Lecture 22: RHP Zero

- **Announcements**:
- Design Project Checkpoint:
  - Due Tuesday, Nov. 19, 11:59 p.m.
  - Send to your TA a spice file for your op amp design that simulates correctly, i.e., that reaches a stable bias point where all transistors are saturated (except MOS R's)
  - It doesn't need to meet the project specs, but it should simulate correctly
- HW#10 online, due next Wednesday @ 8 a.m.
- **Lecture Topics**:
  - Nulling the RHP Zero
- --
- **Last Time**:

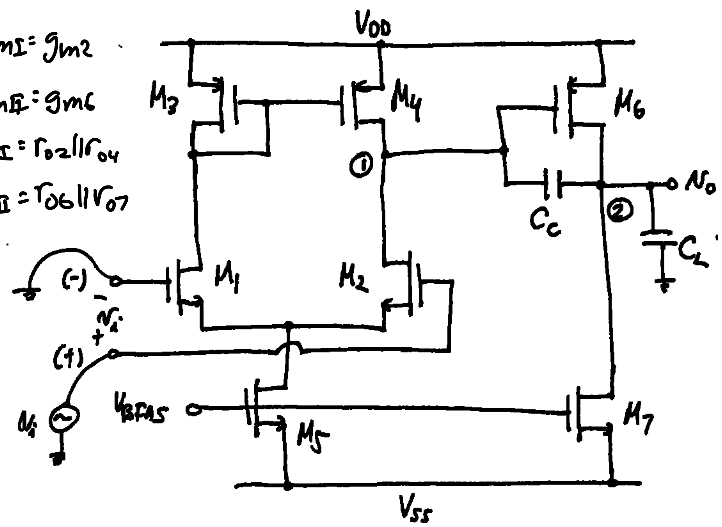
$\boxed{\text{CMOS 2-Stage Op Amp Compensation}}$ (Summary)

$g_{mI} = g_{m2}$

$g_{mII} = g_{m6}$

$R_I = r_{o2} \| r_{o4}$

$R_{II} = r_{o6} \| r_{o7}$



From our previous analysis:

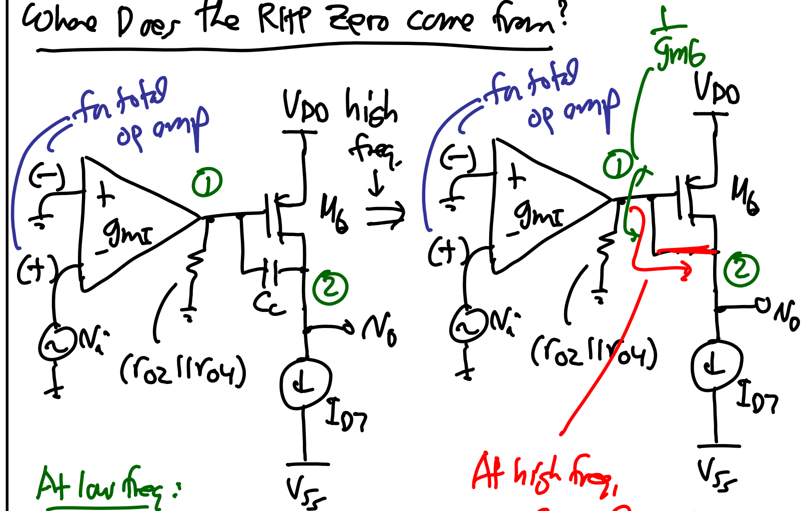
$$p_1 = -\frac{1}{g_{mII} R_I R_{II} C_c}$$

$[C_c \gg C_I \text{ or } C_{II}]$ $\quad [C_L \gg C_I]$

$$p_2 = -\frac{g_{mII} C_c}{C_I C_{II} + C_c (C_I + C_{II})} \approx -\frac{g_{mII}}{C_I + C_{II}} \approx -\frac{g_{m6}}{C_L}$$

$$z = +\frac{g_{mII}}{C_c} \leftarrow \text{RHP zero (this will cause problems)}$$

Where Does the RHP Zero come from?



