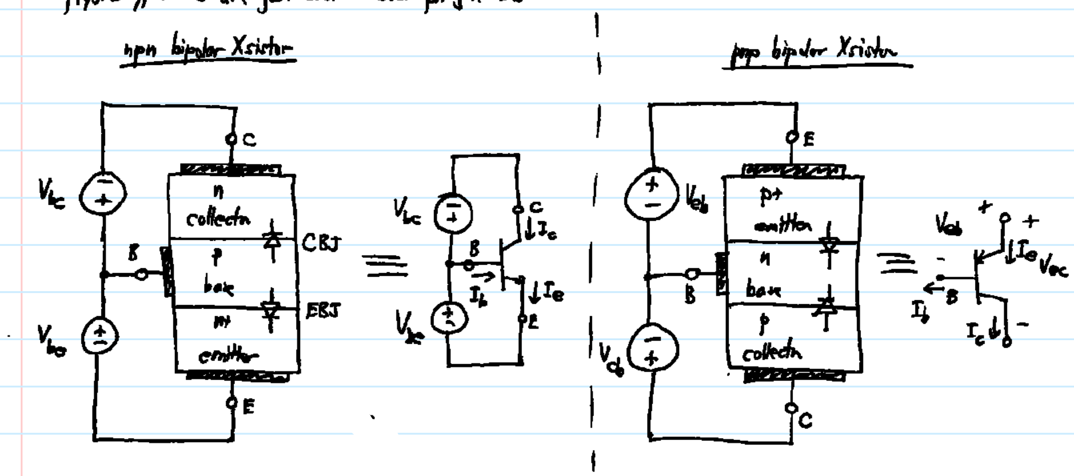
EE 140/240A | BJT Modeling | CTN | 1

Modeling the Bipolar Junction Transistor (BJT)

→ physically, BJT's are just back-to-back pn junctions



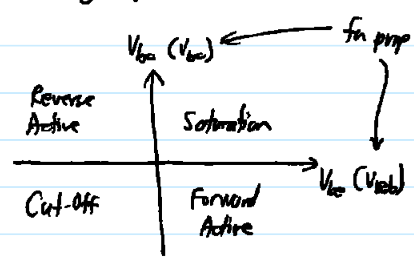
Region of Bipolar Xsistor Operation

→ Key: R ≜ reverse-biased, F ≜ forward-biased

EBJ	CBJ	Mode
R	R	Cut-off (both diodes off)
F	R	Forward Active (widely used in analog amplifier ckts)
R	F	Reverse Active
F	F	Saturation

⇒ can also think of this in a convenient graphical sense:

→ for npn (pnp):

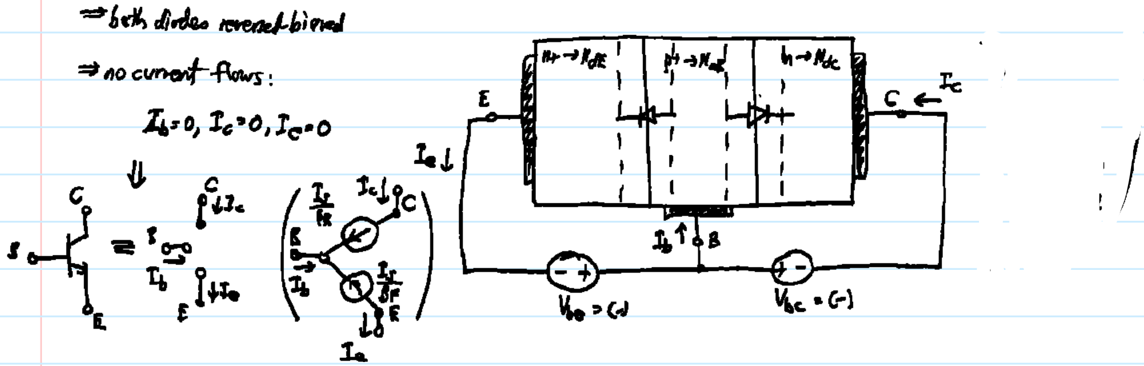


① Cut-Off Region - (npn transistor)

