

Ideal Voltage Amplifier

→ ideal when $\frac{V_o}{V_s} = A_v$; i.e., when source and load R 's do not influence the gain of the amplifier.

For this to occur, the voltage division at the input & output must be eliminated.

This happens when:

$$\left. \begin{matrix} R_i = \infty \\ R_o = 0 \end{matrix} \right\} \text{These resistance values define an ideal voltage amplifier.}$$

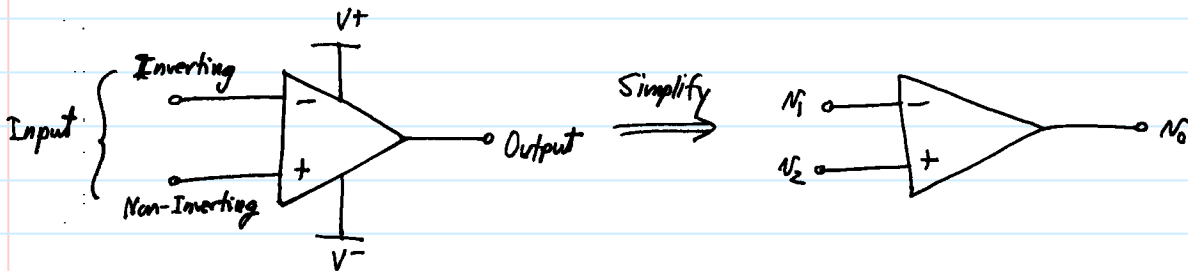
We'll look at other amplifier types later.

→ This, then, naturally leads us to:

Ideal Operational Amplifiers (Op Amps)

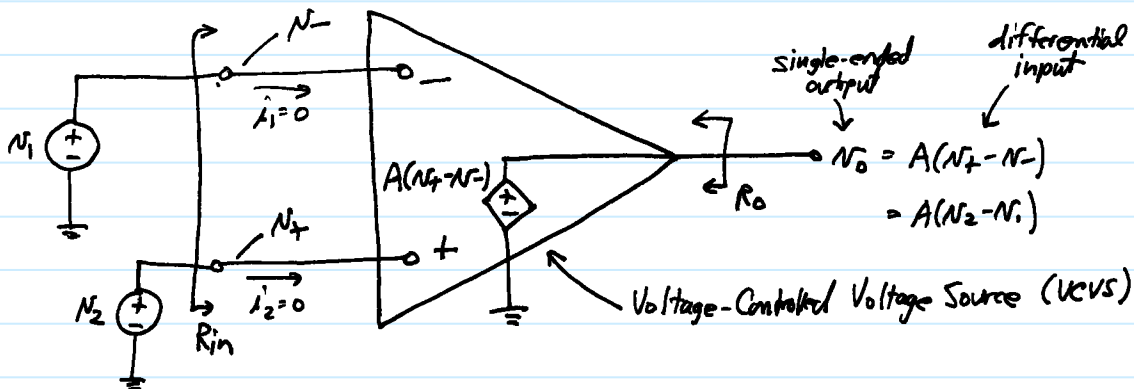
↳ The work horse of analog electronics → combinations of op amps w/ feedback components allow the implementation of analog computers, sampled-data systems, analog filters, A/D converters, DAC's, instrumentation amplifiers

In general, have a minimum of 5 terminals:



Perhaps the best way to define an op amp is thru its equivalent ckt:

Equivalent Ckt. of an Ideal Op Amp:



Properties of Ideal Op Amps:

① $R_{in} = \infty$ leads to ④ $i_+ = i_- = 0$

② $R_o = 0$

③ $A = \infty$ leads to ⑤ $V_+ = V_-$, assuming $N_o = \text{finite}$

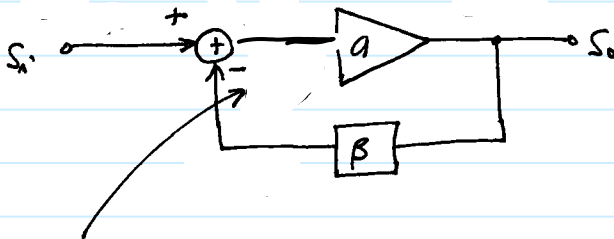
↳ Why? Because for $\infty(N_+ - N_-) = N_o = \text{finite}$

$$\therefore \underbrace{N_+ - N_-}_{\frac{N_o}{\infty}} = 0 \rightarrow N_+ = N_- \Rightarrow \text{virtual short ckt. (virtual ground)}$$

Big assumption! ($N_o = \text{finite}$)

How can we assume this? \Rightarrow only when there is an appropriate negative feedback path!

Negative Feedback



where S could be a current, voltage, displacement, etc.,...

