EE 140 | Op Amp Circuits

EE 140/240A
Lecture 13
10/9/14
CTN
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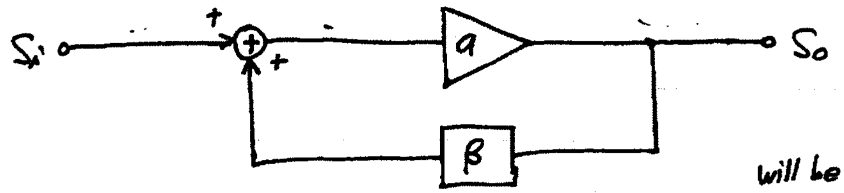
Contrast w/ Positive Feedback

These pages courtesy of C. Nguyen
EE 140/240A Fa 2013



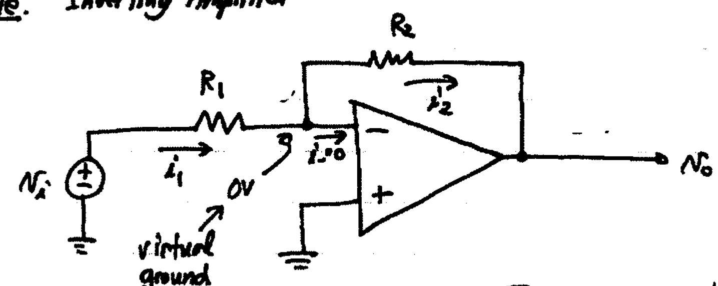
⇒ output blow up!
(for $a\beta > 1$)
will be the case for $a = \infty$

⇒ usually get oscillation!

Thus, for a bounded, controllable function, need negative FB around an op amp.

Op Amp Ckts.

Example. Inverting Amplifier



Break the loop and inject test signal.

① Verify that there is negative FB. ✓

(1a) analyze to find v^+ input voltage.

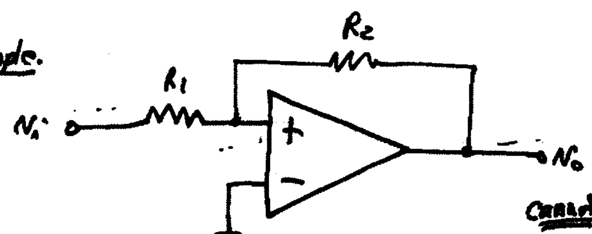
② ∴ v_o = finite ⟶ $v_+ = v_-$ ⟶ node attached to (-) terminal is virtual ground

③ $i_- = 0$ ∴ $i_1 = i_2$

$$\left.\begin{aligned} i_1 &= \frac{v_i - 0}{R_1} = \frac{v_i}{R_1} = i_2 \\ v_o &= 0 - i_2 R_2 = -i_2 R_2 \end{aligned}\right\} \Rightarrow v_o = -\left(\frac{v_i}{R_1}\right)R_2 = -\frac{R_2}{R_1}v_i \quad \therefore \boxed{\frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$

Note: Gain dependent only on R_1 & R_2 (external components), not on the op amp gain.

Example.



$v_o = L^+$ or L^- depending on initial conds.
$v_+ = (+) \longrightarrow L^+$
$v_+ = (-) \longrightarrow L^-$.

Cannot analyze using ideal op amp method!

① Verify that there is neg. FB ✗

∴ $v_o \neq$ finite, $v_+ \neq v_-$
⇒ this ckt. will "rail out"

EE 140 | Basic Op Amp Design

CTN

How does one make an op amp? (It turns out, you already know!)

