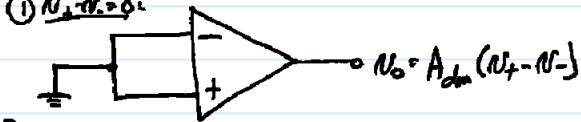


Device Mismatch Effects in Diff. Amplifiers

⇒ up to this point, we assumed that  $Q_1$  &  $Q_2$  are perfectly matched  
 ⇒ in actual ckt., got device mismatches due to processing variations

The Result:

①  $N_+ = N_- = 0$  → Output not zero when Input is zero →  $N_{out} \neq 0$  when  $N_{in} = 0$ !



Ideal Case:  $N_o = 0$

Reality:  $N_o \neq 0$ , even w/  $(N_+ - N_-) = 0$ !

② Input  $I_{B1} \neq I_{B2}$  if  $Q_1$  &  $Q_2$  not matched. (fn BJT & JFET only.)

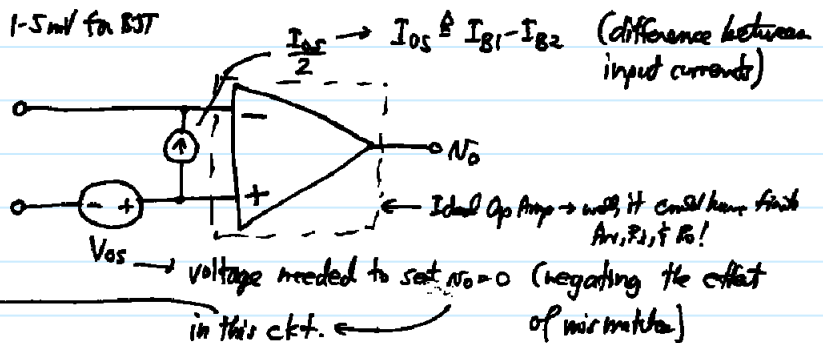
To model these effects, introduce:

① Input Offset Voltage,  $V_{os}$

② Input Offset Current,  $I_{os}$

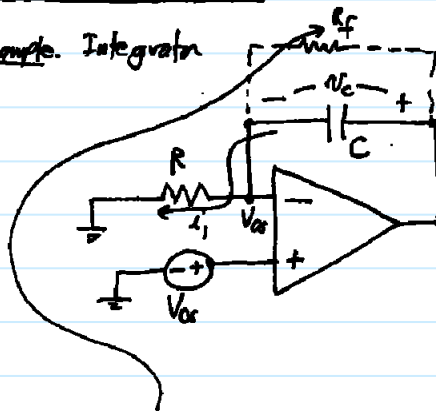
Typ. 1-5 mV for BJT

Typ.  $I_{os} = 10$  nA for BJT



Effect of  $V_{os}$  on Op Amp Ckt. -

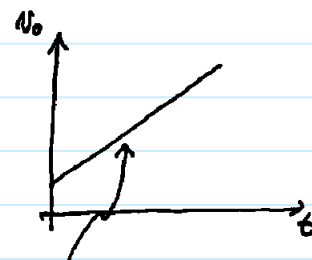
Example. Integrator



$$N_o = V_{os} + \frac{1}{C} \int_0^t i_i dt$$

$$= V_{os} + \frac{1}{C} \int_0^t \frac{V_{os}}{R} dt$$

$$= V_{os} \left(1 + \frac{t}{RC}\right) + N_o|_{t=0}$$



Fix: Place an  $R_f$  in shunt w/ the C

→ then  $N_o = V_{os} \left(1 + \frac{R_f}{R}\right)$ , and railing doesn't happen

