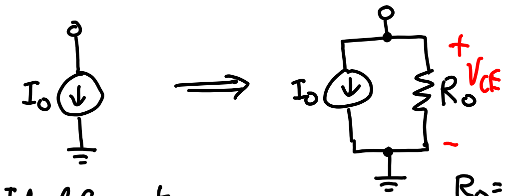
Lecture 9: Current Sources

- **Announcements:**
  - HW#4 online
  - Lab#1 in progress: make sure you have the updated lab version
- **Lecture Topics:**
  - Current Source Review
  - Widlar Current Source
  - Supply & Temperature Independent Biasing
- ---------------------------------------
- **Last Time:**
- Finished active load inspection analysis examples
- Now, start a new topic: Current Sources

over

Transistor Current Sources

How can a transistor implement a current source?



bipolar

$R_O = \infty \rightarrow$ ideal

Ideal Current Source

Actual Current Source

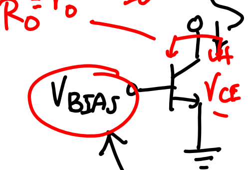
Bipolar

Forward-Active Xsistor Current Source

$V_A \sim 100\text{-}300k\Omega$

$R_O = r_o \cong \frac{V_A}{I_C}$

$$I_O = I_C = I_S \exp\left(\frac{V_{BIAS}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$



Very stable if V_{BIAS} = const. & V_A = large

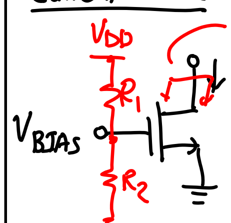
MOS

Note that V_{BIAS} must be very accurate due to exponential to several sig. figs.

e.g., $V_{BIAS} = 0.68745V$

Saturated MOS Xsistor Current Source

$R_O = r_o = \frac{1}{\lambda I_O}$



$$I_O = I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{BIAS}-V_t)^2(1+\lambda V_{DS})$$

Again, very stable if V_{BIAS} = const. & λ = small

Problem: need to generate a stable V_{BIAS}

We now focus on methods for generating V_{BIAS}.

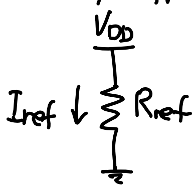
But how do we get this degree of precision using a transistor ckt?

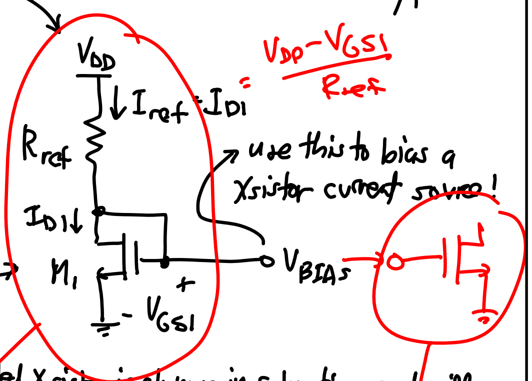
Solution:

$\boxed{\text{Replica Biasing}}$ (a simple & effective approach)

1. Generate the desired current.
2. Push the current through a Xsistor and allow it to reach a stable bias pt.
3. Use this stable bias pt. as V_{BIAS} ← this can be very precise!

One simple approach:





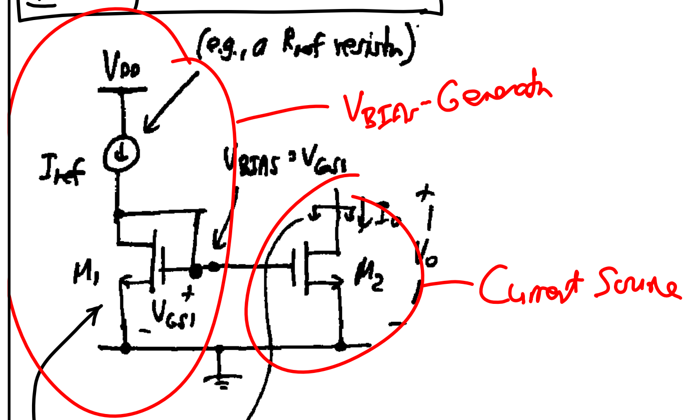
A diode connected Xsistor is always in saturation and will basically bias itself to support the needed current!

$$I_{ref} = I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_t)^2 (1 + \lambda V_{DS1})$$

V_{BIAS} ← V_{GS1}

Now, can distribute this V_{BIAS} to the gates of many MOS transistor current sources!