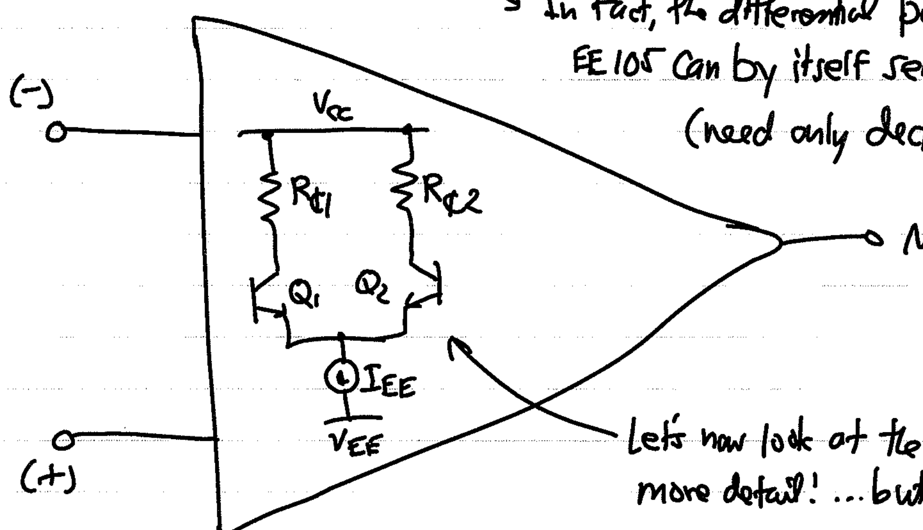
⇒ <u>Basic Needed Attributes</u>:

① Gain (voltage gain).

② Two inputs, (+) and (−).

③ One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE 105 can by itself serve as an op amp! (need only decide which of the inputs is the (+) and (−) terminals)

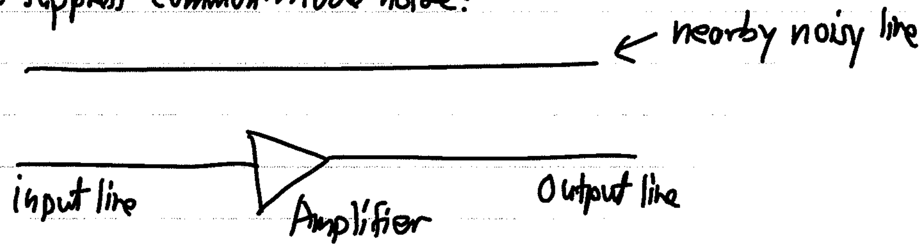


Let's now look at the diff. pair in more detail! ...but first

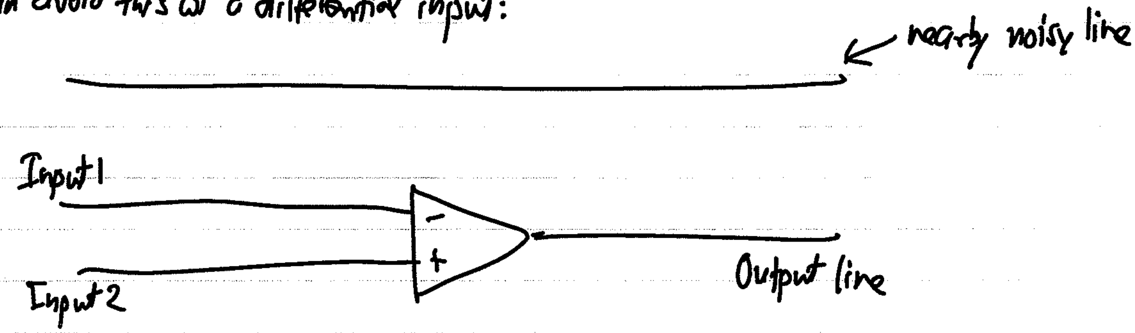
Why have 2 inputs?

① To get a virtual short for op amp ckts.

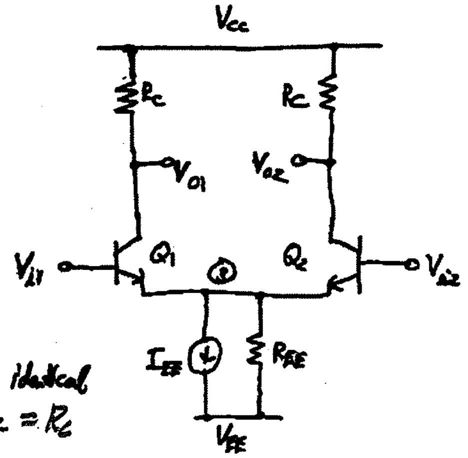
② To suppress common-mode noise:



Can avoid this w/ a differential input:



Differential Pair (Emitter Coupled Pair)



Assume:
Q_1 & Q_2 identical
$R_{c1} = R_{c2} = R_c$

Purpose: Amplify the difference between two signals regardless of their common-mode DC values (or their common-mode values in general)

Definition: $V_{id} = V_{i1} - V_{i2}$ (differential input)