⇒ both diodes reverse-biased

⇒ no current flows:

$I_b = 0,\ I_c = 0,\ I_e = 0$



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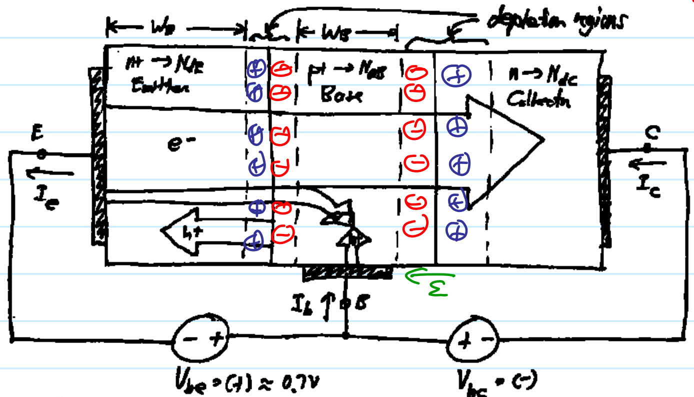
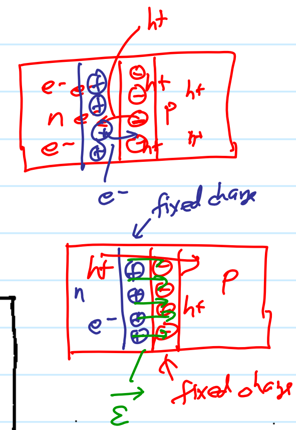
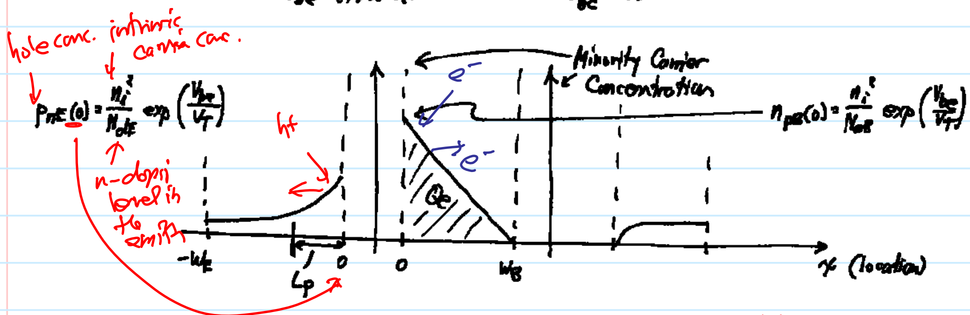
② Forward-Active Region - (npn transistor)

⇒ BEJ Forward-Biased (i.e., diode on), BCJ Reverse-Biased (i.e., diode off)







Forward biasing of the BEJ generates three current components:

① e⁻'s injected from emitter to base: $I_{nB} = -A J_{nB}^{diff}$ that make it to the collector

② h⁺'s injected from base to emitter: $I_{pE} = A J_{pE}^{diff}$

③ recombination of e⁻'s & h⁺'s in base: I_{rB}

$I_c = I_{nB} = ①$

$I_e = I_{nB} + I_{pE} + I_{rB} = ① + ② + ③$

$I_b = I_{pE} + I_{rB} = ② + ③$

$$I_{nB} = -A J_{nB}^{diff} = -Aq \underbrace{D_{nB}}_{\text{diffusion constant for e}^-\text{'s in B}} \underbrace{\frac{dn_{pB}(x)}{dx}}_{\text{slope}} = -qAD_{nB}\frac{[n_{pB}(W_B) - n_{pB}(0)]}{W_B} = \boxed{qAD_{nB}\frac{n_i^2}{N_{aB}W_B}\exp\left(\frac{V_{be}}{V_T}\right) = ①} \;*$$