Ex. Simple MOS Current Source



$$R_o = r_o = \frac{1}{\lambda I_o}$$

Diode-connected Xsistor → saturation:

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{BIAS} - V_t)^2 (1 + \lambda V_{DS1})$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{BIAS} - V_t)^2 (1 + \lambda V_{DS2})$$

In general, $V_{DS1} \neq V_{DS2}$, but if λ = small, then little difference in I_{D1} & I_{D2}

1. Case: matched M_1 & M_2 ⟹ $I_o = I_{ref}$

2. Case: M_1 & M_2 scaled w/r to each other

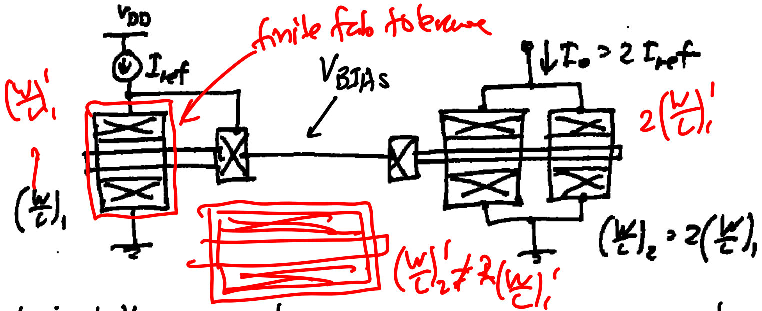
$$\Rightarrow \boxed{I_o = I_{ref}\frac{(W/L)_2}{(W/L)_1}}$$

⟹ use $L_1 = L_2$ for better accuracy, then:

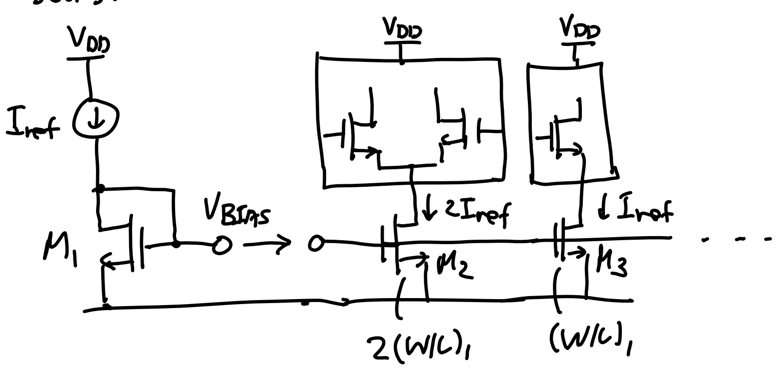
$$\boxed{\frac{I_o}{I_{ref}} = \frac{W_2}{W_1}}$$

Note: For better accuracy, should use multiple copies of one device when scaling currents → reduces edge effects!

Ex: Layout for a Doubling Current Source

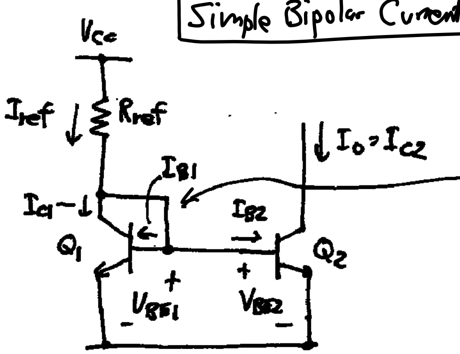


A single V_{BIAS} generator can now serve numerous current sources:



How about bipolar?

Simple Bipolar Current Source



KCL here to get $I_{C2} = f(I_{ref})$

Assume Q_1 & Q_2 are matched.

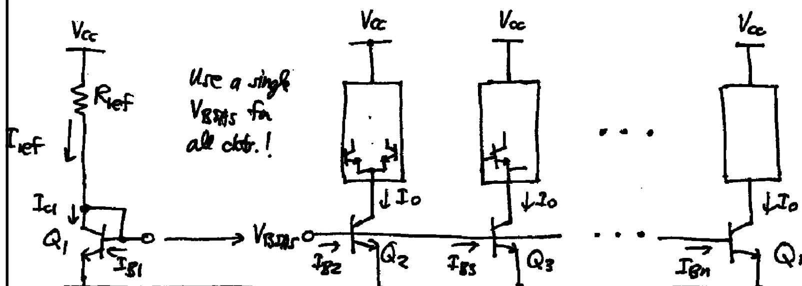
$V_{BE1} = V_{BE2} \longrightarrow I_{C1} = I_{C2} = I_o$ (neglecting V_A's) i.e., $V_A = \infty$