At low freq:

$$\frac{v_②}{v_①} = -g_{m6}(r_{o6} \| r_{o7})$$

(-)

$$\text{total gain} = \frac{v_①}{v_i} \cdot \frac{v_②}{v_①} = (+)$$

(-) (-)

At high freq, got feed-forward path → creates the RHP zero

$$\frac{v_o}{v_i} = -\frac{g_{m1}}{g_{m6}}$$

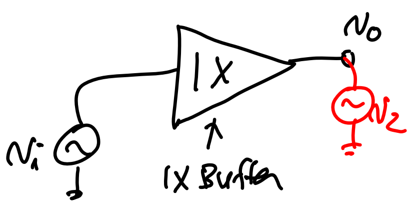
total gain = (-) → 180° phase shift!

Observation.

Miller effect compensation requires FB path.

BUT: The feedforward path (that creates the zero) is not needed!

Solution: ① Kill the feedforward path
② Keep the feedback path



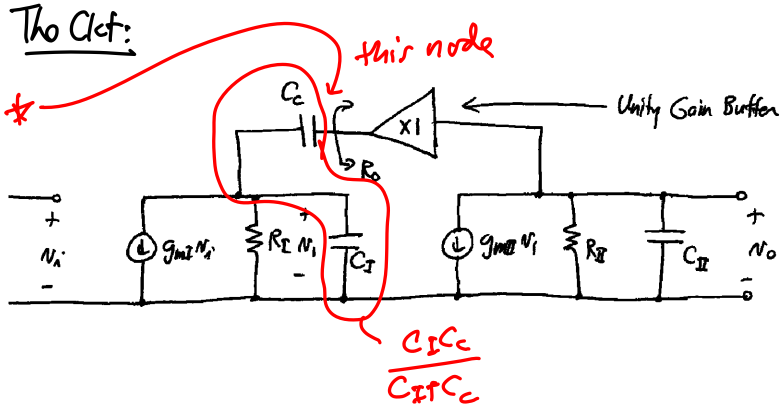
$\frac{v_o}{v_i} = 1$

$\frac{v_i}{v_2} << 1$

∴ buffer is directional!

Solution: Put a 1x buffer in series w/ C_c to prevent feedforward, but allow FB!

The Ckt:



Apply KCL:

$p_1 \cong -\frac{1}{g_{mII} R_I R_{II} C_c}$ (same as before)

$p_2 \cong -\frac{g_{mII} C_c}{C_{II}(C_I + C_c)} \cong -\frac{g_{mII}}{C_{II}}$ $[C_c >> C_I]$

$p_3 \cong -\frac{1}{R_o\underbrace{(C_I C_c/(C_I + C_c))}_{\text{series combination of } C_I \text{ \& } C_c}} \cong -\frac{1}{R_o C_I}$ comes from ✱

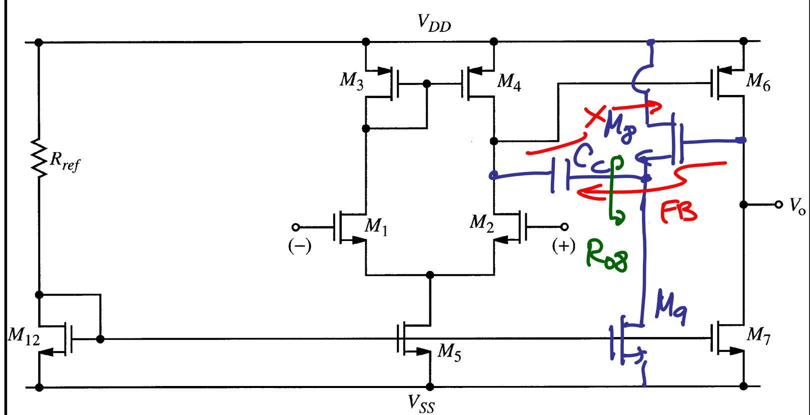
$z \cong -\frac{1}{R_o C_c}$ ← LHP zero! Good!

Remarks.

① An additional pole $p_3 = -\frac{1}{R_o C_I}$ has been created! But since R_o = small (for a buffer) and C_I = small, p_3 is at a very high freq. → contributes very little phase @ $\omega_{u\ell g}$, where $|T(j\omega)| = 1$.