$$\left[\begin{array}{l} n_{pB}(W_B) = \frac{n_i^2}{N_{aB}}\exp\left(\frac{V_{bc}}{V_T}\right) \approx 0 \\ n_{pB}(0) = \frac{n_i^2}{N_{aB}}\exp\left(\frac{V_{be}}{V_T}\right) \end{array}\right]$$

$$I_c = I_S \exp\left(\frac{V_{be}}{V_T}\right)$$

$$I_{pE} = A J_{pE}^{diff} = Aq \underbrace{D_{pE}}_{\text{diffusion constant for h}^+\text{'s in E}} \underbrace{\frac{dp_{nE}(x)}{dx}}_{\text{slope}} = qAD_{pE}\frac{[p_{nE}(0) - p_{nE}(-W_E)]}{W_E} = \boxed{qAD_{pE}\frac{n_i^2}{N_{dE}W_E}\exp\left(\frac{V_{be}}{V_T}\right) = ②} \;*$$

$$\left[\begin{array}{l} p_{nE}(0) = \frac{n_i^2}{N_{dE}}\exp\left(\frac{V_{be}}{V_T}\right) \\ p_{nE}(-W_E) \approx 0 \end{array}\right]$$

could also replace by diffusion length, L_p (for h⁺ in n-type material)

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minority-carrier charge in base

$$I_{rB} = \frac{Q_e}{\tau_b} = \frac{1}{\tau_b}\left[\frac{1}{2} n_{pB}(0) W_B qA\right] = \boxed{\frac{1}{2}\frac{n_i^2 W_B qA}{N_B \tau_b}\exp\left(\frac{V_{BE}}{V_T}\right) = ③} \quad *$$

minority carrier lifetime in base

Define Forward Current Gain $=\beta_F$:

$$\beta_F = \frac{I_c}{I_b} = \frac{①}{③+②} = \frac{\frac{qAD_{nB}n_i^2}{N_{aB}W_B}}{\frac{1}{2}\frac{n_i^2 W_B qA}{N_{aB}\tau_b} + \frac{qAD_{pE}n_i^2}{N_{dE}\underset{L_p}{W_E}}} = \left[\frac{W_B^2}{2\tau_b D_{nB}} + \frac{D_{pE}W_B \overset{N_{aB}}{N_A}}{D_{nB}\underset{L_p}{W_E}\underset{N_{dE}}{N_D}}\right]^{-1}$$

To maximize β_F, want: ① W_B = small

② $N_{dE} \gg N_{aB}$ (this is why emitter is n+) → also leads to $D_{pE} \ll D_{nB}$ — which we also want

③ τ_b = large (base Si must be free of impurities/defects to prevent recombination)

More Complete Expression for β_F:

$$\beta_F = \underbrace{\frac{N_{aB}W_B}{D_{nB}}\cdot\frac{D_{pE}}{N_{dE}L_{pE}}}_{\text{Injection Efficiency}} + \underbrace{\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}_{\text{Volume Recombination}} + \underbrace{s\left(\frac{A_s}{A_E}\right)\left(\frac{W_B}{D_{nB}}\right)}_{\text{Surface Recombination}} + \underbrace{\frac{W_E N_{aB} W_B}{2D_{nB}\tau n_i}e^{-\frac{V_{be}}{2V_T}}}_{\text{Recombination in the BE Depletion Region}}$$