KCL: $I_{ref} = I_{C1} + I_{B1} + I_{B2} = I_{C1}\left(1 + \frac{2}{\beta}\right)$

$\therefore I_{C1} = I_{C2} = \boxed{I_o = \frac{I_{ref}}{1 + \frac{2}{\beta}}} \longrightarrow I_o \approx I_{ref}$ can say error $\sim \frac{2}{\beta}$

and $\boxed{I_{ref} = \frac{V_{CC} - V_{BE(on)}}{R_{ref}}}$ $\boxed{R_o = r_{o2}}$

Again, a single V_{BIAS} generator can serve many current sources throughout the IC chip:



$I_{ref} = I_{C1} + I_{B1} + I_{B2} + I_{B3} + \cdots + I_{Bn}$

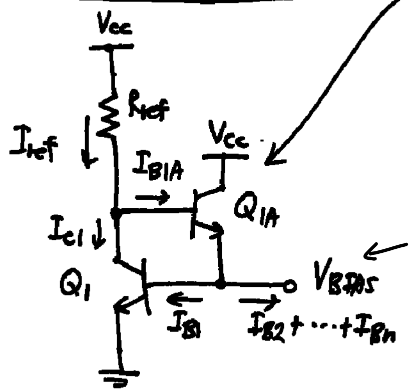
[Identical Xistors] $\Longrightarrow I_{ref} = I_{C1}\left(1 + \frac{n}{\beta}\right)$

$\Longrightarrow \boxed{I_o = I_{C1} = \frac{I_{ref}}{\left(1 + \frac{n}{\beta}\right)}}$

Problem: error $\sim \frac{n}{\beta}$ increases as n
(I_o deviates from I_{ref}, and the deviation depends on n.)

How can one reduce the error?

To reduce the error term, use a $\boxed{\text{Buffered } V_{BIAS}\text{-Generator}}$



Add a buffer Xsistor to attenuate base currents from Xsistor current sources.

← This can now drive the base currents of many bipolar transistor current sources (i.e., active loads).

$I_{ref} = I_{C1} + I_{B1A}$

$$I_{B1A} = \frac{I_{B1} + I_{B2} + \cdots I_{Bn}}{\beta + 1} = \frac{n I_{C1}}{\beta(\beta+1)}$$

[Assuming identical Xsistors]

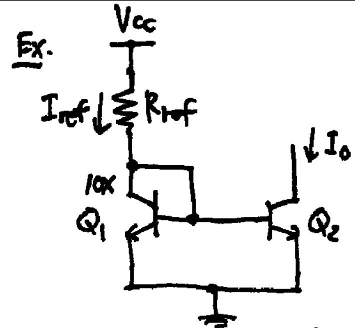
$$I_{ref} = I_{C1}\left(1 + \frac{n}{\beta(\beta+1)}\right)$$

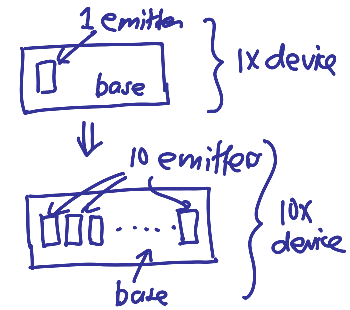
$$\rightarrow \boxed{I_o = I_{C2} = \frac{I_{ref}}{1 + \frac{n}{\beta(\beta+1)}} \approx I_{ref}\left(1 - \frac{n}{\beta^2}\right)}$$

Note: Now, $\boxed{I_{ref} = \frac{V_{CC} - 2V_{BE(on)}}{R_{ref}}}$

Problem: For power savings reasons, oftentimes very small bias currents are needed, on the order of 5μA. This might force too large an R_{ref} in the above bipolar V_{BIAS} generator.

Ex.





i.e., larger emitter area: ↑ Layout like this to get more accurate ratio.

If Q_1 is 10X larger than Q_2.

$\therefore I_{S1} = 10 I_{S2} \rightarrow I_o \cong I_{ref}/10$

$$\therefore I_o = \frac{(V_{CC} - V_{BE(on)})}{10 R_{ref}} \rightarrow R_{ref} = \frac{V_{CC} - V_{BE(on)}}{10 I_o}$$

↑ this helps to lower R_{ref}, but is it enough?