Negative feedback acts to oppose or subtract from input.

$$\left. \begin{aligned} S_o &= a S_\Sigma \\ S_\Sigma &= S_i - \beta S_o \end{aligned} \right\} \Rightarrow \begin{aligned} S_o &= a(S_i - \beta S_o) \\ S_o(1 + a\beta) &= a S_i \end{aligned} \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1 + a\beta}}$$

overall transfer function

$[a \rightarrow \infty] \Rightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$

$\therefore S_o = \frac{1}{\beta} S_i = \text{finite} \checkmark$

(when there is neg. FB around the amplifier)

In Summary:

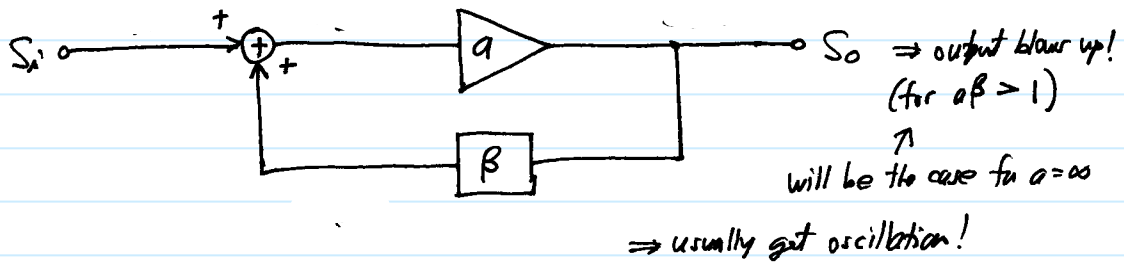
① Neg. FB can insure $S_o = \text{finite}$ even with $a = \infty$.

② ^{Overall} Gain dependent (or overall T.F.) dependent only on external components. (e.g., β)

③ Overall (closed-loop) gain $\frac{S_o}{S_i}$ is independent of amplifier gain a .

↳ very important! \Rightarrow as you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain. i.e., if you're shooting for $a = 50,000$, you might get 47,000 or 60,000 instead.

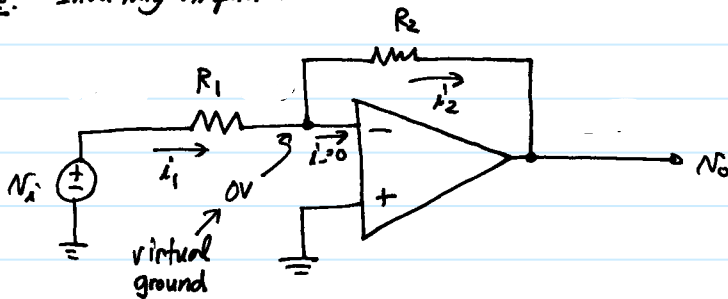
Contrast w/ Positive Feedback



Thus, for a bounded, controllable function, need negative FB around an op amp.

Op Amp Ckts.

Example. Inverting Amplifier

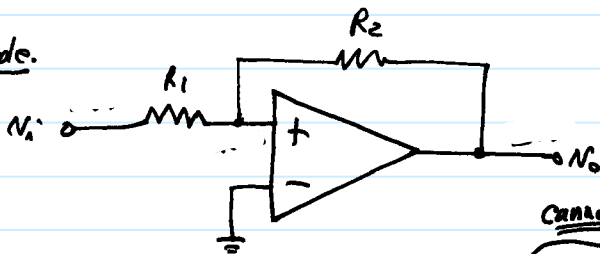


- ① Verify that there is negative FB. ✓
- ② $\therefore N_o = \text{finite} \rightarrow N_+ = N_- \rightarrow$ node attached to (-) terminal is virtual ground
- ③ $i_- = 0 \therefore i_1 = i_2$

$$\left. \begin{aligned} i_1 &= \frac{N_i - 0}{R_1} = \frac{N_i}{R_1} = i_2 \\ N_o &= 0 - i_2 R_2 = -i_2 R_2 \end{aligned} \right\} \Rightarrow N_o = -\left(\frac{R_2}{R_1}\right) N_i = -\frac{R_2}{R_1} N_i \therefore \boxed{\frac{N_o}{N_i} = -\frac{R_2}{R_1}}$$

Note: Gain dependent only on R_1 & R_2 (external components), not on the op amp gain.

Example.



① Verify that there is neg. FB X

Cannot analyze using ideal op amp method!

$\therefore N_o \neq \text{finite}, N_+ \neq N_- \Rightarrow$ this ckt. will "rail out"

$N_o = L^+$ or L^- depending on initial conds.

