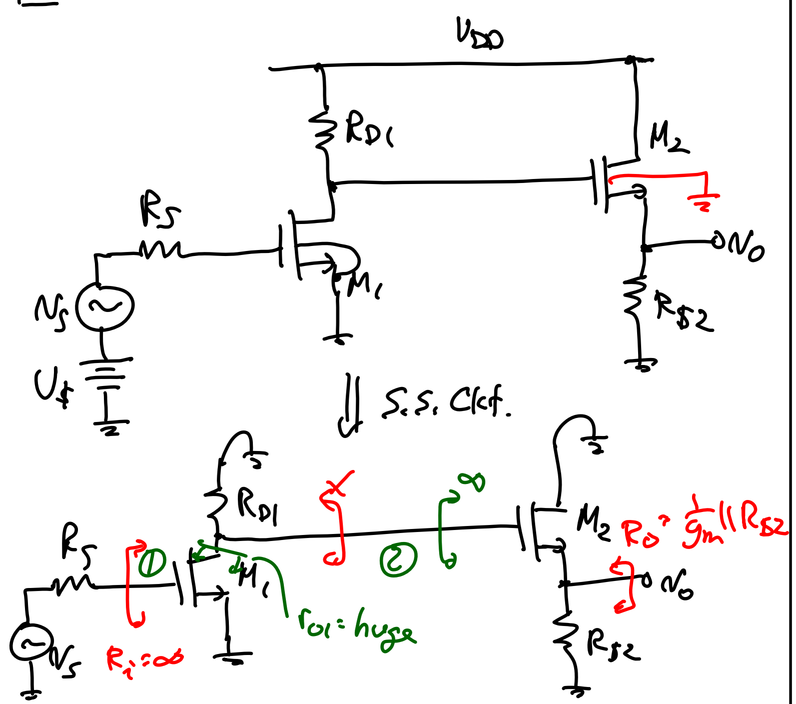
Lecture 6: Frequency Response Inspection Analysis

- **Announcements**:
- HW#2 due tomorrow at 8 a.m.
- HW#3 now online soon; Lab#1 online
- **Lecture Topics**:
  - Amplifier Bode plot
  - Open Circuit Time Constant (OCTC) Analysis
  - Frequency Response Inspection Analysis
- -----------------------------------
- **Last Time**:

$\boxed{\text{MOS Inspection Analysis}}$

Ex. Common-Source Common-Drain Cascode



$$\frac{v_o}{v_s} = \frac{v_{o1}}{v_s}\cdot\frac{v_{o2}}{v_{o1}}\cdot\frac{v_o}{v_{o2}}$$

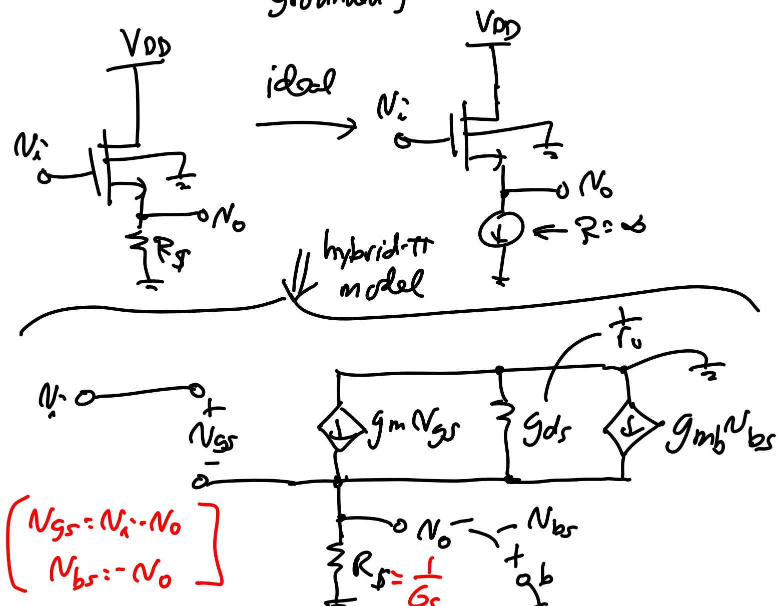
$$= (1)\left(-g_{m1}R_{D1}\right)\left(\frac{R_{S2}}{\frac{1}{g_{m2}}+R_{S2}}\right) = \frac{v_o}{v_s}$$