$V_{icm} = \dfrac{V_{i1} + V_{i2}}{2}$ (common-mode input)

$$\Longrightarrow \begin{cases} V_{i1} = V_{icm} + \dfrac{V_{id}}{2} \\ V_{i2} = V_{icm} - \dfrac{V_{id}}{2} \end{cases}$$

Differential Gain $= A_d = \dfrac{V_{o1} - V_{o2}}{V_{id}} = \dfrac{V_{od}}{V_{id}}$ (want this to be large for this differential amplifier)

Common-Mode Gain $= A_{cm} = \dfrac{V_{o1}}{V_{cm}}$ or $\dfrac{V_{o2}}{V_{cm}}$ (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio $= CMRR = \dfrac{A_{dm}}{A_{cm}}$ (should be very high to favor the differential-mode and reject the common-mode)

⇒ we also want a high Common-Mode Input Range to reject DC input offsets

⇒ Note: No need for bypass capacitors (large) to the inputs or outputs → can just use direct coupling!

Biasing & Large Signal Common-Mode Behavior

Case: $R_{EE} = \infty$ → ideal current source biasing → $I_{E1} = I_{E2} = \frac{I_{EE}}{2}$ → $V_{o1} = V_{o2}$ ⇒ $V_{od} = 0$

If $V_{icm} \uparrow$ → $V_{⓪} \uparrow$, but current drawn from I_{EE} stays constant ∴ I_{c1} & I_{c2} stay constant → bias pt. doesn't change

$g_m = \frac{1}{2}\frac{I_{EE}}{V_T}$

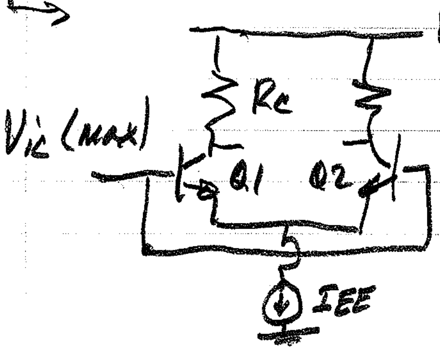
Case: R_{EE} = finite

$V_{⓪} = V_{i1} - V_{BE(on)}$

If $V_{icm} \uparrow$ → $V_{⓪} \uparrow$ → $I_{E1} = I_{E2} \uparrow$ (current draw $= I_{EE} + \frac{V_{⓪}}{R_{EE}}$)

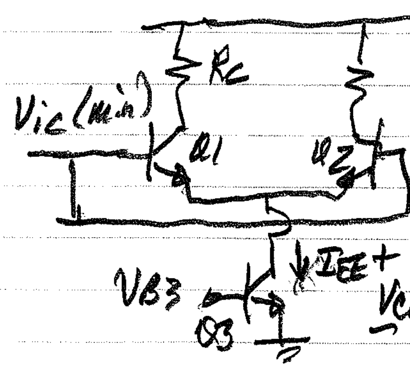
⇒ In general, R_{EE} will be large, so this component won't be large, and the bias pt. won't Δ much

Input common-mode range:



Q1 & Q2 at edge of sat: KVL

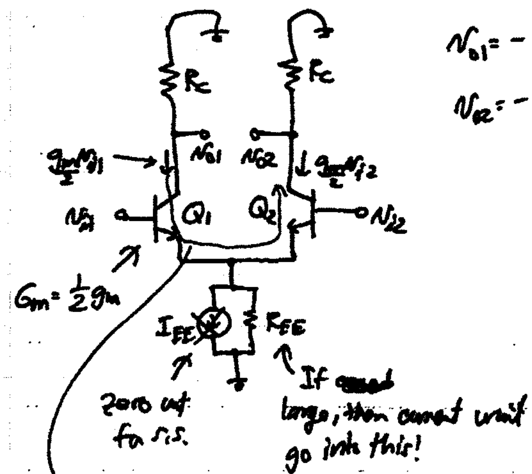
$V_{ic}(max) = V_{cc} - \frac{I_{EE}}{2} R_c - V_{CE(SAT)} + V_{BE(ON)}$