→ but, usually  $R_f$  is large to allow the C to dominate

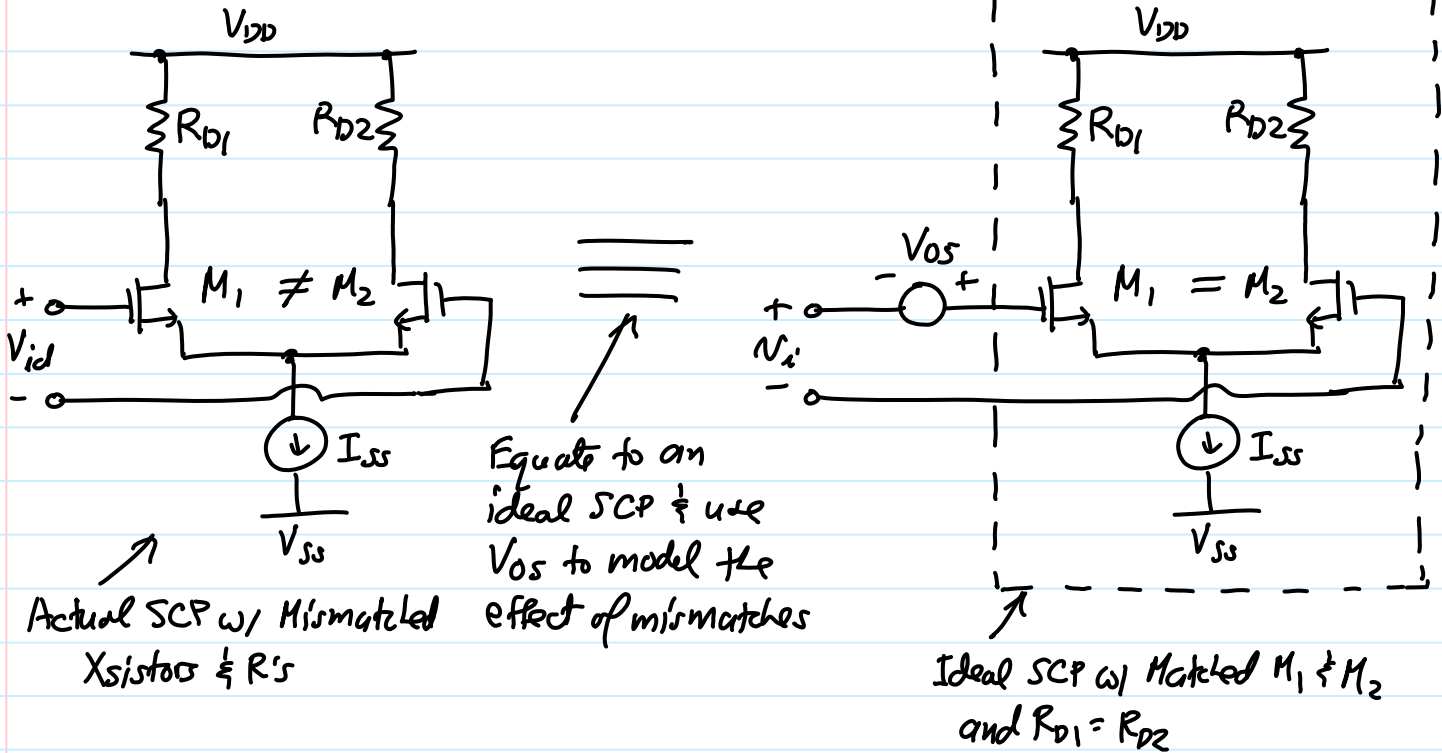
the integrator Xfer function  $\therefore N_o = V_{os} \left(1 + \frac{R_f}{R}\right)$  can be quite large → still want  $V_{os} = \text{small}$

will continue to increase until op amp hits the voltage rails

$V_{os}$  is even more important in setting the resolution of AD converters and other precision ckt.

$V_{OS}$  of a Mismatched SCP

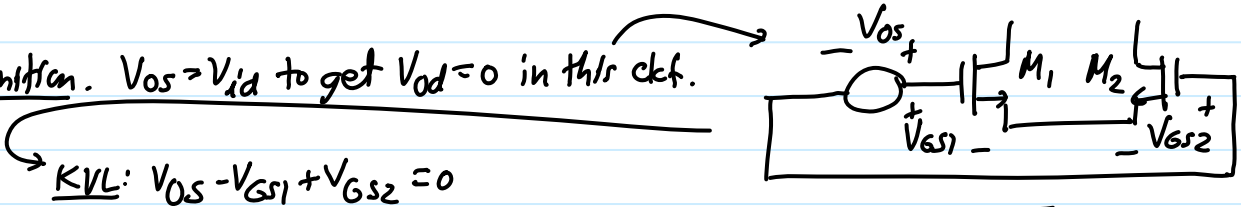
Objective: Derive an expression for  $V_{OS}$ .



Input offset voltage  $V_{OS}$  arises due to variations in:

- ① Xsistors,  $M_1 \neq M_2 \rightarrow \frac{W}{L}$  and  $V_t$  vary
- ②  $R_{D1} \neq R_{D2} \rightarrow$  causes gain variation

Definition.  $V_{OS} = V_{id}$  to get  $V_{od} = 0$  in this ckt.