$N_+ = (+) \rightarrow L^+$
 $N_+ = (-) \rightarrow L^-$

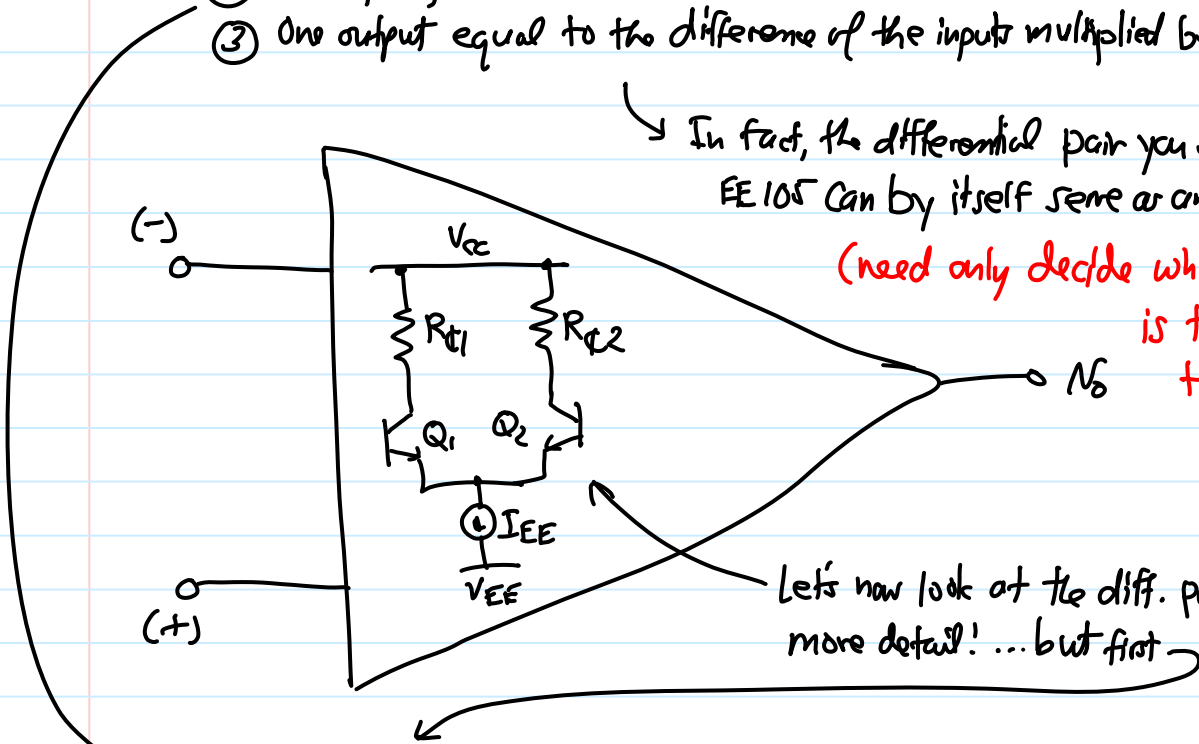
How does one make an op amp? (It turns out, you already know!)

⇒ Basic Needed Attributes:

- ① Gain (voltage gain).
- ② Two inputs, (+) and (-).
- ③ One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE 105 can by itself serve as an op amp!

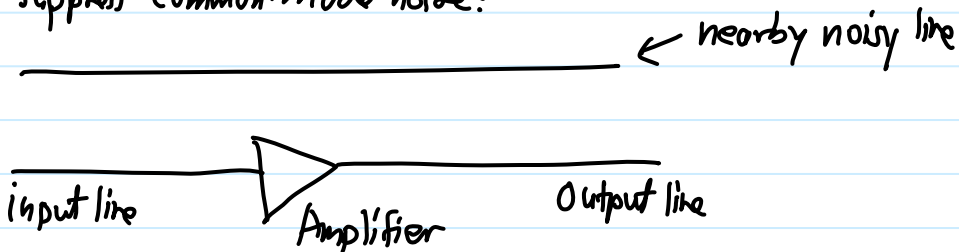
(need only decide which of the inputs is the (+) and (-) terminals)



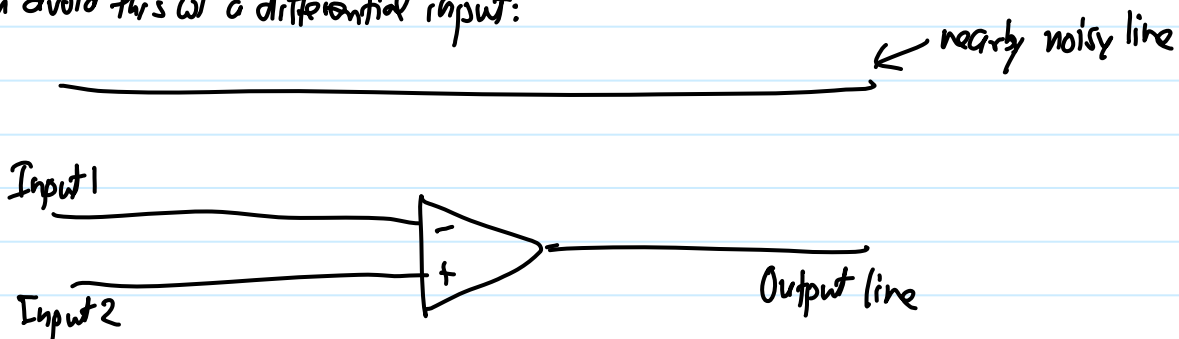
Let's now look at the diff. pair in more detail! ... but first

Why have 2 inputs?

