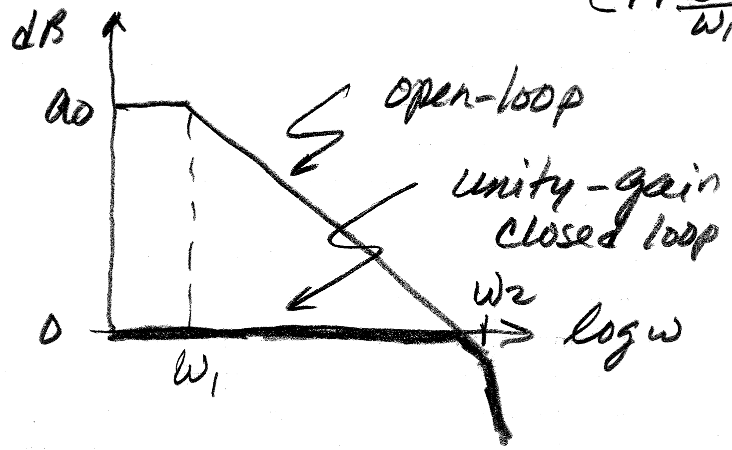
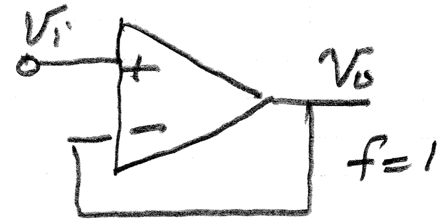
An aside: Frequency / time response of a two-pole system

$$a(s) = \text{open-loop gain} = \frac{a_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$





Worst-case for closed-loop stability

$$A(s) = \text{closed-loop gain} = \frac{a(s)}{1+a(s)} = \frac{A_0}{(s/\omega_0)^2 + 2k(s/\omega_0) + 1}$$

Where $A_0 = \dfrac{a_0}{a_0+1}$

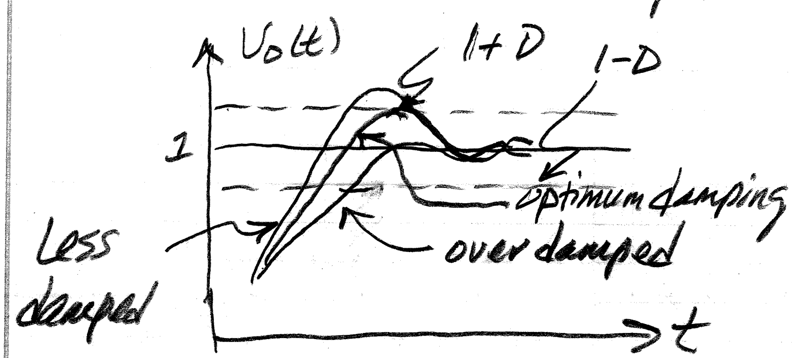
$$\omega_0 = \left[\omega_1 \omega_2 (1+a_0)\right]^{1/2}$$

And $k = \dfrac{\omega_1 + \omega_2}{2\omega_0}$ with $\beta = \omega_2/\omega_1$ $\equiv$ pole separation factor