KVL:  $V_{OS} - V_{GS1} + V_{GS2} = 0$

$$\therefore V_{OS} = V_{GS1} - V_{GS2} = V_{t1} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} (W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} (W/L)_2}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} (W/L)_1}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} (W/L)_2}} \quad (1)$$

Define difference and average quantities:

$\rightarrow \Delta I_D = I_{D1} - I_{D2}$	$\Delta \left(\frac{W}{L}\right) = \left(\frac{W}{L}\right)_1 - \left(\frac{W}{L}\right)_2$	$\Delta V_t = V_{t1} - V_{t2}$	$\Delta R_D = R_{D1} - R_{D2}$
$\rightarrow I_D = \frac{I_{D1} + I_{D2}}{2}$	$\left(\frac{W}{L}\right) = \frac{1}{2} \left[ \left(\frac{W}{L}\right)_1 + \left(\frac{W}{L}\right)_2 \right]$	$V_t = \frac{V_{t1} + V_{t2}}{2}$	$R_D = \frac{R_{D1} + R_{D2}}{2}$

Rearranging:

$$I_{D1} = I_D + \frac{\Delta I_D}{2} \quad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \frac{\Delta(W/L)}{2} \quad V_{t1} = V_t + \frac{\Delta V_t}{2}$$

$$I_{D2} = I_D - \frac{\Delta I_D}{2} \quad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \frac{\Delta(W/L)}{2} \quad V_{t2} = V_t - \frac{\Delta V_t}{2}$$

Substituting into (1):

$$V_{OS} = \Delta V_t + \sqrt{\frac{2(I_D + \frac{\Delta I_D}{2})}{\mu_n C_{ox} \left[ \left(\frac{W}{L}\right) + \frac{1}{2} \Delta\left(\frac{W}{L}\right) \right]}} - \sqrt{\frac{2(I_D - \frac{\Delta I_D}{2})}{\mu_n C_{ox} \left[ \left(\frac{W}{L}\right) - \frac{1}{2} \Delta\left(\frac{W}{L}\right) \right]}}$$

$\left[ V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox} (W/L)}} \right]$

$$= \Delta V_t + (V_{GS} - V_t) \left\{ \sqrt{\frac{1 + \frac{\Delta I_D}{2I_D}}{1 + \frac{1}{2} \frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1 - \frac{\Delta I_D}{2I_D}}{1 - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)}}} \right\}$$

Binomial Theorem:

$$(1 + nx)^m \xrightarrow{n = \text{small}} 1 + mnx$$

$$V_{OS} = \Delta V_t + (V_{GS} - V_t) \left\{ \left(1 + \frac{1}{4} \frac{\Delta I_D}{I_D}\right) \left(1 - \frac{1}{4} \frac{\Delta(W/L)}{(W/L)}\right) - \left(1 - \frac{1}{4} \frac{\Delta I_D}{I_D}\right) \left(1 + \frac{1}{4} \frac{\Delta(W/L)}{(W/L)}\right) \right\}$$

$$= \Delta V_t + (V_{GS} - V_t) \left( \frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta(W/L)}{(W/L)} \right)$$

$$\therefore V_{OS} = \Delta V_t + \frac{1}{2} (V_{GS} - V_t) \left\{ \frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

When  $V_{id} = V_{OS} \rightarrow V_{od} = 0 \therefore I_{D1} R_{D1} = I_{D2} R_{D2} \rightarrow$  mismatch in  $I_D$  must be opposite that of  $R_D$

$$V_{OS} = \Delta V_t + \frac{1}{2} (V_{GS} - V_t) \left\{ -\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