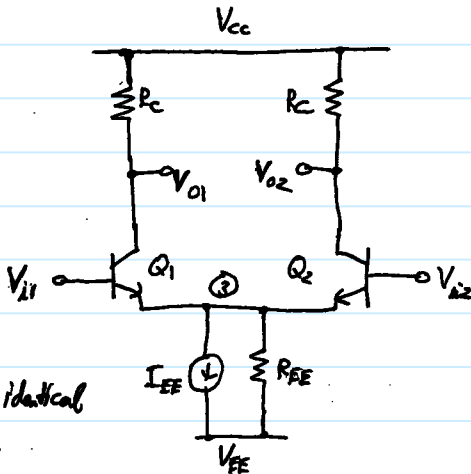
- ① To get a virtual short for op amp ckt.
- ② To suppress common-mode noise:



Can avoid this w a differential input:



Differential Pair (Emitter-Coupled Pair)



Assume:
 $Q_1 + Q_2$ identical
 $R_{C1} = R_{C2}$

Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

Definition: $V_{id} = V_{i1} - V_{i2}$ (differential input)
 $V_{icm} = \frac{V_{i1} + V_{i2}}{2}$ (common-mode input)

$$\Rightarrow \begin{cases} V_{i1} = V_{icm} + \frac{V_{id}}{2} \\ V_{i2} = V_{icm} - \frac{V_{id}}{2} \end{cases}$$

Differential Gain = $A_d = \frac{V_{O1} - V_{O2}}{V_{id}} = \frac{V_{od}}{V_{id}}$ (want this to be large for this differential amplification)

Common-Mode Gain = $A_{cm} = \frac{V_{O1}}{V_{cm}}$ or $\frac{V_{O2}}{V_{cm}}$ (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio = $CMRR = \frac{A_{dm}}{A_{cm}}$ (should be very high to favor the differential mode and reject the common-mode)

\Rightarrow we also want a high Common-Mode Input Range to reject DC input offsets
 \Rightarrow Note No need for bypass capacitors (large) to the inputs or outputs \rightarrow can just use direct coupling!

Biasing & Large Signal Common-Mode Behavior

Case: $R_{EE} = \infty \rightarrow$ ideal current source biasing $\rightarrow I_{E1} = I_{E2} = \frac{I_{EE}}{2} \rightarrow V_{O1} = V_{O2} \Rightarrow V_{od} = 0$

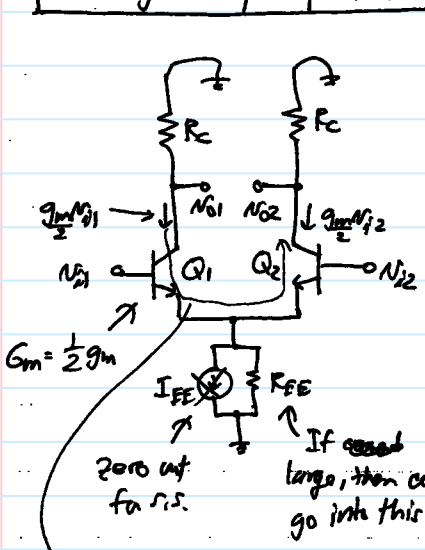
If $V_{cm} \uparrow \rightarrow V_{O1} \uparrow$, but current draw from I_{EE} stays constant $\therefore I_{C1}$ & I_{C2} stay constant \rightarrow bias pt. doesn't change
 $g_{m1} = \frac{1}{2} \frac{I_{EE}}{V_T}$

Case: $R_{EE} = \text{finite} \rightarrow V_{O1} = V_{i1} - V_{BE(on)}$

If $V_{icm} \uparrow \rightarrow V_{O1} \uparrow \rightarrow I_{E1} = I_{E2} \uparrow$ (current draw = $I_{EE} + \frac{V_{O1}}{R_{EE}}$)

\Rightarrow in general, R_{EE} will be large, so this component won't be large, and the bias pt. won't Δ much

Small-Signal Analysis of Diff. Pair



$$N_{o1} = -\frac{1}{2} g_m N_{i1} R_c + \frac{1}{2} g_m N_{i2} R_c$$

$$N_{o2} = -\frac{1}{2} g_m N_{i2} R_c + \frac{1}{2} g_m N_{i1} R_c$$

$$N_{o1} = -\frac{1}{2} g_m R_c (N_{i1} - N_{i2})$$

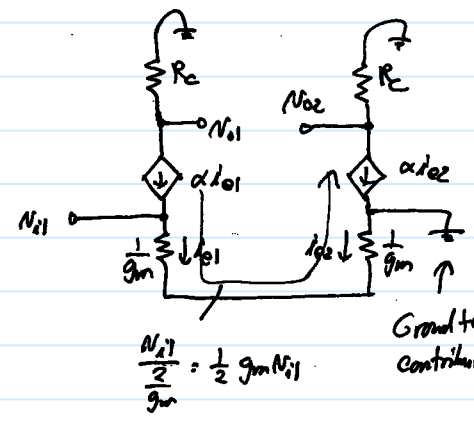
$$N_{o2} = +\frac{1}{2} g_m R_c (N_{i1} - N_{i2})$$

$$\therefore N_{od} = N_{o1} - N_{o2} = -g_m R_c (N_{i1} - N_{i2})$$

$$\frac{N_{od}}{N_{id}} = A_{dm} = -g_m R_c$$

zero w/ for s.s.
If R_{EE} large, then current won't go into this!