② A LHP zero now emerges, $z_I = -\frac{1}{R_o C_c}$.
This helps stability as discussed before. (by contributing (+) phase shift → PM↑)

Actual Implementation of Buffer-Based Zero Cancellation



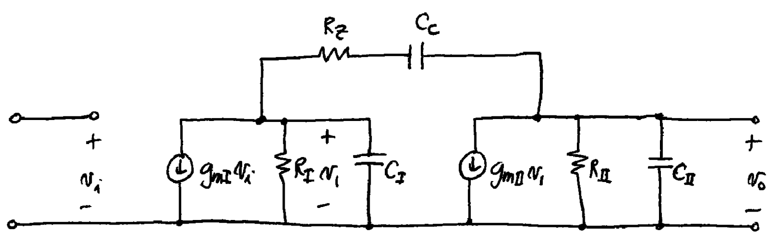
$R_{o8} \cong \frac{1}{g_{m8}} = \frac{1}{\sqrt{2\mu_p C_{ox}(\frac{W}{L})_p I_{D8}}}$ → Want this sufficiently small to drive $|p_3|\uparrow$

increase I_{D8}, or increase $(\frac{W}{L})_8$

Problems: more power ✗ more area ✗ → cost!

Solution: Better technique!

$\boxed{\text{Nulling Resistor in Series w/ } C_c}$



Doing KCL:

$$p_1 = -\frac{1}{g_{mII}R_I R_{II} C_c}$$

$$p_2 \cong \frac{-g_{mII}C_c}{C_I C_{II} + C_c(C_I + C_{II})} \cong -\frac{g_{mII}}{C_{II}}$$

same as before

$$p_3 = -\frac{1}{R_z C_I}$$ ← pole due to R_z

$$z_1 = \frac{1}{C_c(\frac{1}{g_{mII}} - R_z)}$$ ← relocated zero (function of R_z)

Note: The position of the zero depends upon the value of the "nulling resistor" R_z

If $R_z < \frac{1}{g_{mII}}$ then z_1 is in the RHP

If $R_z > \frac{1}{g_{mII}}$ then " " LHP!

↳ This is great! → can convert the zero to a LHP one!

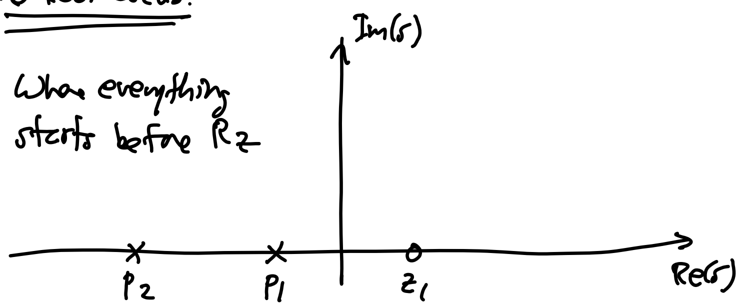
↳ can even stick the zero on top of a pole to eliminate it!

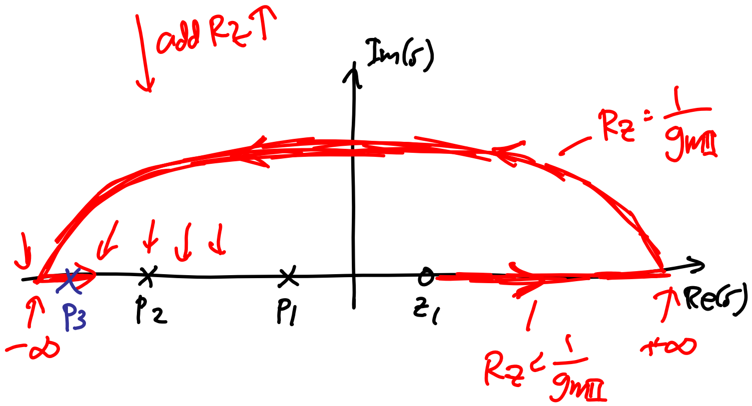
$$H(s) = \cdots \frac{\cancel{(s-z_1)}}{\cancel{(s-p_1)}}$$

if $z_1 = p_1$

The Root Locus:

When everything starts before R_Z





Zero Placement Strategies:

① Eliminate $z_1 \rightarrow$ move it to ∞:

$$z_1 = \frac{1}{C_c\left(\frac{1}{g_{mII}} - R_Z\right)} \rightarrow \infty \quad \text{when} \quad \boxed{R_Z = \frac{1}{g_{mII}}}$$