Recombination in the BE Depletion Region ← significant @ low current levels

where: s = Surface Recombination Velocity

D_i = Diffusion constant

n_i = intrinsic carrier concentration

N_i = carrier concentration

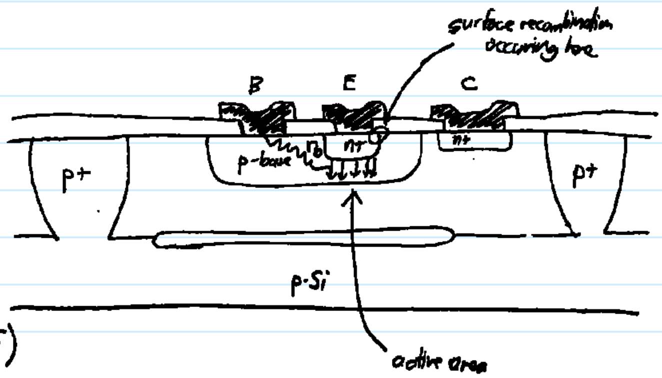
A_E = total emitter area

A_s = sidewall emitter area

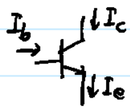
τ = minority carrier lifetime

L_i = diffusion length $(L_i = \sqrt{D_i \tau})$

W_B = active base width



So β relates I_b & I_c. To relate I_c & I_e, use KCL:

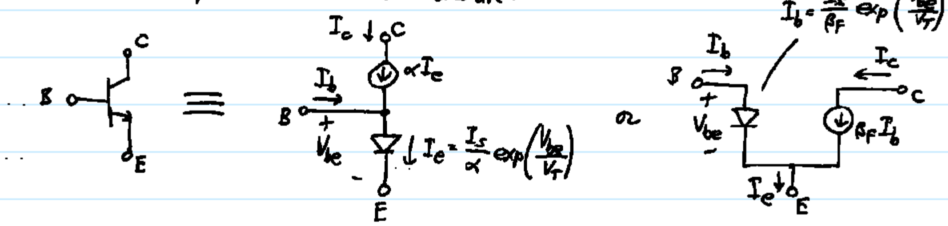


$$I_e = I_c + I_b = I_c + \frac{I_c}{\beta} = \left(1+\frac{1}{\beta}\right) I_c$$

$$\Rightarrow I_c = \left(\frac{1}{1+\frac{1}{\beta}}\right) I_e = \left(\frac{\beta}{\beta+1}\right) I_e = \alpha I_e, \text{ where } \boxed{\alpha = \frac{\beta}{\beta+1}} \Rightarrow \boxed{\beta = \frac{\alpha}{1-\alpha}}$$

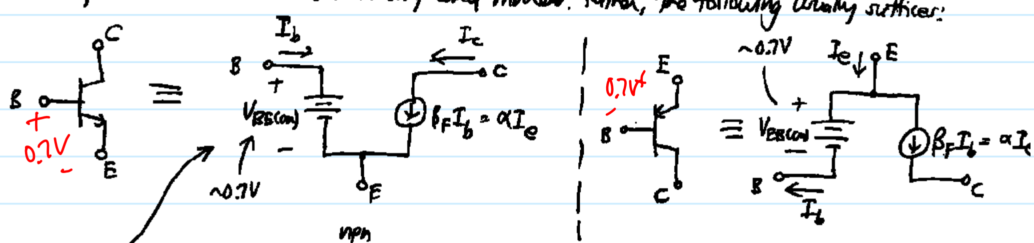
Equivalent Large Signal Ckt. Models for Forward-Active BJTs

There are several of them. The most useful ones are:

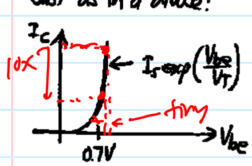


Common Base (CCCS) Common Emitter (CCCS)

But usually one doesn't have to use those complicated models. Rather, the following usually suffices:



Just as in a diode:



You should already be used to using approximate models like this ⇒ the more complicated models are a waste of time in comparison

③ Reverse-Active Region –

⇒ very similar to forward-active region except now: BEJ reverse-biased
BCJ forward-biased