Problem: Simulate in SPICE → the gain will be 80–90% of what is calculated using

the problem is w/ g_{mb} in the source follower

this is the difference between bipolar & MOS hybrid-π models

Source Follower: (w/ substrate grounded)



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$$g_m(v_i - v_o) = v_o(g_{ds} + G_s + g_{mb})$$

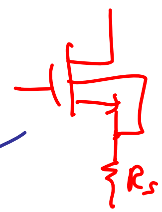
$$\Rightarrow A_v = \frac{v_o}{v_i} = \frac{g_m}{g_m + g_{mb} + g_{ds} + G_s}$$

$$\begin{bmatrix} R_s \to \infty \to G_s = 0 \\ g_{ds} << g_m + g_{mb} \end{bmatrix}$$

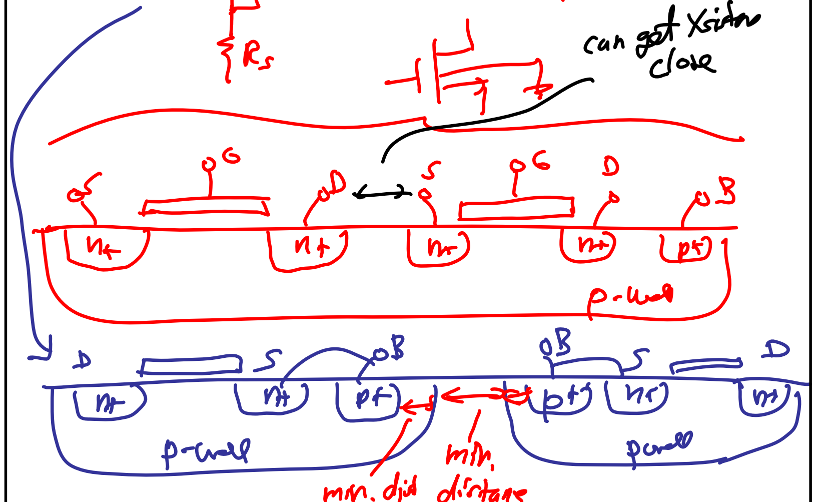
Body factor

$$\boxed{A_v \approx \frac{g_m}{g_m + g_{mb}} = \underbrace{\frac{1}{1+\eta}}_{\neq 1}, \quad \eta = \frac{g_{mb}}{g_m} = \frac{\gamma}{2\sqrt{V_{SB} + 2\phi_f}}}$$

To make it '1', do this:

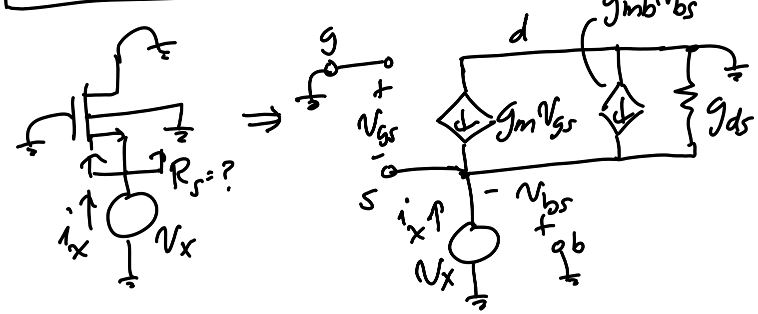


$\Rightarrow$ not generally practical



X Need min. distances between wells when Xsistors have their own wells $\rightarrow$ takes more chip area $\rightarrow$ {bad} $\rightarrow$ cost!

Effect of g_{mb} on Impedance



$$\begin{bmatrix} v_{gs} = -v_x = v_{bs} \\ v_{ds} = -v_x \end{bmatrix}$$

$$R_o = \frac{1}{g_m + g_{mb} + g_{ds}} = \frac{1}{g_m} \| \frac{1}{g_{mb}} \| r_o$$

