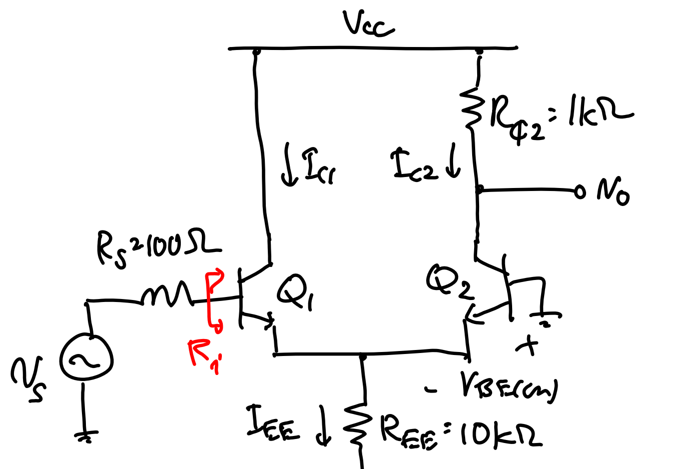
## Lecture 5: MOS Inspection Analysis

- **Announcements**:
- Handed out computer account sheets
- **Lecture Topics**:
  - Multi-Tx Amplifier Examples
  - MOS Inspection Analysis
- ----------------------------------------
- **Last Time**:
- Multi-TX inspection analysis
- Continue with this

$\boxed{\text{Inspection Analysis of a Multi-Transistor Ckt.}}$

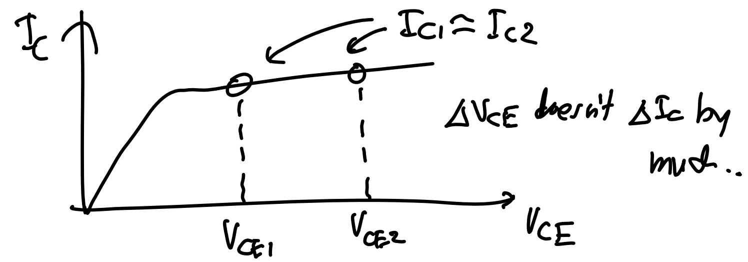



Assume $Q_1$ & $Q_2$ identical.
Want $R_i, R_o$, gain.

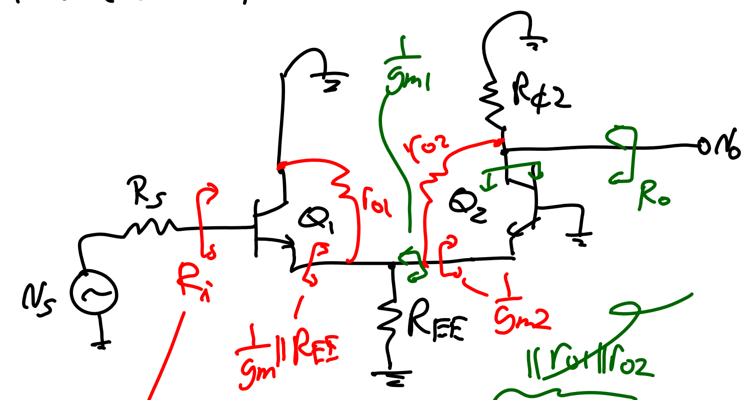
Find the DC operating pt.

$$I_{EE} = \frac{-V_{BE(on)} - V_{EE}}{R_{EE}} \quad (V_{BE(on)} = 0.7V)$$

$$I_{C1} = I_{C2} = \frac{I_{EE}}{2} \rightarrow \begin{cases} r_o = r_{o1} = r_{o2} \\ g_m = g_{m1} = g_{m2} \\ r_\pi = r_{\pi1} = r_{\pi2} \end{cases}$$




Draw the S.S. Ckt:




$$R_i = r_\pi + (\beta+1)R_E = r_{\pi1} + (\beta+1)\left(R_{EE} \| \frac{1}{g_{m2}}\right)$$