Q3 at edge of sat: KVL

$V_{ic}(min) = V_{CE(SAT)} + V_{BE}$

Small-Signal Analysis of Diff. Pair



$$v_{o1} = -\frac{1}{2} g_m v_{i1} R_C + \frac{1}{2} g_m v_{i2} R_C$$

$$v_{o2} = -\frac{1}{2} g_m v_{i2} R_C + \frac{1}{2} g_m v_{i1} R_C$$

$$v_{o1} = -\frac{1}{2} g_m R_C (v_{i1} - v_{i2})$$

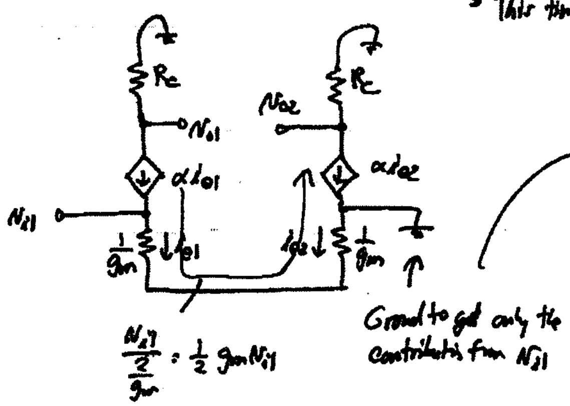
$$v_{o2} = +\frac{1}{2} g_m R_C (v_{i1} - v_{i2})$$

$$\therefore v_{od} = v_{o1} - v_{o2} = -g_m R_C (v_{i1} - v_{i2})$$

$$\therefore \boxed{\frac{v_{od}}{v_{id}} = A_{dm} = -g_m R_C}$$

⇒ Easiest to see this happening using the T-model: (for those who must see the model ckt.)

↳ This time also get the $\frac{v_{o1} - v_{o2}}{v_{i1}}$ gain!



$$v_{o1} = -\frac{1}{2} g_m R_C v_{i1}$$

$$v_{o2} = +\frac{1}{2} g_m R_C v_{i1}$$

$$v_{o1} - v_{o2} = -g_m R_C v_{i1}$$

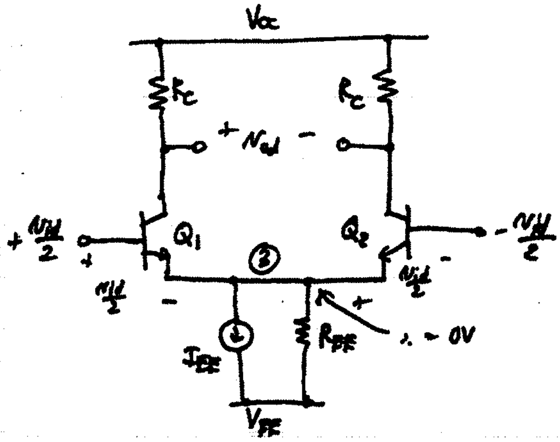
$$\therefore \boxed{\frac{v_{od}}{v_{i1}} = -g_m R_C}$$

$$\boxed{\frac{v_{o2}}{v_{i1}} = \frac{1}{2} g_m R_C}$$

$$\boxed{\frac{v_{o1}}{v_{i1}} = -\frac{1}{2} g_m R_C}$$

Diff. Mode Analysis

Assume a ckt. w/ only diff. input:



Total current thru I_{EE} = const.

⟶ V_E = const. as input changes

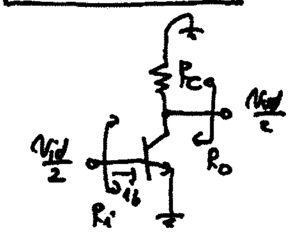
⟶ ③ acts as an incremental ground! ⟶ $v_③ = 0V$ (always!)

∴ we can ground ③, and then have a $\boxed{\text{Differential Half Ckt.}}$

↓

__Note:__ Can really only do this for a purely symmetrical ckt.!

Differential Half Ckt.