Threshold Mismatch



bias independent

Geometric (i.e., Layout) Variation

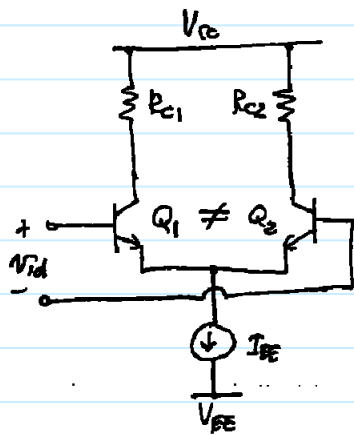
↪ scale w/ overdrive

that of  $R_D$ 

$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D}$$

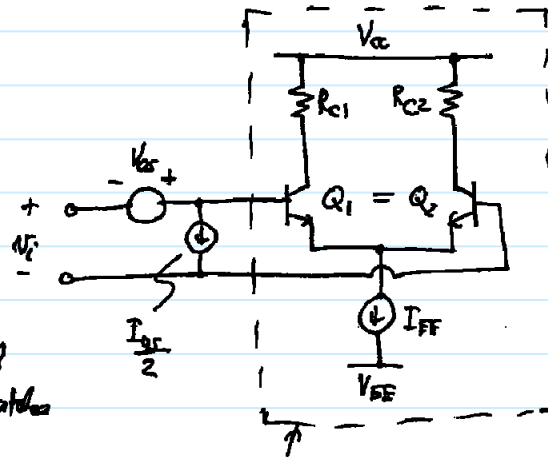
$V_{OS}$  in a Mismatched ECP

Objective: Derive an expression for  $V_{OS}$ .



Actual ECP w/ Mismatched  $\beta$  and  $R$ 's

Equivalent to an ideal ECP + use  $V_{OS}$  +  $I_{EE}$  to model the effect of mismatches



Ideal ECP w/ Matched  $Q_1$  &  $Q_2$  and  $R_{C1} = R_{C2}$

Input Offset Voltage  $V_{OS}$  arises due to variations in:

①  $\beta$  mismatch,  $Q_1 \neq Q_2 \rightarrow I_{S1} \neq I_{S2}$  &  $\beta$  vary:

$$I_S = \frac{q N_A^2 D_n A}{N_A W_B (V_{CB})}$$

↑  
"function of"

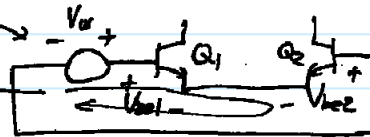
$I_{S1} \neq I_{S2}$  can be caused by:

- (i)  $A_1 \neq A_2$  (etching tolerance limits)
- (ii)  $N_A1 \neq N_A2$  (doping variations of base)
- (iii)  $W_B = f(V_{CB})$  (width variations exacerbated by  $V_{CB}$  diff)

②  $R_{C1} \neq R_{C2} \rightarrow$  cause gain variation

Definition:  $V_{OS} = V_{id}$  to get  $V_{od} = 0$ , which occurs when:

KVL:  $V_{OS} - V_{be1} + V_{be2} = 0$



$$V_{OS} = V_{be1} - V_{be2} = V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} = V_T \ln \left( \frac{I_{C1}}{I_{C2}} \frac{I_{S2}}{I_{S1}} \right)$$

find  $\frac{I_{C1}}{I_{C2}}$  in terms of design elements:

[When  $V_{id} = V_{OS} \rightarrow V_{od} = 0V$ ]  $\rightarrow V_{od} = (V_{CC} - I_{C1}R_{C1}) - (V_{CC} - I_{C2}R_{C2}) = 0$

$$I_{C1}R_{C1} = I_{C2}R_{C2} \rightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}}$$

$$V_{OS} = V_T \ln \left( \frac{R_{C2}}{R_{C1}} \frac{I_{S2}}{I_{S1}} \right)$$

This is an exact equation for  $V_{OS}$ . It's often more useful & intuitive to express this in terms of percent variations (and eventually standard deviations).

Convert to percent Variation Form -

$$\text{Define: } \left. \begin{aligned} R_c &= \frac{R_{c1} + R_{c2}}{2}, \Delta R_c = R_{c1} - R_{c2} \\ I_s &= \frac{I_{s1} + I_{s2}}{2}, \Delta I_s = I_{s1} - I_{s2} \end{aligned} \right\} \text{Objective: Express Var in terms of percent variations } \frac{\Delta R_c}{R_c} \text{ \& } \frac{\Delta I_s}{I_s}.$$

$$\downarrow$$

In general:  $\left. \begin{aligned} \Delta X: X_1 - X_2 \\ X = \frac{X_1 + X_2}{2} \end{aligned} \right\} \begin{aligned} X_1 &= X + \frac{\Delta X}{2} \\ X_2 &= X - \frac{\Delta X}{2} \end{aligned} \Rightarrow \text{Thus: } \begin{aligned} R_{c1} &= R_c + \frac{\Delta R_c}{2}, R_{c2} = R_c - \frac{\Delta R_c}{2} \\ I_{s1} &= I_s + \frac{\Delta I_s}{2}, I_{s2} = I_s - \frac{\Delta I_s}{2} \end{aligned}$