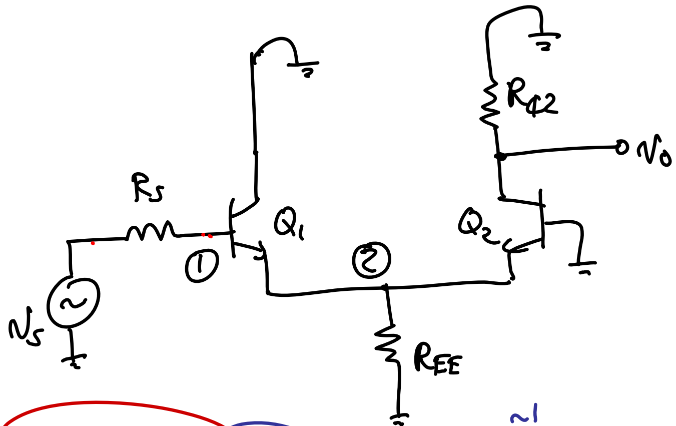
$$\left[R_{EE} \gg \frac{1}{g_{m2}}\right] \rightarrow R_i \approx r_{\pi1} + \underbrace{\frac{(\beta+1)}{g_{m2}}}_{r_{\pi2}}$$

$$R_i = 2r_\pi$$

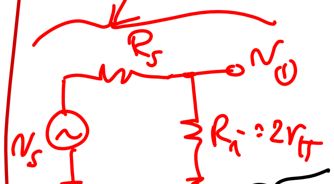
$$R_o = R_{C2} \| r_{o2}\left(1 + \frac{g_{m2}\left(\frac{1}{g_{m1}} + \cancel{\frac{R_S}{\beta+1}}\right)}{1 + \cancel{1/r_\pi}}\right)$$ assuming $R_S$ small

$$= R_{C2} \| (2r_{o2}) \approx R_{C2}$$

500kΩ–MΩ's $[2r_{o2} \gg R_{C2}]$

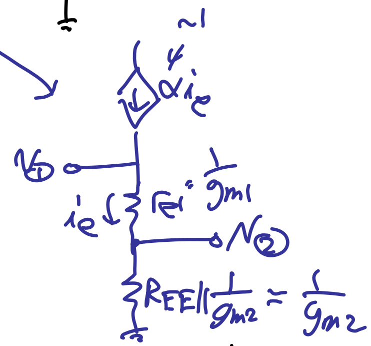



$$\frac{v_o}{v_s} = \frac{v_1}{v_s} \cdot \frac{v_2}{v_1} \cdot \frac{v_o}{v_2}$$




loading going forward to ground

$$\frac{v_1}{v_s} = \frac{2r_\pi}{2r_\pi + R_s}$$



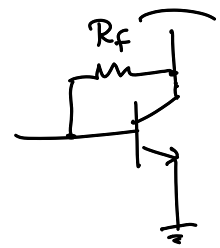

Just a voltage divider: $\frac{v_2}{v_1} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} = \frac{1}{2}$


$\frac{v_o}{v_2} = -G_{m2} R_{C} = g_m R_{C2}$ — negl. $r_{o2}$

$$G_{m2} = -\left(\frac{r_{\pi 2}}{r_{\pi 2} + 0}\right) g_m = -g_m$$

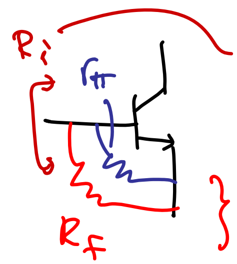
$$\frac{v_o}{v_s} = \frac{1}{2}\left(\frac{2r_\pi}{2r_\pi + R_S}\right) g_{m2} R_{C2} \approx \frac{1}{2} g_m R_{C2}$$

Reminder: Inspection analysis might not work when there is feedback:




feedback → not really inspectable (until the last week of class)