$k > 1 \rightarrow$ overdamped

$k = 1 \rightarrow$ critically damped

$k < 1 \rightarrow$ underdamped



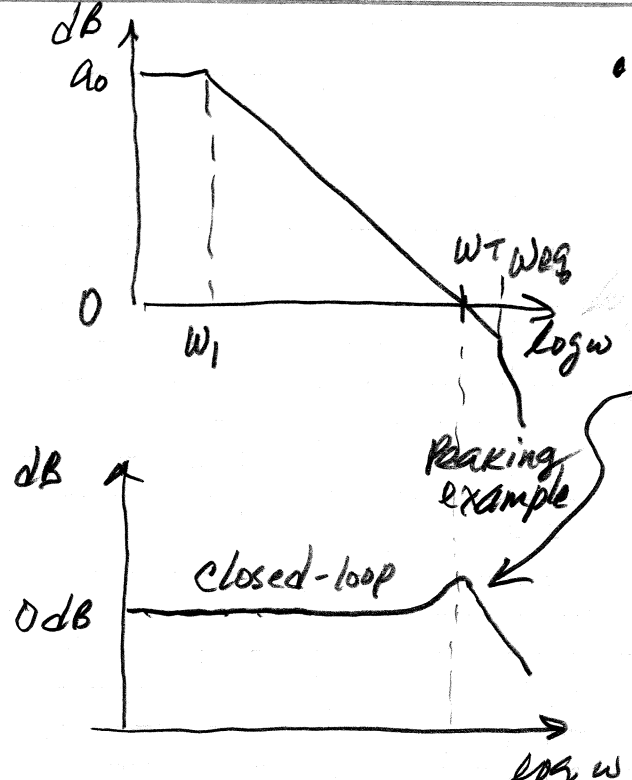
$\beta_0 \equiv$ optimum pole separation factor

$$\simeq \frac{4(1+a_0)}{1+(\pi/\ln D)^2}$$

$\phi_m(mst) \equiv$ phase margin for minimum settling time

$$= 90° - \tan^{-1}\left[\frac{1+(\pi/\ln D)^2}{4}\right]$$

- See: Yang & Allstot, IEEE Trans. Circuits and Systems, March 1990.



- It can be shown that

$$\frac{1}{\omega_{eq}} = \frac{1}{\omega_2} + \frac{1}{\omega_3} + \cdots$$

ϕ_m	ω_T/ω_{eq}	Q	Peaking
55°	0.700	0.925	1.08 dB
60°	0.580	0.817	0.72 dB
65°	0.470	0.717	0.40 dB
70°	0.360	0.622	0.12 dB
75°	0.270	0.527	$\approx$ 0 dB

- Canonic Forms for denominator of second-order System:

$$A(s) = \frac{K}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

where Q = quality factor
ω_0 = resonant frequency

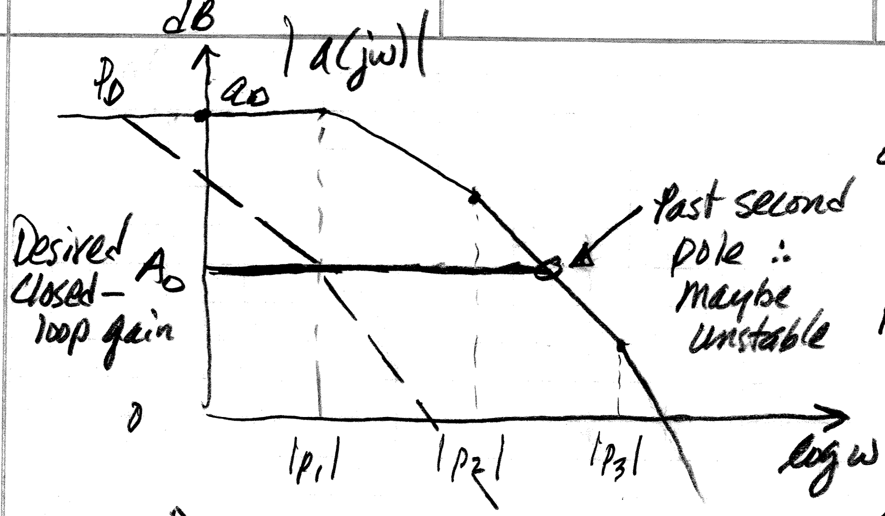
or

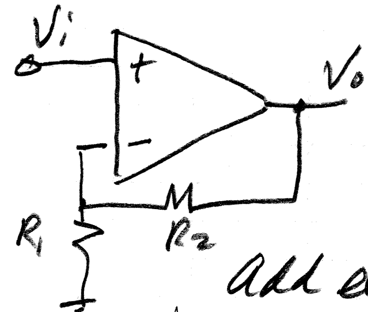
$$A(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ζ = damping factor
ω_n = natural frequency