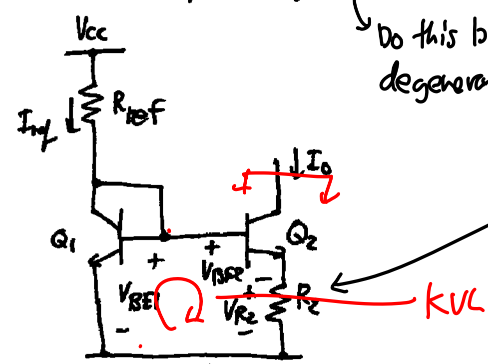
Ex. $I_o = 5\mu A$, $V_{CC} = 30V$

$R_{ref} \cong \frac{30}{5\mu} = \underline{600k\Omega}$ ← That's way too big! (Yes, there's only one of them on the chip, but this takes up too much space!)

The Low Current Solution: $\boxed{\text{Widlar Current Source}}$

⇒ scale $I_{C2} = I_o$ by reducing V_{BE2} (relative to V_{BE1}):

↳ Do this by emitter degenerating Q_2 via R_2



$$V_{BE1} = V_{BE2} + V_{R_2} = V_{BE2} + \frac{1}{\alpha} I_{C2}R_2 \approx V_{BE2} + I_{C2}R_2$$

$$I_{C2}R_2 = V_{BE1} - V_{BE2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}}$$

$$I_{C2}R_2 = V_T \ln \frac{I_{C1}}{I_{C2}}$$ [Assuming Q_1 & Q_2 are matched.]

$$\boxed{I_o R_2 = V_T \ln \frac{I_{ref}}{I_o}}$$

Rule of Thumb:

$V_{R_2} = I_{C2}R_2$	$I_{C2} = I_o$
18mV	$\frac{1}{2} I_{ref}$
42mV	$\frac{1}{5} I_{ref}$
60mV	$\frac{1}{10} I_{ref}$
120mV	$\frac{1}{100} I_{ref}$

5μA example again

Ex. scale by 100X using Widlar source

$$V_{BE1} - V_{BE2} = 120mV \rightarrow R_2 = \frac{120mV}{5\mu A} = 24k\Omega$$

$$I_{ref} = 500\mu A \rightarrow R_{ref} = \frac{30V}{500\mu A} = 60k\Omega$$

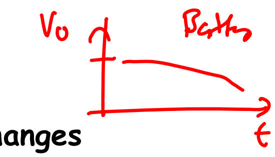
More reasonable than 6000kΩ before.
If want smaller, scale by 1000X instead.

Another advantage of the Widlar: larger R_o ∴ a more ideal current source:

$$R_o = r_{o2}(1 + g_{m2}R_2)$$

Supply & Temperature Independent Biasing

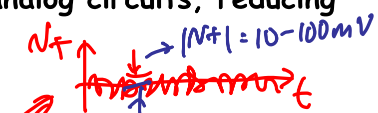
- Why is it necessary?
- For battery-operated systems, battery voltages vary over time
  - Amplifier gains change
  - Power consumption changes
  - Frequency of oscillators changes
  - In summary: long-term stability degrades
  - Large uncertainty in biasing translates to overdesign that wastes power

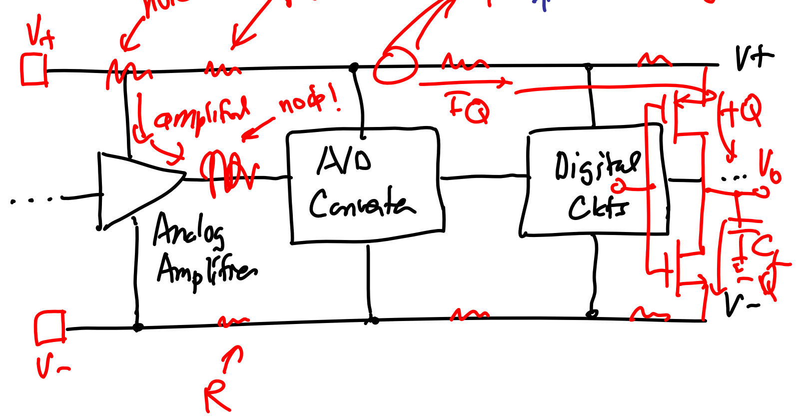


- Same issues as above when temperature varies with time

$$I_C = I_S \exp\left(\frac{V_{BE}}{\frac{kT}{q}}\right)$$

- Short-term supply variations
  - In mixed signal circuits, i.e., both analog and digital together, digital switching generates noise on the supply lines
  - Noise can couple to analog circuits, reducing their dynamic range