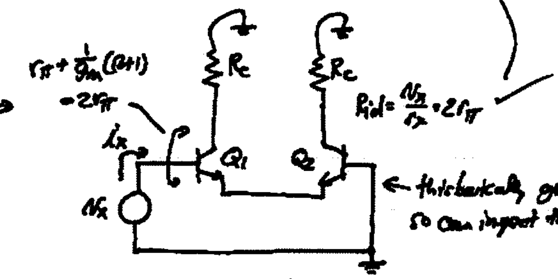
By inspection: $\frac{v_{od}/2}{v_{id}/2} = \frac{v_{od}}{v_{id}} = \boxed{A_{dm} = -g_m R_C}$

$\frac{v_{id}/2}{i_b} = r_\pi \longrightarrow R_{id}' = \frac{v_{id}}{i_b} = \boxed{2r_\pi = R_{id}}$

$\frac{v_{od}/2}{i_o} = r_o \| R_C \longrightarrow R_{od} = \frac{v_{od}}{i_o} = 2(r_o \| R_C) \approx \boxed{2R_C = R_{od}}$

S.S. params. determined w/ $I_C = \frac{I_{EE}}{2}$

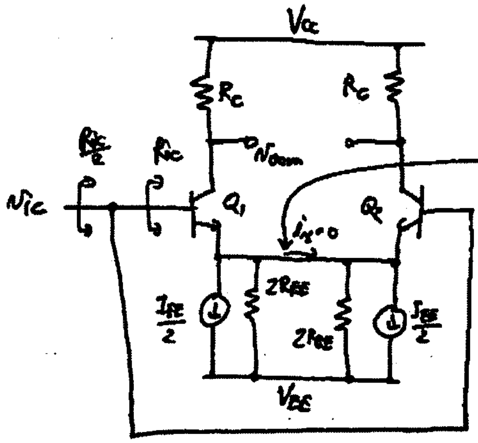
First define



← theoretically grounded, so can import this

Common-Mode Analysis

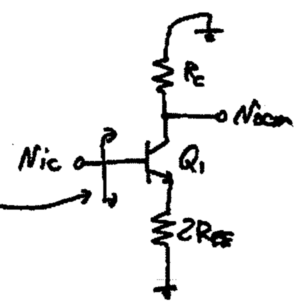
Assume a pure CM input → tie inputs together



By symmetry, $i_x = 0$. ⟹ thus, really have the equivalent of an open ckt. here

∴ ⟹ can split the ckt. into CM half-ckts.!

S.S. CM Half-Ckt.



$R_{ic} = r_\pi + (\beta+1)(2R_{EE})$ @ each input

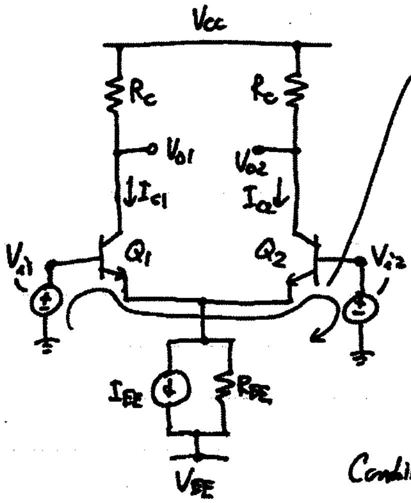
$$\boxed{A_{cm} = \frac{v_{ocm}}{v_{ic}} = -\frac{g_m R_C}{1 + g_m(2R_{EE})} \approx -\frac{R_C}{2R_{EE}}}$$

Want small for large CMRR ∴ want R_{EE} = large!

Common-Mode Rejection Ratio = CMRR = $\frac{A_{dm}}{A_{cm}} = \dfrac{-g_m R_C}{-\frac{g_m R_C}{1+g_m(2R_{EE})}}$ ⟹ $\boxed{\text{CMRR} = 1 + 2g_m R_{EE}}$

Having looked at S.S. parameters, we now turn to large signal performance. Here, we'll be particularly interested in the linear range of the ECP.

Large Signal ECP Performance



Find I_{c1} & I_{c2}:

KVL: $V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$

$I_{c1} = I_{s1}\exp\left(\frac{V_{be1}}{V_T}\right) \rightarrow V_{be1} = V_T \ln\left(\frac{I_{c1}}{I_{s1}}\right)$, $V_{be2} = V_T \ln\left(\frac{I_{c2}}{I_{s2}}\right)$

$$V_{i1} - V_T \ln\left(\frac{I_{c1}}{I_{c2}}\frac{I_{s2}}{I_{s1}}\right) - V_{i2} = 0 \rightarrow \ln\frac{I_{c1}}{I_{c2}} = \frac{V_{i1}-V_{i2}}{V_T} = \frac{V_{id}}{V_T}$$