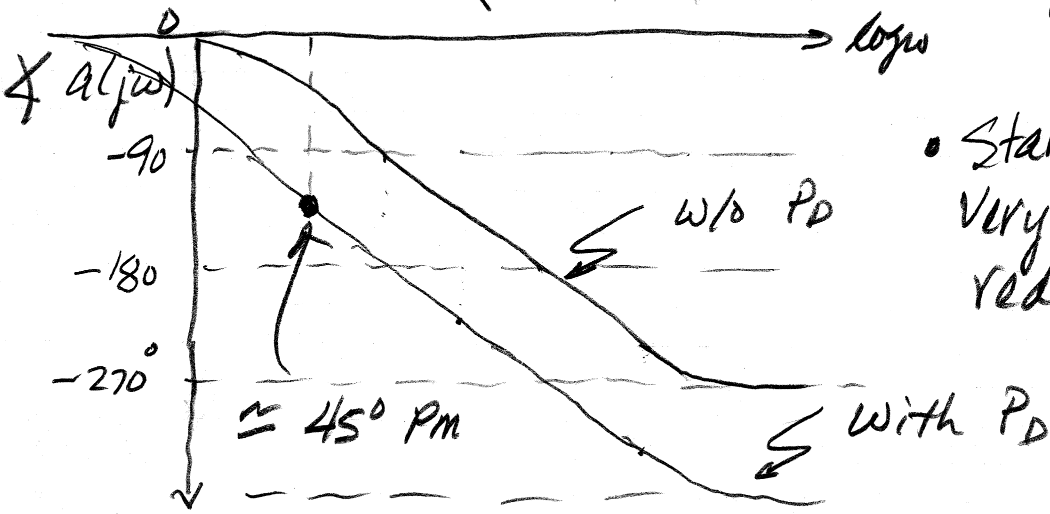
- <u>Compensation Techniques:</u>

(i) <u>Narrowbanding</u> – Introduce a new dominant pole, P_D, into the system so that there is sufficient separation between P_D and P_1. P_1 becomes second pole. Want $P_1/P_D = T_{o\,max}$



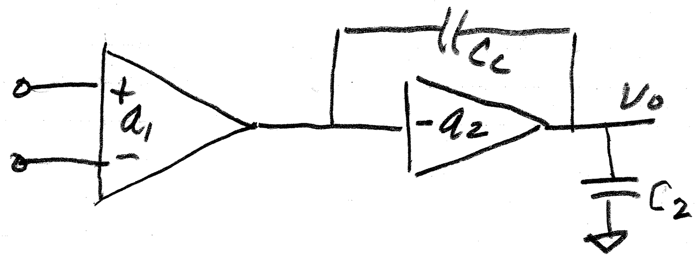


Add elements to create new pole at P_D

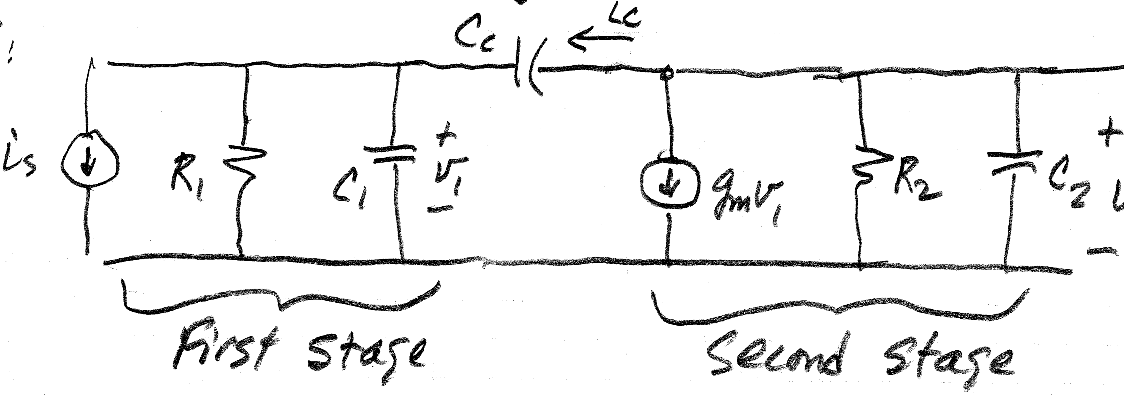


- Stable but very much reduced BW

(ii) <u>Pole-splitting compensation</u>: Take advantage of Miller multiplication of compensation capacitance