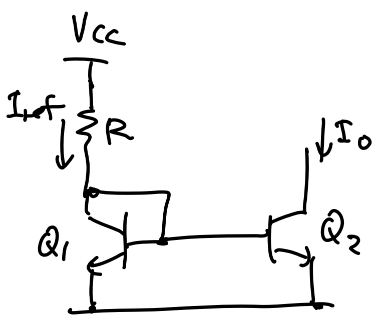


Define: Sensitivity of Y to X

$$S_X^Y = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{X}{Y}\frac{\Delta Y}{\Delta X} = \frac{X}{Y}\frac{\partial Y}{\partial X}$$

For supply dependence, we want $S_{V_{CC}}^{I_O} = 0$.

$\boxed{\text{Simple Current Source}}$



Neglecting base currents:

$$I_O = I_{ref} = \frac{V_{CC} - V_{BE(on)}}{R}$$

$$I_O \simeq \frac{V_{CC}}{R} \quad [V_{CC} \gg V_{BE(on)}]$$

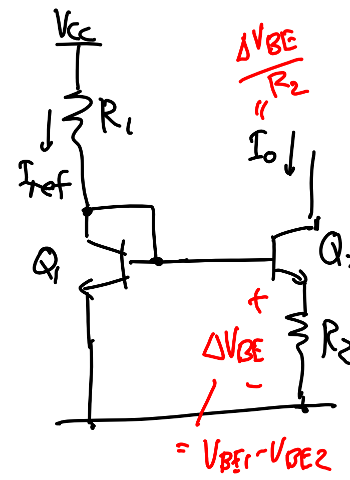
Thus:

$$S_R^{I_O} = \frac{R}{I_O}\frac{\partial I_O}{\partial R} = \frac{R^2}{V_{CC}}\left(-\frac{V_{CC}}{R^2}\right) \longrightarrow S_R^{I_O} = -1$$

$$S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{\partial I_O}{\partial V_{CC}} = R\left(\frac{1}{R}\right) \Rightarrow S_{V_{CC}}^{I_O} = 1$$

∴ a 10% change in V_{CC} leads to 10% change in I_O! terrible!

$\boxed{\text{Widlar Current Source}}$ (Any better?)



$$V_T \underbrace{\ln\frac{I_{ref}}{I_O}}_{(\ln I_{ref} - \ln I_O)} = I_O R_2$$

↓ Differentiate w/r to V_{CC}!

$$V_T\left(\frac{1}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}} - \frac{1}{I_O}\frac{\partial I_O}{\partial V_{CC}}\right) = R_2\frac{\partial I_O}{\partial V_{CC}}$$

↓ math

$$\frac{\partial I_O}{\partial V_{CC}} = \frac{\frac{V_T}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}}}{\left(R_2 + \frac{V_T}{I_O}\right)}$$

$$\therefore S_{V_{CC}}^{I_O} = \frac{V_{CC}}{I_O}\frac{\partial I_O}{\partial V_{CC}} = \frac{V_T\left(\frac{V_{CC}}{I_{ref}}\frac{\partial I_{ref}}{\partial V_{CC}}\right)}{I_O R_2 + V_T}$$

$S_{V_{CC}}^{I_{ref}} = 1$

$$\Rightarrow S_{V_{CC}}^{I_O} = \left(\frac{1}{1 + \frac{I_O R_2}{V_T}}\right) S_{V_{CC}}^{I_{ref}}$$

Since $I_{ref} = \frac{V_{CC} - V_{BE(on)}}{R_1} \simeq \frac{V_{CC}}{R_1} \Rightarrow S^{I_{ref}}_{V_{CC}} = 1$

$$\therefore \boxed{S^{I_O}_{V_{CC}} = \frac{1}{1 + \frac{I_O R_2}{V_T}}}$$

For $I_{ref} = 1mA$, $I_O = 10\mu A$, $R_2 = 11.9k\Omega$, then

$10\%\ \Delta$ in $V_{CC} \rightarrow 1.3\%\ \Delta$ in I_O

(better than the simple current source!)

How can we do better?

→ Use another voltage reference:

✓ ① V_{BE} → base emitter junction voltage

② V_Z → Zener diode

③ V_t → threshold voltage (MOS)

✓ ④ $V_T = \frac{kT}{q}$ → thermal voltage

✓ ⑤ E_g → bandgap