⇒ Easiest to see this happening using the T-model: (for those who must see the model ckt.)



↳ This time also get the $\frac{N_{o1}-N_{o2}}{N_{i1}}$ gain!

$$N_{o1} = -\frac{1}{2} g_m R_c N_{i1}$$

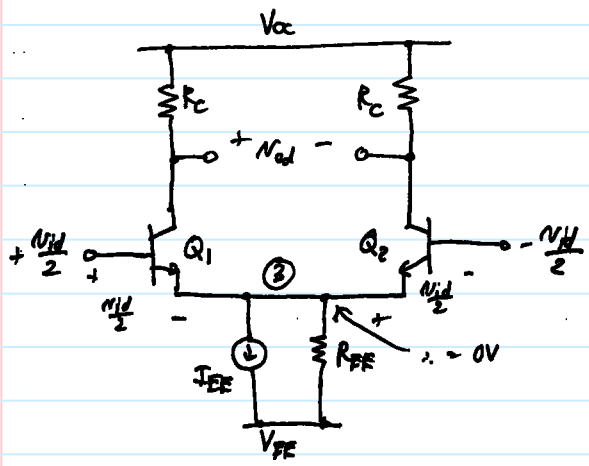
$$N_{o2} = +\frac{1}{2} g_m R_c N_{i1}$$

$$N_{o1} - N_{o2} = -g_m R_c N_{i1}$$

$$\frac{N_{od}}{N_{i1}} = -g_m R_c$$

Diff. Mode Analysis

Assume a ckt. w/ only diff. input:



Total current thru I_{EE} = const.

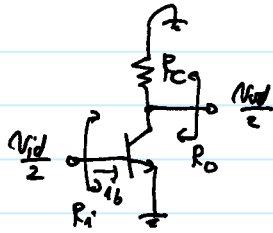
- V_E = const. as input changes
- (3) act as an incremental ground! → $V_{(3)} = 0V$ (always!)

∴ we can ground (3), and thus have

a **Differential Half Ckt.**

Note: Can really only make this for a purely symmetrical ckt!

Differential Half Ckt.



By inspection: $\frac{V_{od}/2}{V_{id}/2} = \frac{V_{od}}{V_{id}} = A_{dm} = -g_m R_c$

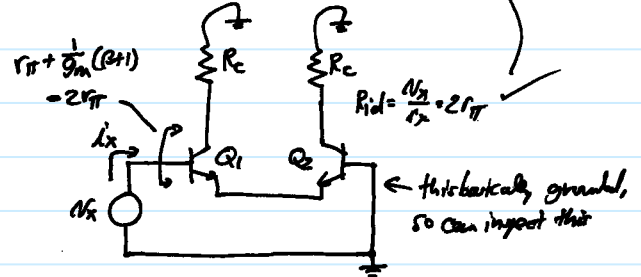
$\frac{V_{id}/2}{i_b} = r_{\pi} \rightarrow R_{id} = \frac{V_{id}}{i_b} = 2r_{\pi} = R_{id}$

$\frac{V_{od}/2}{i_o} = r_o || R_c \rightarrow R_{od} = \frac{V_{od}}{i_o} = 2(r_o || R_c) \approx 2R_c = R_{od}$

S.S. params. determined w/ $I_c = \frac{I_{EE}}{2}$

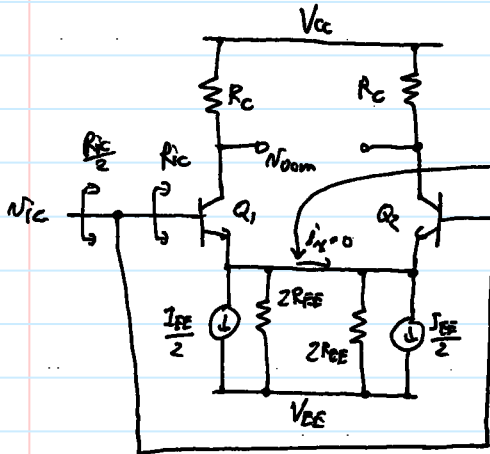
Common-Mode Analysis

Assume a pure CM input \rightarrow tie inputs together

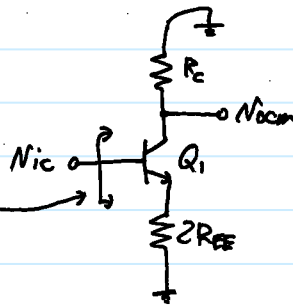


By symmetry, $i_x = 0 \Rightarrow$ thus, really have to equivalent of an open ckt. here

$\therefore \Rightarrow$ can split the ckt. into CM half-ckt.!



S.S. CM Half-Ckt.