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{id}}{V_T}\right) \quad (1)$$

$$I_{EE} = I_{e1} + I_{e2} = \frac{1}{\alpha}(I_{c1} + I_{c2}) \quad (2)$$

Combine (1) & (2) to get:

$$I_{C1} = \frac{\alpha I_{EE}}{1+\exp\left(-\frac{V_{id}}{V_T}\right)}, \quad I_{C2} = \frac{\alpha I_{EE}}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \quad (3)$$

Find V_{od}:

$$\left.\begin{aligned} V_{o1} &= V_{cc} - I_{c1}R_c \\ V_{o2} &= V_{cc} - I_{c2}R_c \end{aligned}\right\} \quad V_{od} = V_{o1} - V_{o2} = (I_{c2} - I_{c1})R_c$$

using (3)

$$= \alpha_F I_{EE} R_c \left\{ \frac{1}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \times \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)} - \frac{1}{1+\exp\left(-\frac{V_{id}}{V_T}\right)} \times \frac{\exp\left(+\frac{V_{id}}{2V_T}\right)}{\exp\left(+\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)+\exp\left(\frac{V_{id}}{2V_T}\right)} - \frac{\exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(\frac{V_{id}}{2V_T}\right)+\exp\left(-\frac{V_{id}}{2V_T}\right)} \right\}$$

$$= \alpha_F I_{EE} R_c \left\{ \frac{\exp\left(-\frac{V_{id}}{2V_T}\right) - \exp\left(\frac{V_{id}}{2V_T}\right)}{\exp\left(-\frac{V_{id}}{2V_T}\right)+\exp\left(\frac{V_{id}}{2V_T}\right)} \right\} = \alpha_F I_{EE} R_c \frac{\sinh\left(-\frac{V_{id}}{2V_T}\right)}{\cosh\left(-\frac{V_{id}}{2V_T}\right)}$$

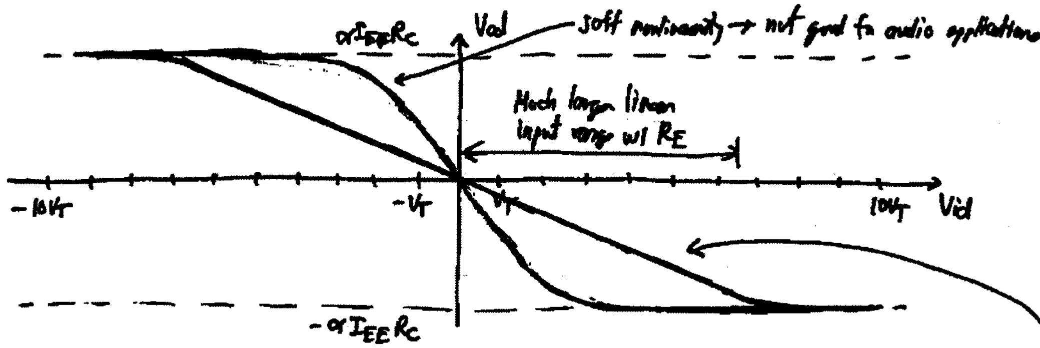
$$\left[\left.\begin{aligned} \sinh u &= \tfrac{1}{2}(e^u - e^{-u}) \\ \cosh u &= \tfrac{1}{2}(e^u + e^{-u}) \end{aligned}\right\} u = -\frac{V_{id}}{2V_T} \right]$$

$$\therefore \boxed{V_{od} = \alpha_F I_{EE} R_c \tanh\left(-\frac{V_{id}}{2V_T}\right)}$$

From our knowledge of the Taylor series for

$\tanh x \approx x - \frac{x^3}{3} + \frac{2}{15}x^5 - \ldots$

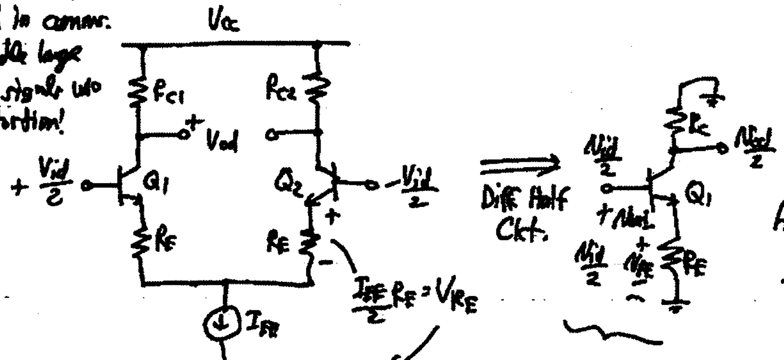
this is fairly linear for small V_{id}, but gets nonlinear abruptly when V_{id} approachs a threshold value!