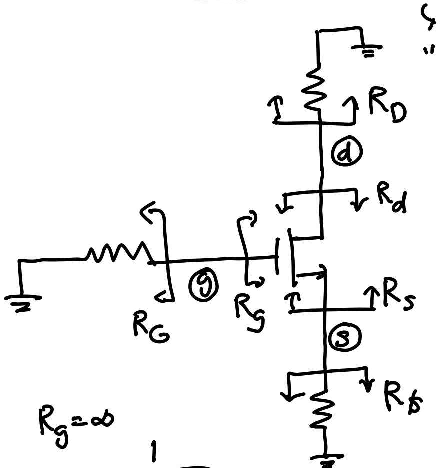
$$[g_{ds} << g_m + g_{mb}]$$

$$\boxed{R_o \approx \frac{1}{g_m + g_{mb}}}$$

$\Rightarrow$ more extensive analysis shows that other inspection formulas change to accommodate a grounded body by replacing "g_m" ~~in the denominator~~ w/ "$g_m + g_{mb}$"

$\rightarrow$ end up w/ the following:

MOS Inspection Formulas w/ Substrate Grounded



only difference from "substrate tied to source" case is that g_m is replaced by g_m+g_{mb} in some of the formulas → particularly ones where the source is involved!

$R_g = \infty$

$R_s = \frac{1}{g_m+g_{mb}}$

$R_d = r_o[1+(g_m+g_{mb})R_S]$

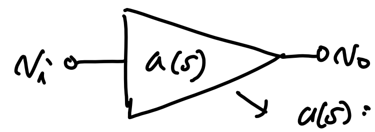
$\frac{v_d}{v_g} = -G_m R_{(d)}$, $G_m = \frac{g_m}{1+(g_m+g_{mb})R_S}$

$\frac{v_d}{v_s} = -G_m R_{(d)}$, $G_m = -(g_m+g_{mb})$

$\frac{v_s}{v_g} = \frac{g_m R_S}{1+(g_m+g_{mb})R_S}$

Remark. When the substrate is tied to the source, $g_{mb} = 0$.

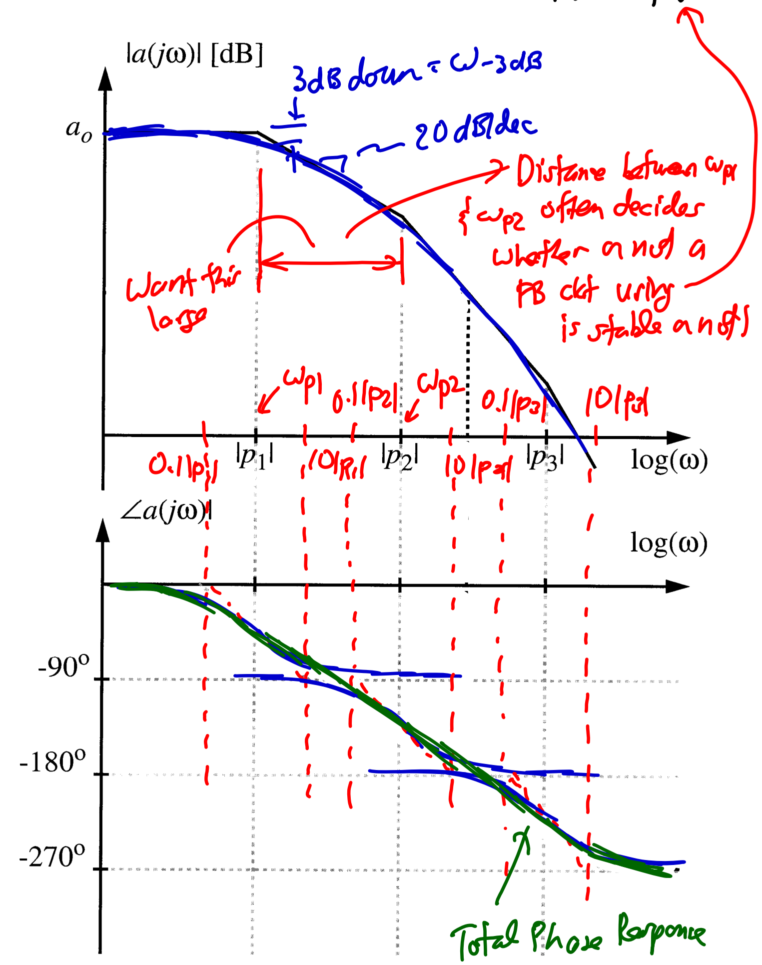
Poles & Zeros → Bode Plots



complex variable → $j\omega$

$$a(s) = \frac{v_o}{v_i}(s) = \frac{a_o}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})\cdots}$$