$R_f$ just in parallel w/ $r_\pi$



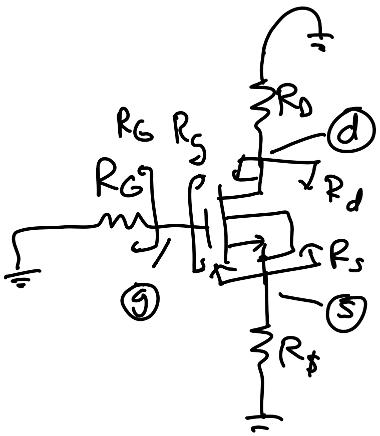

$R_i = r_\pi' + (\beta' + 1) R_E$ (not the same as $\beta$ & you figure out what it is in your HW)

$r_\pi' = r_\pi \| R_F$

## MOS Xsistor Ckts.

⇒ for now, ignore Body effect (i.e., ignore $g_{mb}$)

↳ use the same inspection formulas as bipolar, but use $\beta \to \infty$, $r_\pi = \frac{\beta}{g_m} \to \infty$




⇒ referring to the bipolar "Inspection Formula" sheet:

| Bipolar | | MOS |
|---|---|---|
| $R_b = \left(\frac{1}{g_m} + R_E\right)(\beta+1)$ | $\xrightarrow[\beta \to \infty]{}$ | $R_g = \infty$ |
| $R_e = \frac{1}{g_m} + \frac{R_B}{\beta+1}$ | $\xrightarrow[\beta \to \infty]{}$ | $R_s = \frac{1}{g_m}$ |
| $R_c = r_o\left[1 + \frac{g_m R_E}{1 + R_B/r_\pi}\right]$ | $\xrightarrow[r_\pi \to \infty]{}$ | $R_d = r_o(1 + g_m R_S)$ |

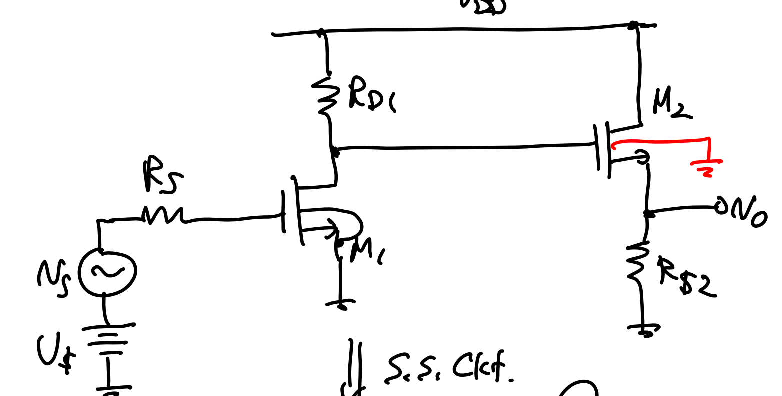
$$\frac{v_d}{v_g} = -G_m R_d \,, \quad G_m = \frac{g_m}{1 + g_m R_S}$$

$$\frac{v_d}{v_s} = -G_m R_d \,, \quad G_m = -g_m$$

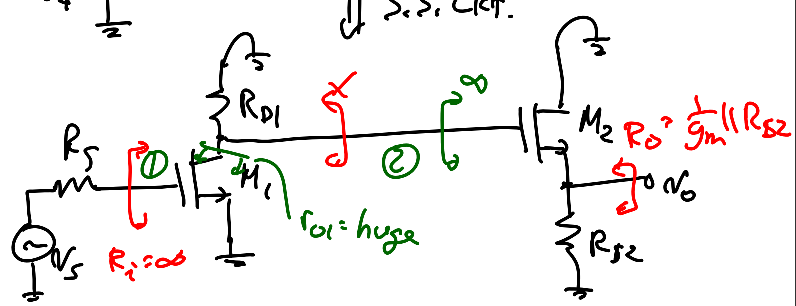
$$\frac{v_s}{v_g} = \frac{g_m R_S}{1 + g_m R_S} = \frac{R_S}{\frac{1}{g_m} + R_S}$$