⇒ one important difference: $\beta_R \propto \frac{N_{dC} W_C D_{nB}}{N_{aB} W_B D_{pC}}$ (L_{pC}) → $N_{dC} \ll N_{aB}$ → $D_{nB} \ll D_{pC}$ (since collector is n⁻)

∴ β_R is much smaller than β_F

⇒ poor device performance

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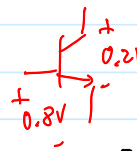
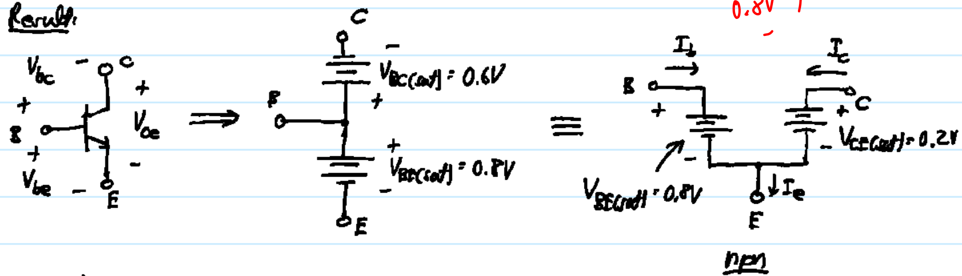
④ Saturation Region:

BEJ forward-biased → $V_{BE(on)}^{sat} \sim 0.8V$ (higher than 0.7V in saturation)

BCJ forward-biased → $V_{BC(on)}^{sat} \sim 0.6V$



Result:



⇒ currents now determined by the attached elements & KCL:

$I_e = I_b + I_c$; no longer have $I_b = \frac{I_c}{\beta}$ or $I_c = \alpha I_e$

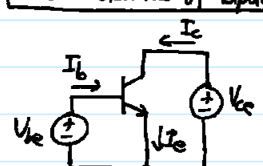
These no longer apply when BJT is in saturation.

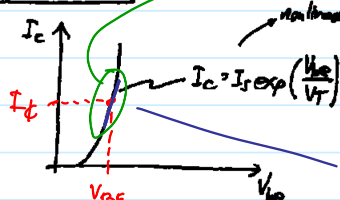
When determining DC operating point:

Pass { ① Assume forward-active → check for cut-off (enough V_{be}?)
② Determine V_{ce}.

③ If $V_{ce} > V_{CE(sat)} = 0.2V$, then ok (i.e., it's forward-active) ... otherwise, must do the analysis over assuming saturation.

IV Characteristics of Bipolar Junction Transistors



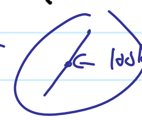


Region of Operation → if voltages (signals) are small, then this curve looks linear in the region of operation.

$I_c = I_S \exp\left(\frac{V_{be}}{V_T}\right)$ (a diode-like characteristic)

nonlinear → not easy to work with since we can't use linear system theory → thus need to linearize!

digital ckts. → piecewise lin.
analog ckts. → small-signal models

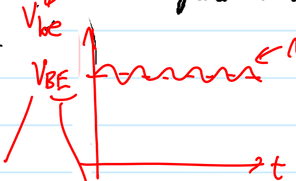


← looks linear in zoom-in!

upper case variable
lower case "be" (subscript) } total signal

$V_{be} = V_{BE} + v_{be}$

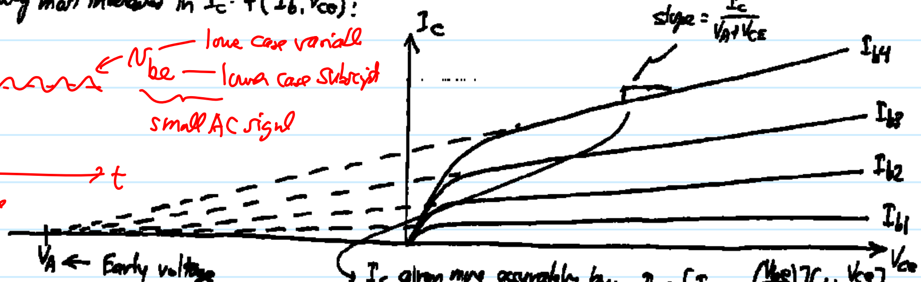
⇒ really most interested in $I_c = f(I_b, V_{ce})$:



v_{be} — lower case variable, — lower case subscript

small AC signal

upper case variable, upper case subscript: Large DC voltage (DC Bias Pt.)