$R_{ic} = r_{\pi} + (\beta+1)(2R_{EE})$
@ each input

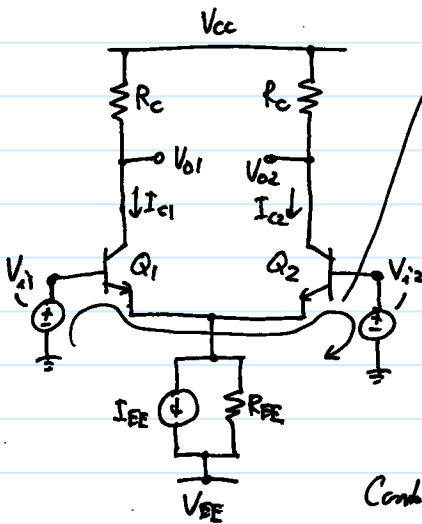
$A_{cm} = \frac{N_{oom}}{N_{ic}} = -\frac{g_m R_c}{1 + g_m (2R_{EE})} \approx -\frac{R_c}{2R_{EE}}$

Want small for large CMRR \therefore want $R_{EE} = \text{large!}$

Common-Mode Rejection Ratio = $CMRR = \frac{A_{dm}}{A_{cm}} = \frac{-g_m R_c}{-\frac{g_m R_c}{1 + g_m (2R_{EE})}} \Rightarrow CMRR = 1 + 2g_m R_{EE}$

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the ECP.

Large Signal ECP Performance

Find I_{C1} & I_{C2} :

$$\text{KVL: } V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$$

$$I_{C1} = I_{S1} \exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right), \quad V_{be2} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right)$$

$$V_{i1} = V_T \ln\left(\frac{I_{C1}}{I_{S2}} \frac{I_{S2}}{I_{S1}}\right) - V_{i2} = 0 \rightarrow \ln \frac{I_{C1}}{I_{C2}} = \frac{V_{i1} - V_{i2}}{V_T} = \frac{V_{id}}{V_T}$$

$$\frac{I_{C1}}{I_{C2}} = \exp\left(\frac{V_{id}}{V_T}\right) \quad (1)$$

$$I_{EE} = I_{C1} + I_{C2} = \frac{1}{\alpha} (I_{C1} + I_{C2}) \quad (2)$$

Combine (1) & (2) to get:

$$I_{C1} = \frac{\alpha I_{EE}}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)}, \quad I_{C2} = \frac{\alpha I_{EE}}{1 + \exp\left(\frac{V_{id}}{V_T}\right)} \quad (3)$$

Find V_{od} :

$$V_{o1} = V_{CC} - I_{C1} R_C$$

$$V_{o2} = V_{CC} - I_{C2} R_C$$

$$V_{od} = V_{o1} - V_{o2} = (I_{C2} - I_{C1}) R_C$$

using (3)

$$= \alpha I_{EE} R_C \left\{ \frac{1}{1 + \exp\left(\frac{V_{id}}{V_T}\right)} - \frac{1}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)} \right\}$$

$$\times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)} \quad \times \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right)}$$

$$= \alpha I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right) + \exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha I_{EE} R_C \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right) + \exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha I_{EE} R_C \frac{\sinh\left(-\frac{V_{id}}{2V_T}\right)}{\cosh\left(-\frac{V_{id}}{2V_T}\right)}$$

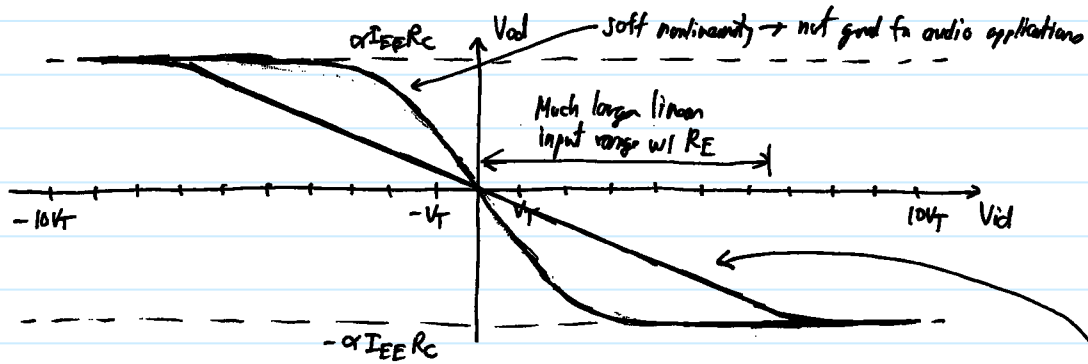
$$\left. \begin{aligned} \sinh u &= \frac{1}{2} (e^u - e^{-u}) \\ \cosh u &= \frac{1}{2} (e^u + e^{-u}) \end{aligned} \right\} u = -\frac{V_{id}}{2V_T}$$