In the above curve, the $\frac{V_{od}}{V_{id}}$ Xfer function is really only linear for $V_{id} \ll V_T$ → beyond V_T, start to enter the nonlinear realm of curve → causes signal distortion: e.g., phone breaking up, television static

To linearize: add emitter degeneration (same trick as used before for single Xistor amplifiers)

Needed in comm. to handle large input signals w/o distortion!



$\frac{I_{EE}}{2}R_E = V_{RE}$

$A_{dm} = -\frac{g_m R_C}{1 + g_m R_E}$

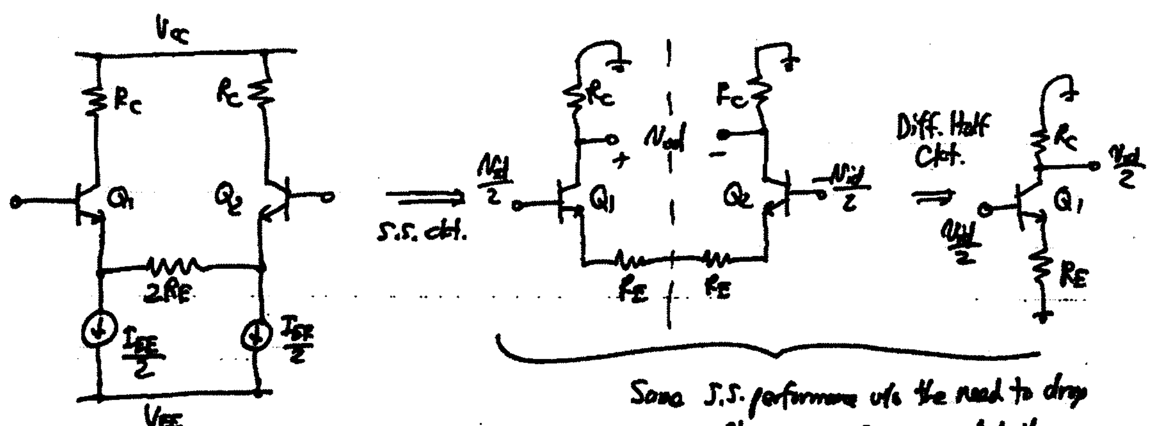
⇒ s.s. gain reduced, but the linear range is increased

If I_{EE} = large, then this can force large supply voltages

$\frac{v_{id}}{2} = v_{be1} + v_{RE}$

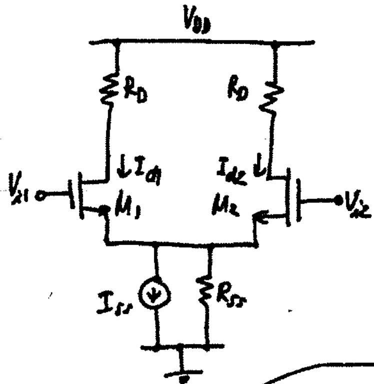
This can still be $v_{be1} \ll V_T$ if this absorbs some of the input voltage!

Alternative Biasing Technique If Need Larger DC Current:–



Same s.s. performance w/o the need to drop a DC voltage across R_E → ~~get better~~ can use lower V_{CC} & V_{EE}.

MOSFET Source-Coupled Pair



Assume: M_1 & M_2 are identical.

Find $\Delta I_d = I_{d1} - I_{d2} = f(V_{id})$.

$\Longrightarrow$ approach: get $V_{id} = f(\Delta I_d) \longrightarrow$ then invert to get $\Delta I_d = f(V_{id})$

$$I_{d1} = \underbrace{\tfrac{1}{2}\mu_n C_{ox}\left(\tfrac{W}{L}\right)_1}_{k}(V_{gs1} - V_t)^2 \Longrightarrow V_{gs1} = V_t + \sqrt{\frac{2I_{d1}}{k}}$$

$$\therefore\ V_{id} = V_{gs1} - V_{gs2} = \sqrt{\frac{2I_{d1}}{k}} - \sqrt{\frac{2I_{d2}}{k}}$$