With these formulations:

$$V_{OS} = V_T \ln \left[ \frac{R_{c2} I_{s2}}{R_{c1} I_{s1}} \right] = V_T \ln \left\{ \frac{R_c - \frac{\Delta R_c}{2}}{R_c + \frac{\Delta R_c}{2}} \frac{I_s - \frac{\Delta I_s}{2}}{I_s + \frac{\Delta I_s}{2}} \right\} = V_T \ln \left\{ \frac{1 - \frac{\Delta R_c}{2R_c}}{1 + \frac{\Delta R_c}{2R_c}} \frac{1 - \frac{\Delta I_s}{2I_s}}{1 + \frac{\Delta I_s}{2I_s}} \right\}$$

$$\left[ \ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right] \xrightarrow{\uparrow} V_{OS} \approx V_T \left[ -\frac{\Delta R_c}{2R_c} - \frac{\Delta R_c}{2R_c} - \frac{\Delta I_s}{2I_s} - \frac{\Delta I_s}{2I_s} \right]$$

taking the first term assuming  $\Delta R \ll R_c$  &  $\Delta I_s \ll I_s$

$$V_{OS} = V_T \left[ -\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s} \right]$$

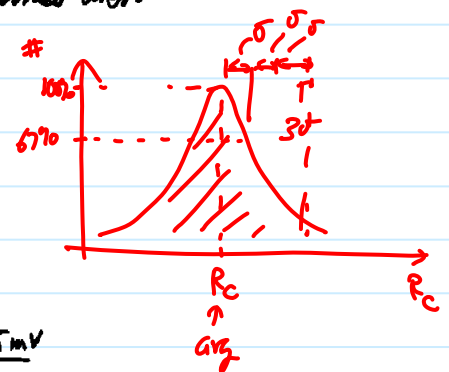
Since  $\frac{\Delta R_c}{R_c}$  and  $\frac{\Delta I_s}{I_s}$  are statistically <sup>varying</sup> parameters for a given process run & layout, one usually expresses terms in the form of variances when specifying V<sub>OS</sub>:

→ since  $\frac{\Delta R_c}{R_c}$  &  $\frac{\Delta I_s}{I_s}$  are uncorrelated, their variances add like powers:

$$\sigma_{Var} = V_T \sqrt{\sigma_{\Delta R_c/R_c}^2 + \sigma_{\Delta I_s/I_s}^2}$$

Ex: Typ.  $\sigma_{\Delta R_c/R_c} \sim 0.01$ ,  $\sigma_{\Delta I_s/I_s} \sim 0.05$

$$\therefore \sigma_{V_{OS}} = (26m) \sqrt{(0.01)^2 + (0.05)^2} = 1.3mV \quad \text{Typ. Var for BJT} \sim 1-5mV$$



V<sub>OS</sub> Drift w/ Temperature

$$\frac{dV_{OS}}{dT} = \frac{kT}{q} \left\{ \underbrace{-\frac{\Delta R_c}{R_c} - \frac{\Delta I_s}{I_s}}_{\text{indep. of } T} \right\} \frac{1}{T} = \frac{Var}{T} \quad \text{[In kelvin]}$$

Ex:  $\frac{dV_{OS}}{dT} = \frac{1.3m}{300K} = 4.3 \mu V/K$  <sup>drift</sup> around  $T = 300K$ .

$I_{OS}$  in a Mismatched ECP

By Definition:  $I_{OS} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2} = I_{OS}$

To express in percent variations:

$$\begin{cases} I_{C1} = I_C + \frac{\Delta I_C}{2} \\ I_{C2} = I_C - \frac{\Delta I_C}{2} \end{cases} \quad \begin{cases} \beta_1 = \beta + \frac{\Delta \beta}{2} \\ \beta_2 = \beta - \frac{\Delta \beta}{2} \end{cases}$$

$$\therefore I_{OS} = \frac{I_C + \frac{\Delta I_C}{2}}{\beta + \frac{\Delta \beta}{2}} - \frac{I_C - \frac{\Delta I_C}{2}}{\beta - \frac{\Delta \beta}{2}} = \frac{I_C}{\beta} \left\{ \frac{1 + \frac{\Delta I_C}{2I_C}}{1 + \frac{\Delta \beta}{2\beta}} - \frac{1 - \frac{\Delta I_C}{2I_C}}{1 - \frac{\Delta \beta}{2\beta}} \right\}$$