V_A ← Early voltage

I_c given more accurately by: $I_c = \left[I_S \exp\left(\frac{V_{be}}{V_T}\right)\right]\left(1 + \frac{V_{ce}}{V_A}\right)$

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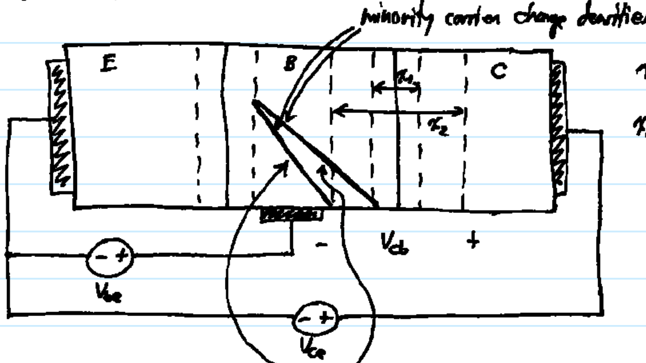
What is happening physically?



$x_1 \triangleq$ depl. region width for $V_{ce} = V_{ce1}$

$x_2 \triangleq$ depl. region width for $V_{ce} = V_{ce2} > V_{ce1}$

① Case: $V_{ce} = V_{ce1} \rightarrow x_1 \rightarrow I_{c1} \propto$ slope of this line

② Now, increase $V_{ce1} \rightarrow V_{ce2} \rightarrow V_{cb}\uparrow \rightarrow x\uparrow$ to $x_2 \rightarrow I_{c2} \propto$ slope of this line

$\therefore I_{c2} > I_{c1}$

Thus, $V_{ce}\uparrow \rightarrow I_c\uparrow$ due to $x_{depl.}\uparrow$

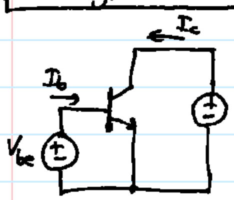
Result: $I_c = f(I_b, V_{ce})$ in forward-active!

$$I_c = \left[I_s \exp\left(\frac{V_{be}}{V_T}\right)\right]\left[1 + \frac{V_{ce}}{V_A}\right]$$

This, then, is a more accurate I_c equation.

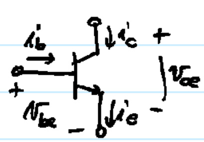
Small-Signal Models for Forward-Active Bipolar Xsistors

Y-Parameter Model

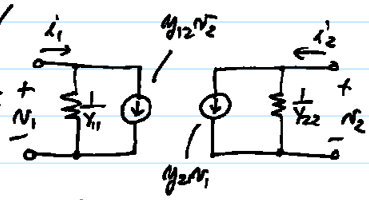


S.S. Ckt $\Longrightarrow$

zero out all DC sources



$\equiv$

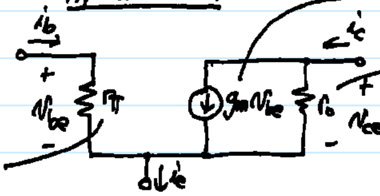


$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0}$ $y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$

$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$ $y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0}$

If only interested in the forward direction

Hybrid-π Model



$r_\pi = \frac{v_{be}}{i_b}$

$g_m = \frac{i_c}{v_{be}}$

$r_o = \frac{v_{ce}}{i_c}$

Specified by the bias point.