Define.
$$\left.\begin{aligned} \Delta I_d &= I_{d1} - I_{d2} \\ I_d &= \frac{I_{d1} + I_{d2}}{2} \end{aligned}\right\} \quad \begin{aligned} I_{d1} &= I_d + \frac{\Delta I_d}{2} \\ I_{d2} &= I_d - \frac{\Delta I_d}{2} \end{aligned}$$

$$V_{id} = \sqrt{\frac{2\left(I_d + \frac{\Delta I_d}{2}\right)}{k}} - \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{k}} \Longrightarrow \frac{k}{2}V_{id}^2 = I_d + \cancel{\frac{\Delta I_d}{2}} - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2} + I_d - \cancel{\frac{\Delta I_d}{2}}$$

$$\frac{k}{2}V_{id}^2 = 2I_d - 2\sqrt{I_d^2 - \left(\frac{\Delta I_d}{2}\right)^2}$$

$\Longrightarrow$ now rearrange to get ΔI_d (algebra)

Solve for ΔI_d:
$$\Delta I_d = \frac{k}{2}V_{id}\left(\frac{2I_{ss}}{k/2} - V_{id}^2\right)^{1/2} = \boxed{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{id}\sqrt{\left(\frac{2I_{ss}}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\right) - V_{id}^2} = \Delta I_d}$$

Large Signal Equation for Differential Output Current

Valid so long as the devices stay saturated:

V_{GS} for $I_D = \frac{I_{ss}}{2}$

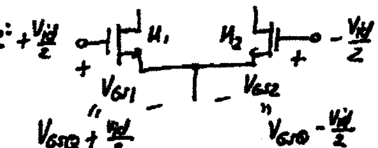
$$|V_{id}| \le \sqrt{\frac{2I_{ss}}{k}} = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{2}\,(V_{GS} - V_t)$$

if true then input devices are both saturated

Thus, to extend the linear input range:

1. $I_{ss}\uparrow \longrightarrow (V_{GS} - V_t)\uparrow$
2. W/L
3. $L\uparrow$

To derive this:



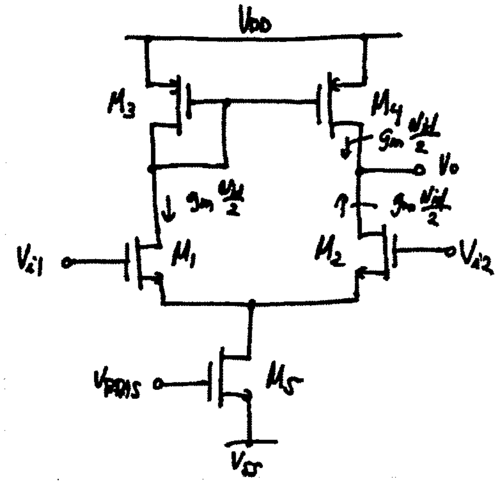
When $\frac{V_{id}}{2} \ge V_{GSQ} - V_t = \Delta V$ then M_2 will cut-off

$\therefore\ V_{id} \le 2(V_{GSQ} - V_t) \longrightarrow$ to maintain saturation

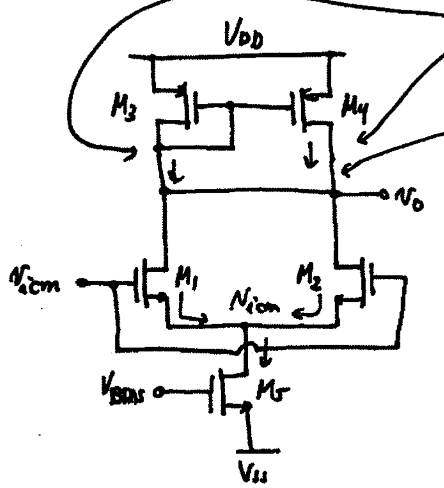
$$V_{GSQ} - V_t = \sqrt{\frac{2I_{d2}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\left(I_d - \frac{\Delta I_d}{2}\right)}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \frac{V_{id}}{2}$$

then plug in ΔI_d & solve for V_{id}

MOS Differential Stage w/ Current Mirror Load



CMRR -



$V_{D3} = V_{D4}$ (they track each other) → provided the ckt. is completely symmetrical.

⇒ Thus, can short the drains of M_3 & M_4

Common-Mode Input Range - Range of input voltages for which all devices remain in saturation.

Low End - must keep M_5 saturated