$$\left[ \frac{1}{1+x} \approx 1 - x + x^2 - \dots \right] \rightarrow = \frac{I_C}{\beta} \left\{ \left(1 + \frac{\Delta I_C}{2I_C}\right) \left(1 - \frac{\Delta \beta}{2\beta}\right) - \left(1 - \frac{\Delta I_C}{2I_C}\right) \left(1 + \frac{\Delta \beta}{2\beta}\right) \right\}$$

$$= \frac{I_C}{\beta} \left\{ 1 + \frac{\Delta I_C}{2I_C} - \frac{\Delta \beta}{2\beta} - \frac{\Delta I_C}{2I_C} \frac{\Delta \beta}{2\beta} - 1 + \frac{\Delta I_C}{2I_C} - \frac{\Delta \beta}{2\beta} + \frac{\Delta I_C}{2I_C} \frac{\Delta \beta}{2\beta} \right\}$$

$$I_{OS} = \frac{I_C}{\beta} \left\{ \frac{\Delta I_C}{I_C} - \frac{\Delta \beta}{\beta} \right\}$$

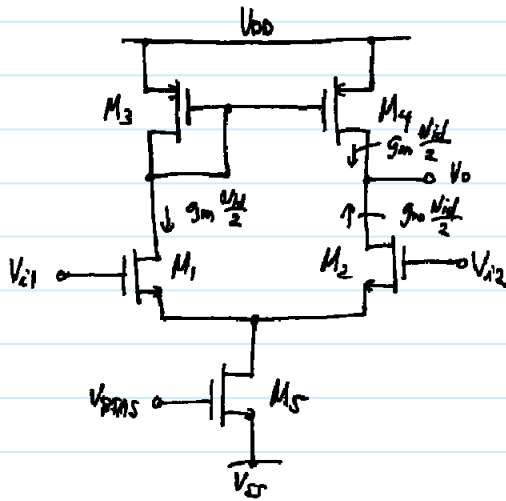
But for  $V_{od} = 0V \rightarrow \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}} \rightarrow \frac{\Delta I_C}{I_C} = -\frac{\Delta R_C}{R_C}$

$$\therefore I_{OS} = -\frac{I_C}{\beta} \left( \frac{\Delta R_C}{R_C} + \frac{\Delta \beta}{\beta} \right)$$

Ex. Typ:  $\sigma_{\Delta \beta/\beta} = 0.1$ ,  $\sigma_{\Delta R_C/R_C} = 0.01$

$$\rightarrow I_{OS} = -\frac{I_C}{\beta} \left[ \sigma_{\Delta R_C/R_C}^2 + \sigma_{\Delta \beta/\beta}^2 \right]^{1/2} \approx -0.1 \frac{I_C}{\beta} \approx -0.1 I_B = I_{OS}$$

## MOS Differential Stage w/ Current Mirror Load



Small-Signal Gain: (similar to BJT)

$$\frac{V_o}{V_{id}} = g_{m3,4}(r_{o2} \parallel r_{o4}) = \frac{g_{m3,4}}{g_{m1,2} + g_{m3,4}} = \frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_{D2}}}{\lambda_2 I_{D2} + \lambda_4 I_{D4}}$$

$$= \frac{\sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_{D2}}}{\frac{I_{D2}}{2} (\lambda_2 + \lambda_4)} \Rightarrow \frac{V_o}{V_{id}} = \frac{2}{\lambda_2 + \lambda_4} \sqrt{\frac{\mu_n C_{ox} (W/L)_2}{I_{D2}}}$$

→ all the same PS effects, etc...

$$\left[ \frac{\Delta(W/L)_{3,4}}{(W/L)_{1,2}} - \frac{\Delta(W/L)_{3,4}}{(W/L)_{3,4}} \right]$$

Offset Voltage -  $V_{OS} = V_{GS1} - V_{GS2}$  when  $V_{OD} = 0V$

$$V_{OS} = \Delta V_{t1,2} + \Delta V_{E3,4} \left( \frac{g_{m3,4}}{g_{m1,2}} \right) + \frac{(V_{GS} - V_t)_{1,2}}{2} \left[ \frac{\Delta k_{1,2}}{k_{1,2}} + \frac{\Delta k_{3,4}}{k_{3,4}} \right]$$

Via similar derivation to what we just did

For small  $V_{OS}$ : ① small  $(V_{GS} - V_t)$

$$\text{② } g_{m3,4} < g_{m1,2} \rightarrow k_{3,4} < k_{1,2} \quad \frac{1}{2} \left( \frac{W}{L} \right)_{3,4} < \left( \frac{W}{L} \right)_{1,2}$$