$$\therefore V_{od} = \alpha I_{EE} R_C \tanh\left(-\frac{V_{id}}{2V_T}\right)$$

From our knowledge of the Taylor series for

$$\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15} x^5 - \dots$$

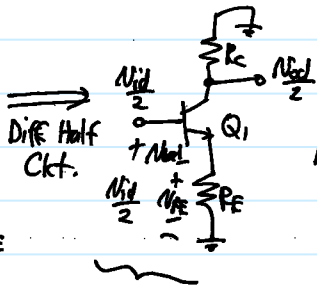
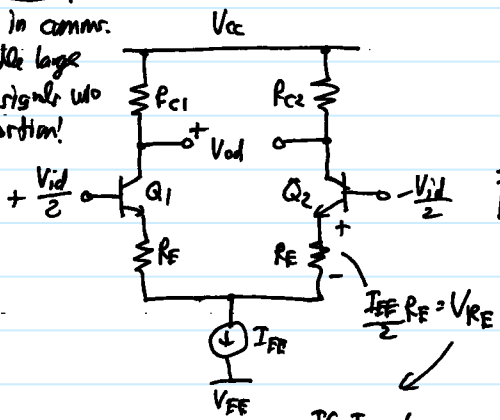
This is fairly linear for small V_{id} , but gets nonlinear abruptly when V_{id} approaches a threshold value!



In the above curve, the $\frac{V_{out}}{V_{id}}$ Xfer function is really only linear for $V_{id} < V_T \rightarrow$ beyond V_T , start to enter the nonlinear realm of curve \rightarrow causes signal distortion: eg, phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xfer amplifiers)

Needed in comm. to handle large input signals w/o distortion!



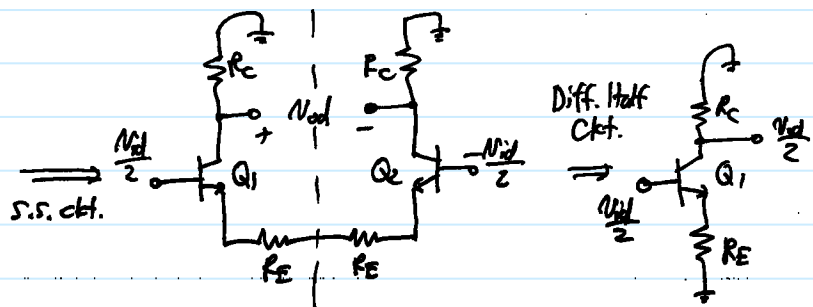
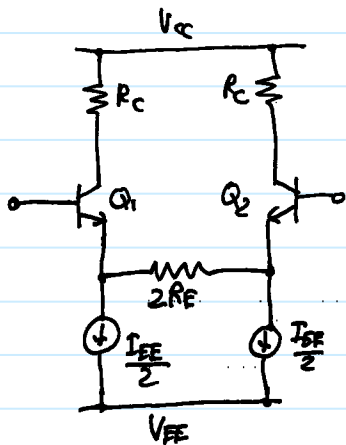
$$A_{dm} = -\frac{g_m R_C}{1 + g_m R_E}$$

\Rightarrow s.s. gain reduced, but the linear range is increased

If I_{EE} is large, then this can force large supply voltages

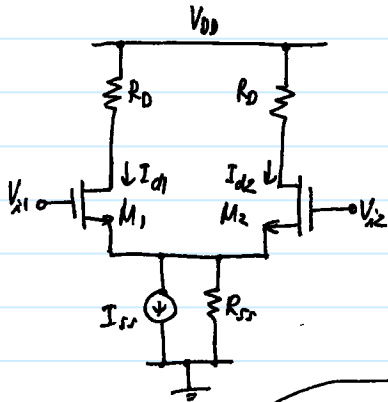
$\frac{N_{id}}{2} = N_{id1} + V_{RE}$
 This can still be $N_{id1} < V_T$ if this absorbs some of the input voltage!

Alternative Biasing Technique if Need Larger DC Currents:-



Same J.S. performance w/o the need to drop a DC voltage across $R_E \rightarrow$ get better
 Can use lower V_{CC} & V_{EE} .

MOSFET Source-Coupled Pair



Assume: M_1 & M_2 are identical.

Find $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$.

\Rightarrow approach: get $V_{id} = f(\Delta I_d) \rightarrow$ then invert to get $\Delta I_d = f(V_{id})$

$$I_{d1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{gs1} - V_t)^2 \Rightarrow V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{k}}$$

$$\therefore V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}}$$

Define:

$$\begin{cases} \Delta I_d = I_{d1} - I_{d2} \\ I_d = \frac{I_{d1} + I_{d2}}{2} \end{cases} \Rightarrow \begin{cases} I_{d1} = I_d + \frac{\Delta I_d}{2} \\ I_{d2} = I_d - \frac{\Delta I_d}{2} \end{cases}$$

$$V_{id} = \sqrt{\frac{2(I_d + \frac{\Delta I_d}{2})}{k}} - \sqrt{\frac{2(I_d - \frac{\Delta I_d}{2})}{k}} \Rightarrow \frac{k}{2} V_{id}^2 = I_d + \frac{\Delta I_d}{2} - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2} + I_d - \frac{\Delta I_d}{2}$$

$$\frac{k}{2} V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2}$$

\Rightarrow Now rearrange to get ΔI_d (algebra)