## MOS Inspection Analysis

Ex. Common-Source Common-Drain Cascode




⇓ S.S. Ckt.



$$\frac{v_o}{v_s} = \frac{v_{1}}{v_s}\cdot\frac{v_{2}}{v_{1}}\cdot\frac{v_o}{v_{2}}$$

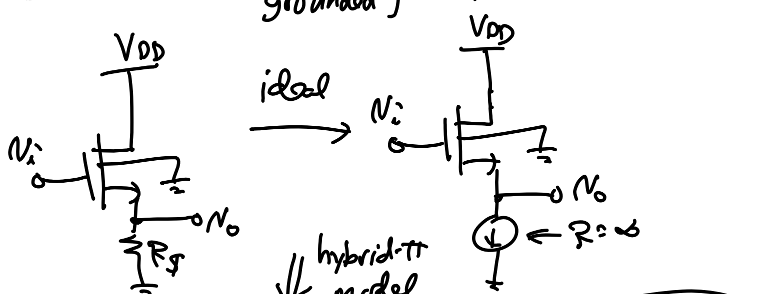
$$= (1)(-g_{m1}R_{D1})\left(\frac{R_{S2}}{\frac{1}{g_{m2}}+R_{S2}}\right) = \frac{v_o}{v_s}$$

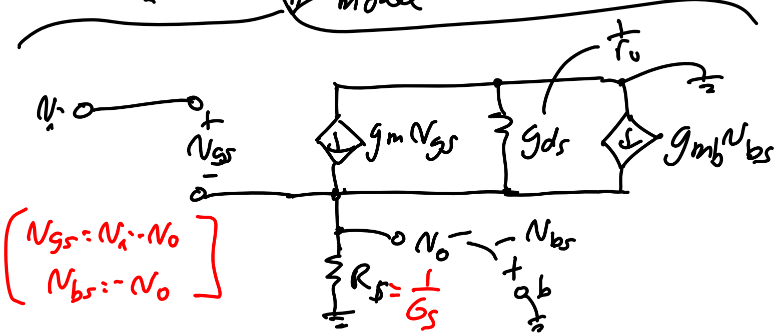
Problem: Simulate in SPICE → the gain will be 80–90% of what is calculated using

the problem is w/ $g_{mb}$ in the source follower

this is the difference between bipolar & MOS hybrid-π models

Source Follower: (w/ substrate grounded)




$v_{gs} = v_i - v_o$
$v_{bs} = -v_o$

$$g_m(v_i - v_o) = v_o(g_{ds}+G_s+g_{mb})$$

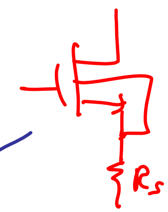
$$\Rightarrow A_v = \frac{v_o}{v_i} = \frac{g_m}{g_m+g_{mb}+g_{ds}+G_s}$$