Load Cap. ↓

After doing this: $p_3 \cong -\frac{g_{mII}}{C_L}$, $p_2 \cong -\frac{g_{mII}}{C_{II}}$ } Usually $C_{II} \gg C_{I}$, so these poles are far apart... but be careful

This is good, but we can do better:

② Eliminate p_3 by placing z_1 on top of it:

$$z_1 = p_3 \Rightarrow \frac{1}{C_c\left(\frac{1}{g_{mII}} - R_Z\right)} = -\frac{1}{R_Z C_L}$$

$$\boxed{R_Z = \frac{1}{g_{mII}\left(1 - \frac{C_L}{C_c}\right)}}$$

After this: ① p_3 gone; p_1 & p_2 left

② Now, can place $\omega_{u\ell f}$ @ p_2 and really get PM = 45° (w/o worrying about the influence of p_3)

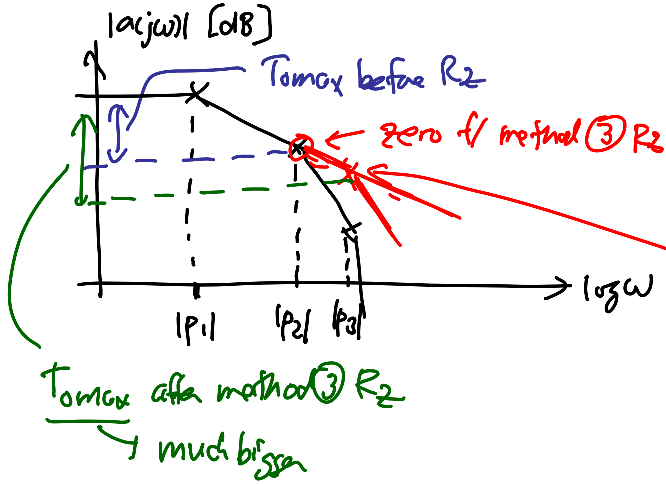
But can still do better

③ Eliminate p_2 by placing z_1 on top of it:

⇒ p_3 becomes the "new" p_2! (higher f; higher T_{omax})

$$z_1 = p_2 \Rightarrow \frac{1}{C_c\left(\frac{1}{g_{mII}} - R_Z\right)} = -\frac{g_{mII}}{C_{II}}$$

$$\boxed{R_Z = \left(\frac{C_c + C_{II}}{C_c}\right)\left(\frac{1}{g_{mII}}\right) = \frac{1}{g_{mII}}\left(1 + \frac{C_{II}}{C_c}\right)}$$



With this choice of R_z:

$$p_3 = -\frac{1}{R_z C_I} = -\frac{1}{\left(\frac{C_c + C_{II}}{C_c}\right)\left(\frac{1}{g_{mII}}\right) C_I}$$

$$\boxed{p_3 = \frac{g_{mII} C_c}{C_I (C_c + C_{II})}}$$

← This becomes the new p_2!

For PM = 45°:

$$C_c = \frac{g_{mI}}{|p_3| A_o} = \frac{g_{mI}}{g_{mII}} \frac{C_I (C_c + C_{II})}{C_c A_o}$$

↑ the "new" $|p_2|$

solve for C_c:

$$\boxed{C_c \cong \sqrt{\frac{g_{mI}}{g_{mII}} \frac{C_I C_{II}}{A_o}}}$$

PM = 45°

For PM = 60°:

$$C_c = \frac{1.73\, g_{mI}}{|p_3| A_o} \longrightarrow \boxed{C_c \cong \sqrt{\frac{1.73\, g_{mI}}{g_{mII}} \frac{C_I C_{II}}{A_o}}}$$

$[C_I \ll C_{II}]$

For PM = 60°

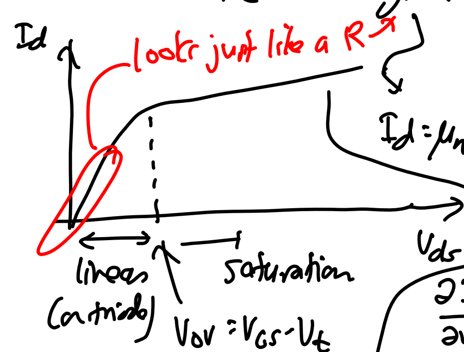
Remark: If settling time is important, then approach ③ may not be the best approach. The reason is that if the zero is not exactly equal to the pole, the a "doublet" ensues, which actually can hurt the settling time.