Solve for ΔI_d :

$$\Delta I_d = \frac{k}{2} V_{id} \left(\frac{2I_{ss}}{k/2} - V_{id}^2 \right)^{1/2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{id} \sqrt{\left(\frac{2I_{ss}}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L}} \right) - V_{id}^2} = \Delta I_d$$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

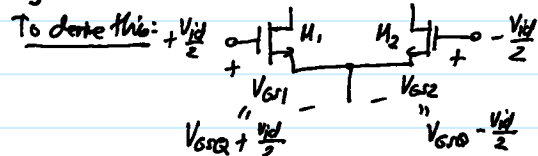
$$|V_{id}| \leq \sqrt{\frac{2I_{ss}}{k}} = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{2} (V_{GS} - V_t)$$

V_{GS} for $I_D = \frac{I_{ss}}{2}$

if true then input devices are both saturated

Thus, to extend the linear input range:

- ① $I_{ss} \uparrow \rightarrow (V_{GS} - V_t) \uparrow$
- ② W/L
- ③ $L \uparrow$



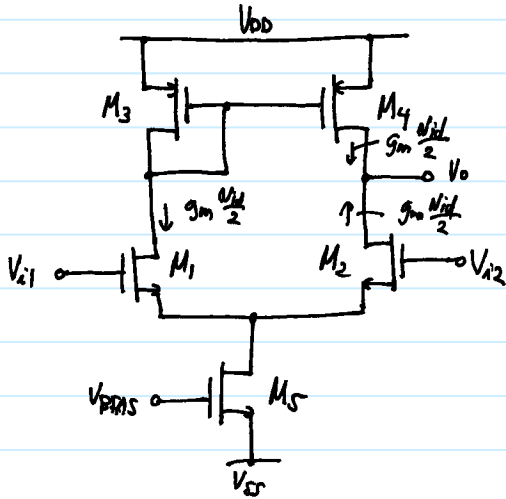
When $V_{id} \geq V_{gs0} - V_t = \Delta V$ then M_2 will cut-off

$\therefore V_{id} \leq 2(V_{gs0} - V_t) \rightarrow$ to maintain saturation

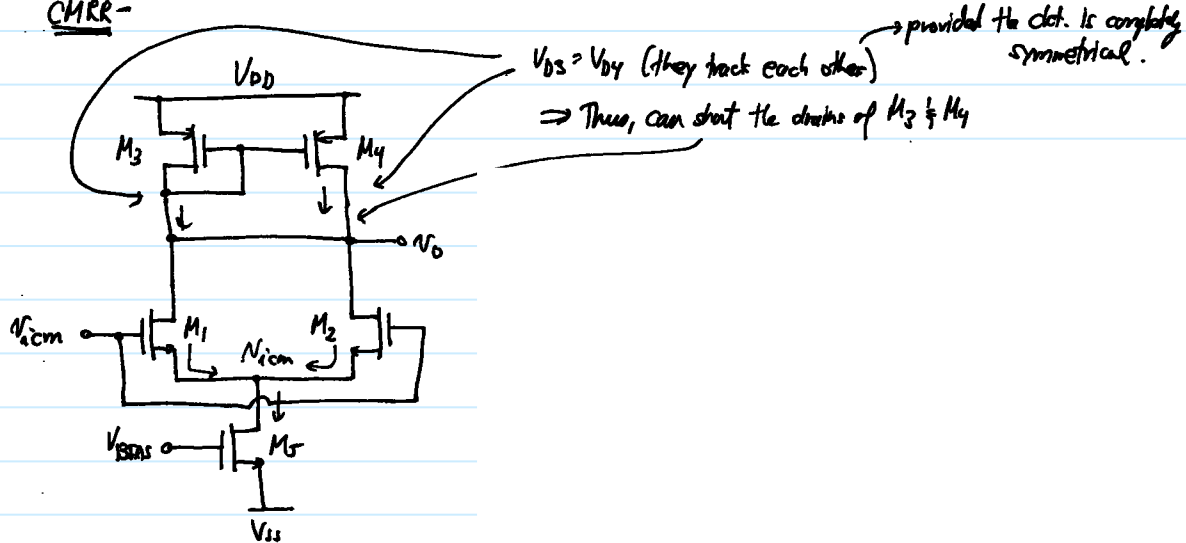
$$V_{gs0} - V_t = \sqrt{\frac{2I_{d2}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{2(I_d - \frac{\Delta I_d}{2})}{\mu_n C_{ox} \frac{W}{L}}} = \frac{V_{id}}{2}$$

Then plug in ΔI_d & solve for V_{id}

MOS Differential Stage w/ Current Mirror Load



CMRR -



Common-Mode Input Range - Range of input voltages in which all devices remain in saturation.

Low End - must keep M_5 saturated

High End - keep M_1, M_2 saturated