← midband gain

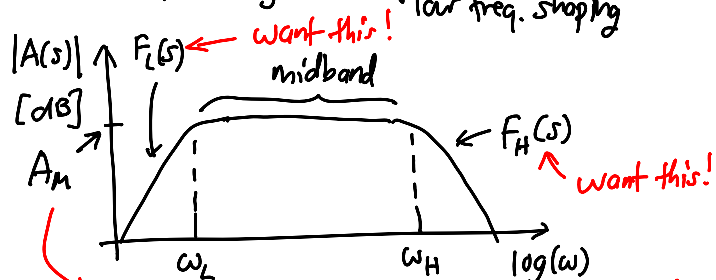


Freq. Response

Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s) \qquad s = j\omega$$

A_M: midband gain; $F_H(s)$: high frequency shaping; $F_L(s)$: low freq. shaping



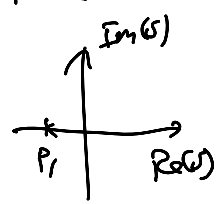
we've already found this using small-signal analysis

High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$



from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

b_1: coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo}$$

where C_j are capacitors in the H.F. ckt., i.e., small ones

$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1µF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$

(ii) $F_H(s) \cong \frac{1}{1 + \frac{s}{\omega_H}}$

(iii) $b_1 \cong \frac{1}{\omega_{p1}} \rightarrow \boxed{\omega_H = \omega_{p1} \cong \frac{1}{b_1} = \frac{1}{\sum_j \tau_{jo}} = \frac{1}{\sum_j C_j R_{jo}}}$

When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

(just FYI)

- Now, go to inspection formula sheet and go over how to use the frequency response parts

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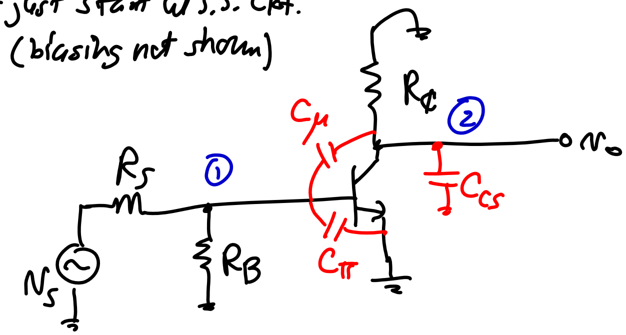
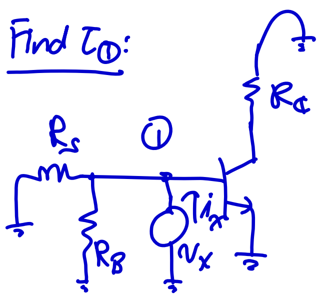
$\boxed{\text{Example H.F. Analysis}}$ (C.E. Ckt.)

→ just start w/ s.s. Ckt.
(biasing not shown)



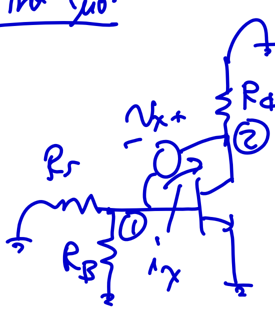
Find τ_1:



$R_1 = R_s \| R_B \| r_\pi$

$\tau_1 = C_\pi(R_s \| R_B \| r_\pi)$

Find $\tau_{\mu o}$:



$R_{\mu o} = \frac{v_x}{i_x} = R_1 + R_2 + g_m R_1 R_2$

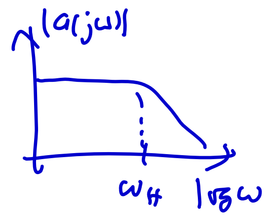
$R_{\mu o} = (R_s \| R_B \| r_\pi) + \cancel{r_o} \| R_C + g_m (R_s \| R_B \| r_\pi) R_C$

