voltage - $V_{OS} = V_{GS1} - V_{GS2}$ when $V_{od} = 0V$

$$V_{OS} = \Delta V_{t1,2} + \Delta V_{t3,4} \left(\frac{g_{m3,4}}{g_{m1,2}} \right) + \frac{(V_{GS} - V_t)_{1,2}}{2} \left[\frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}} \right]$$

Via similar derivation to what we just did

For small V_{OS} : ① small $(V_{GS} - V_t)$

② $g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2} \frac{1}{2} \left(\frac{W}{L}\right)_{3,4} < \left(\frac{W}{L}\right)_{1,2}$