$R_s \to \infty \to G_s = 0$
$g_{ds} \ll g_m + g_{mb}$

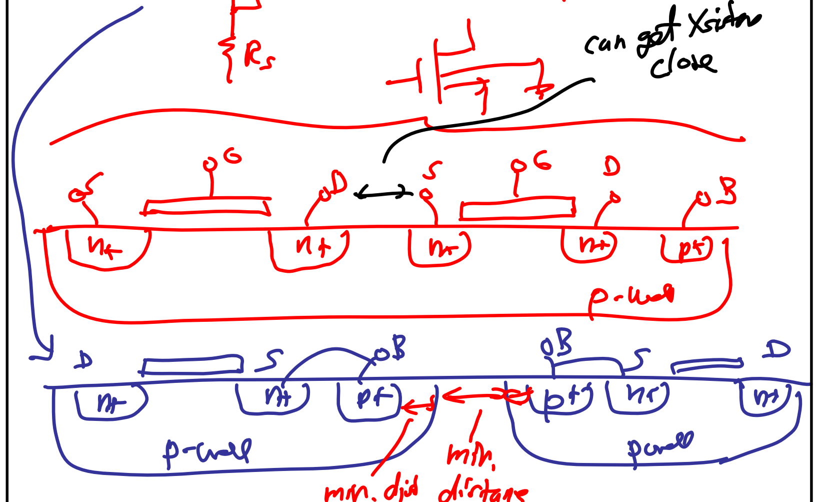
Body factor

$$A_v \cong \frac{g_m}{g_m+g_{mb}} = \frac{1}{\underbrace{1+\eta}_{\neq 1}}, \quad \eta = \frac{g_{mb}}{g_m} = \frac{\gamma}{2\sqrt{V_{SB}+2\phi_F}}$$

To make it '1', do this:

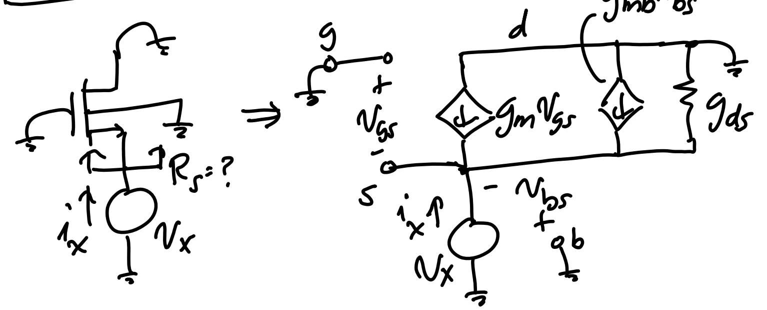



⇒ not generally practical



X Need min. distances between wells when Xsistors have their own wells → takes more chip area → bad → cost!

## Effect of $g_{mb}$ on Impedance




$$\begin{bmatrix} v_{gs} = -v_x = v_{bs} \\ v_{ds} = -v_x \end{bmatrix}$$

$g_m + g_{mb} + g_{ds}$

$$R_o = \frac{1}{g_m + g_{mb} + g_{ds}} = \frac{1}{g_m} \| \frac{1}{g_{mb}} \| r_o$$

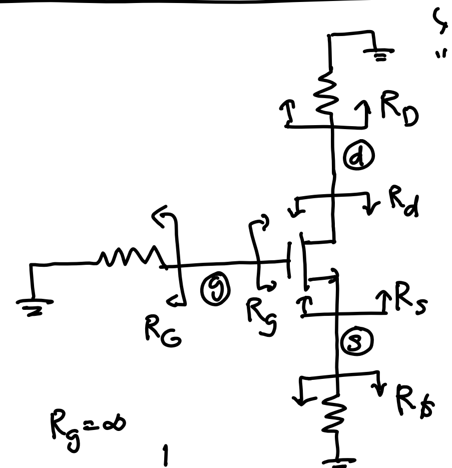
$[g_{ds} \ll g_m + g_{mb}]$

$$R_o \approx \frac{1}{g_m + g_{mb}}$$

⇒ more extensive analysis shows that other inspection formulas change to accommodate a grounded body by replacing "$g_m$" ~~in the denominator~~ w/ "$g_m + g_{mb}$"

⇒ end up w/ the following:

## MOS Inspection Formulas w/ Substrate Grounded




↳ only difference from "substrate tied to source" case is that $g_m$ is replaced by $g_m + g_{mb}$ in some of the formulas

particularly ones where the source is involved!

$R_g = \infty$

$R_s = \frac{1}{g_m + g_{mb}}$

$R_d = r_o[1 + (g_m + g_{mb})R_S]$

$\frac{v_d}{v_g} = -G_m R_d$, $G_m = \frac{g_m}{1 + (g_m + g_{mb})R_S}$

$\frac{v_d}{v_s} = -G_m R_d$, $G_m = -(g_m + g_{mb})$

$\frac{v_s}{v_g} = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S}$

<u>Remark.</u> When the substrate is tied to the source, $g_{mb} = 0$.