$\tau_{\mu o} = C_\mu R_{\mu o}$

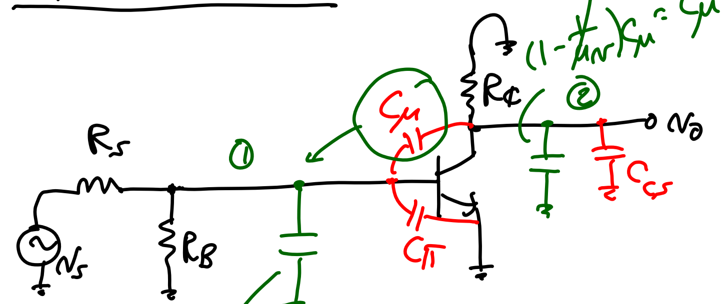
Find τ_2: $\tau_2 = C_{cs}(r_o \| R_C) \approx C_{cs} R_C$

$[r_o \gg R_C]$

$$\omega_H = \frac{1}{\tau_1 + \tau_{\mu o} + \tau_2}$$



Now, use Miller's Theorem:



$(1 - \frac{1}{a_v})C_\mu \approx C_\mu$

$C_M = (1 - a_v)C_\mu = (1 + g_m R_C)C_\mu$

$\tau_1 = (R_s \| R_B \| r_\pi)(C_\pi + (1 + g_m R_C)C_\mu)$

$\tau_2 = (C_\mu + C_{cs})R_C$

$$\omega_H = \frac{1}{\tau_1 + \tau_2}$$

Equal!

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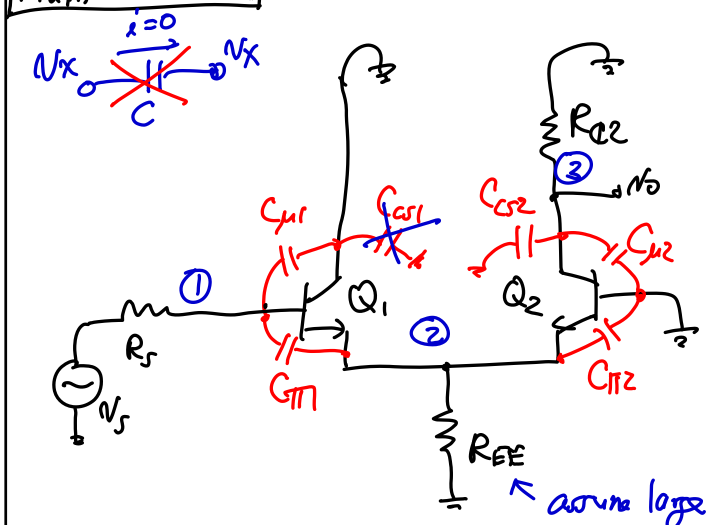
Multi-Xistor Ex.



$\tau_{①} = C_{\mu 1}(R_S \| (2r_\pi))$

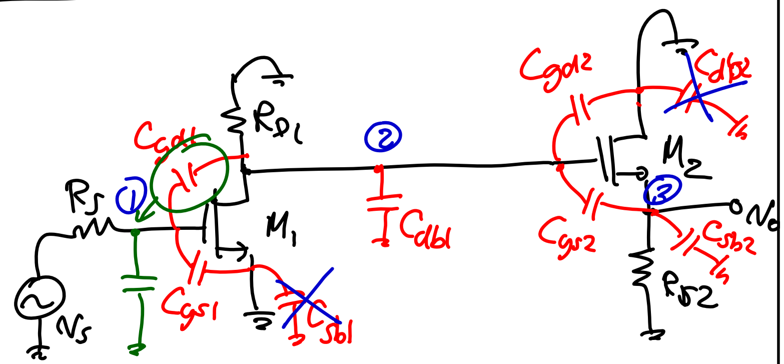
$\tau_{\pi 1} = C_{\pi}\left(r_\pi \,\Big\|\, \dfrac{R_s + \frac{1}{g_{m2}}}{1 + g_{m1}\left(\frac{1}{g_{m2}}\right)}\right)$

$\tau_{②} = C_{\pi 2}\left[\left(\frac{1}{g_{m1}} + \frac{R_s}{\beta+1}\right) \Big\| \frac{1}{g_{m2}} \Big\| \cancel{R_{EE}}\right]$ neg!

$\tau_{③} = (C_{cs2} + C_{\mu 2}) R_{C2}$

$$\omega_H = \frac{1}{\tau_{①} + \tau_{②} + \tau_{③} + \tau_{\pi}}$$

MOS Two-Stage Amplifier



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