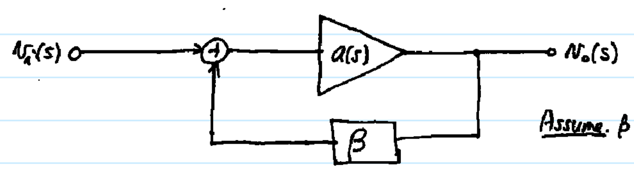
EE 140/240A | Settling Time | CTN | 1

Obtain expressions for overshoot ($V_{overshoot}$) and settling time (T_s) as functions of phase margin, ϕ_m:



Assume: β = const. w/ freq.

$$A(s) = \frac{v_o(s)}{v_i(s)} = \frac{a(s)}{1+a(s)\beta} = \frac{a_o\omega_1\omega_2}{s^2+(\omega_1+\omega_2)s+\omega_1\omega_2(1+a_o\beta)} = \underbrace{\frac{A_o\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{A_o\omega_o^2}{s^2+\left(\frac{\omega_o}{Q}\right)s+\omega_o^2}}_{\text{General Lowpass Biquad Transfer Functions}}$$

$[\omega_o = \omega_n]$

$$\left[a(s) = \frac{a_o}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} = \frac{a_o\omega_1\omega_2}{(s+\omega_1)(s+\omega_2)}\right]$$

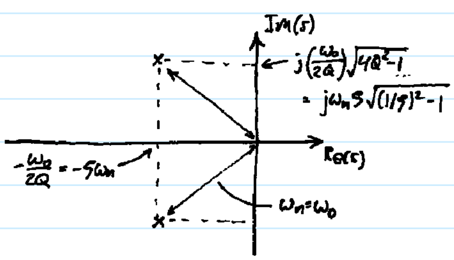
Where by direct comparison:

$$\begin{cases} \omega_n = \sqrt{\omega_1\omega_2(1+a_o\beta)} \\ 2\zeta\omega_n = \omega_1+\omega_2 \longrightarrow \zeta = \dfrac{\omega_1+\omega_2}{2\omega_n} = \dfrac{1}{2}\dfrac{\omega_1+\omega_2}{\sqrt{\omega_1\omega_2(1+a_o\beta)}} \\ A_o\omega_n^2 = a_o\omega_1\omega_2 \longrightarrow A_o = \dfrac{a_o}{1+a_o\beta} \end{cases}$$

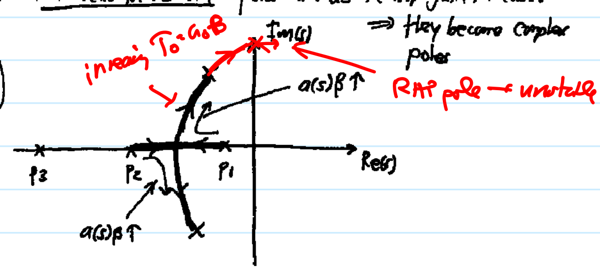
Frequency Response Spectra for various values of ζ on next page

$\boxed{\text{Properties of the General Lowpass Biquad Transfer Function}}$ (a very well-studied function!)

Pole-Zero Diagram:



Root Locus for FB Ckt: poles move as the loop gain increases



Time Domain Behavior:

$$v_o(t) = A_oV_i\left[1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \phi\right)\right], \quad \text{where } \phi = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

For ζ = small → $\phi \simeq \frac{\pi}{2}$

For $\zeta < 1$: (for PM < 90°)

$$\%\text{ Overshoot} = \frac{\text{Peak Value} - \text{Final Value}}{\text{Final Value}} = \exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right] \Rightarrow \boxed{V_{overshoot} = V_o\exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right]}$$

EE 140/240A $T_s = f(f_m)$ CTN 2

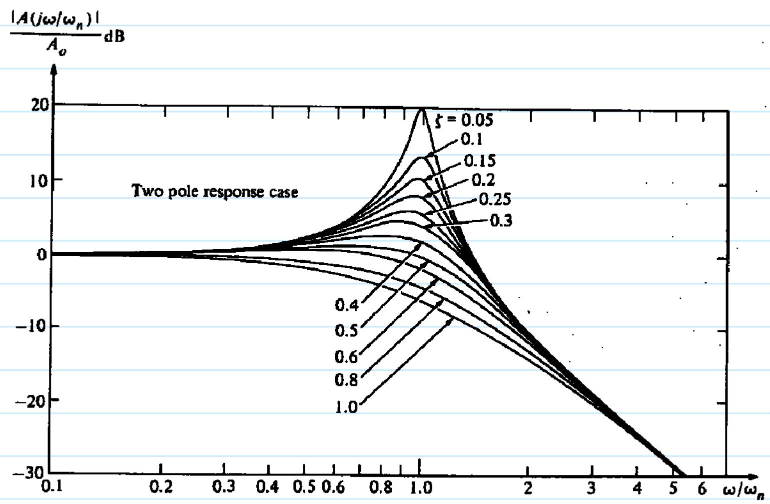
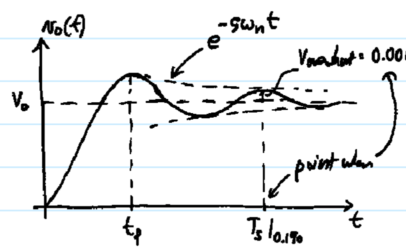


Figure C-2 Gain magnitude response for various values of ζ for a second-order, low-pass system.



Find t_p:

$$\sqrt{1-\zeta^2}\,\omega_n t_p + \phi = \frac{3\pi}{2}$$

$$[\zeta = \text{small} \rightarrow \phi \sim \tfrac{\pi}{2}] \Rightarrow \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

Approximate Determination of 0.1% Settling Time –

$$V_{overshoot}\, e^{-\zeta\omega_n(T_s - t_p)} = 0.001V_o \longrightarrow \boxed{T_s = t_p - \frac{1}{\zeta\omega_n}\ln\left[\frac{0.001V_o}{V_{overshoot}}\right]}$$

$$\longrightarrow \boxed{\zeta = -\frac{1}{\omega_n(T_s - t_p)}\ln\left[\frac{0.001V_o}{V_{overshoot}}\right]}$$

Determine $\phi_m = f(\zeta)$ –

⇒ first get an expression for loop transmission:

$$A(s) = \frac{a(s)}{1 + a(s)\beta} \longrightarrow a(s) = A(s) + a(s)A(s)\beta \longrightarrow a(s) = \frac{A(s)}{1 - \beta A(s)}$$

$$\rightarrow a(s)\beta = \frac{\beta A(s)}{1 - \beta A(s)} = \frac{\beta A_o\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 - \beta A_o\omega_n^2} \underset{\left[\beta \approx \frac{1}{A_o}\right]}{=} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

pole @ origin LHP pole