Discussed in a handout to be posted on the course website. → also, discussed in Razavi, problem 10.19.

Actual Implementation

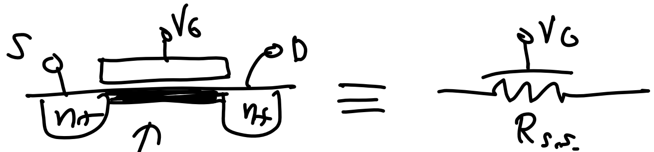
⇒ resistors are too big! → ∴ implement using a much smaller MOS resistor

MOS Resistor: just an MOS Xsistor operated in the linear region



$$I_d = \mu_n C_{ox} \frac{W}{L}\left[(V_{gs} - V_t) V_{ds} - \frac{1}{2} V_{ds}^2\right]$$

$$\frac{\partial I_d}{\partial V_{ds}} = \mu_n C_{ox} \frac{W}{L}(V_{gs} - V_t - V_{ds})$$



uniform inversion charge → just like n+ material

control effective sheet R by Δ'ing gate voltage V_G

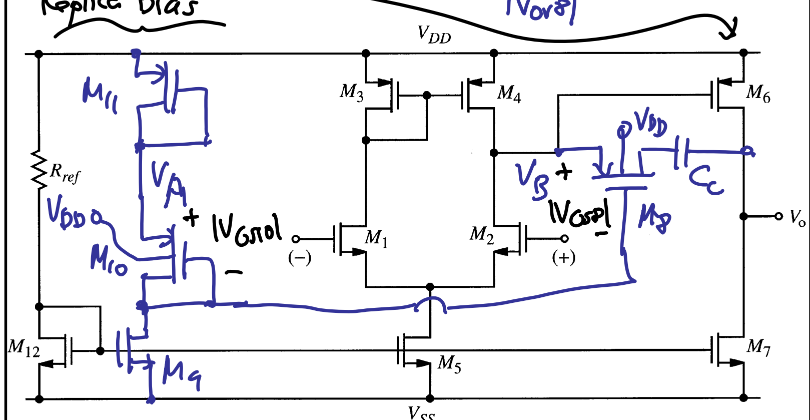
$$\boxed{R_{s.s.} = \left[\frac{dI_d}{dV_{ds}}\right]^{-1}_{V_{gs}=V_{GS},\,V_{ds}=V_{DS}} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t-V_{DS})} = \frac{1}{g_{ds}}}$$

variable resistor controllable by V_{GS}!

Actual Implementation:

$$V_{DS8} = 0V \Rightarrow R_8 = \frac{1}{\mu_p C_{ox}(W/L)_8\underbrace{(|V_{GS8}|-|V_{t8}|)}_{|V_{ov8}|}}$$

Replica Bias



Design:

Need $V_A = V_B \rightarrow |V_{GS11}| = |V_{GS6}|$, know $|V_{t11}| = |V_{t6}|$

$$\sqrt{\frac{2I_{D11}}{\mu_p C_{ox}(\frac{W}{L})_{11}}} = \sqrt{\frac{2I_{D6}}{\mu_p C_{ox}(\frac{W}{L})_6}}$$

$\Downarrow$

$$\boxed{\left(\frac{W}{L}\right)_{11} = \left(\frac{W}{L}\right)_6\frac{I_{D11}}{I_{D6}} = \left(\frac{W}{L}\right)_6\frac{I_{D10}}{I_{D6}}}$$

Also need $|V_{GS10}| = |V_{GS8}|$

Because $V_A = V_B \rightarrow V_{S10} = V_{S8} \rightarrow |V_{t10}| = |V_{t8}|$

$$\therefore |V_{ov10}| = |V_{ov8}| = \sqrt{\frac{2I_{D10}}{\mu_p C_{ox}(\frac{W}{L})_{10}}}$$

Thus:

$$R_8 = \frac{1}{\mu_p C_{ox}(\frac{W}{L})_8\sqrt{\frac{2I_{D10}}{\mu_p C_{ox}(\frac{W}{L})_{10}}}} = \frac{\sqrt{\mu_p C_{ox}(W/L)_{10}}}{\mu_p C_{ox}(\frac{W}{L})_8\sqrt{2I_{D10}}}$$