<u>model</u>:



$$\frac{V_o}{i_s} = \frac{(g_m - sC)R_1R_2}{1 + s[(C_2+C_c)R_2 + (C_1+C_c)R_1 + g_m R_1 R_2 C_c] + s^2 R_1 R_2 (C_1C_2 + C_2C_c + C_1C_c)}$$

$$z = \text{RHP zero} = \frac{g_m}{C_c}$$

This is a two-pole transfer function:

$D(s) \equiv$ Denominator polynomial

$$= \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)$$

$$= 1 - s\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{p_1p_2}$$

Apply a <u>dominant pole approximation</u>: i.e. $p_1 << p_2$

$$\therefore D(s) \simeq 1 - \frac{s}{p_1} + \frac{s^2}{p_1p_2}$$

<u>Equating coefficients</u>:

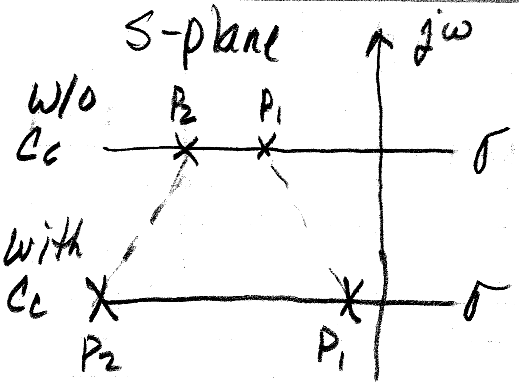
$$p_1 = -\frac{1}{R_1(C_1+C_c) + R_2(C_2+C_c) + g_m R_1 R_2 C_c}$$

$$\boxed{\therefore p_1 \simeq -\frac{1}{g_m R_2 R_1 C_c}}$$ if $g_m R_2 >> 1$ and C_c is large

<u>Now, equate</u> coefficients of s^2 terms:

$$\boxed{p_2 \simeq -\frac{g_m C_c}{C_1C_2 + C_c(C_1+C_2)}}$$

- Let's examine relative to p_1 and p_2 before frequency compensation:



w/o C_c: $p_1 = -\frac{1}{R_1 C_1}$

with C_c: $p_1 \simeq -\frac{1}{g_m R_2 R_1 C_c}$

$$= -\underbrace{\frac{1}{R_1 C_1}}_{\text{original } p_1} \cdot \underbrace{\underbrace{\frac{1}{g_m R_2}}_{\text{second stage gain}} \cdot \left(\frac{C_1}{C_c}\right)}_{\text{Both terms typically} \gg 1}$$

w/o C_c: $p_2 = -\frac{1}{R_2 C_2}$

with C_c: $p_2 \simeq -\frac{g_m C_c}{C_1 C_2 + C_c(C_1 + C_2)}$

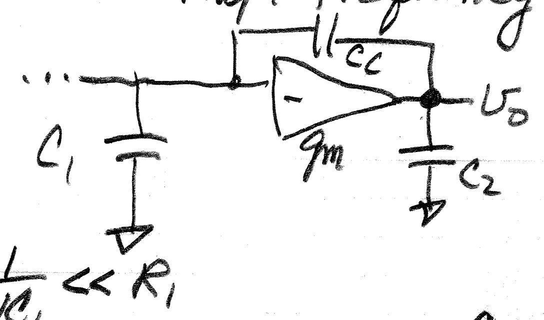
Let's associate p_2 with the output node: $\boxed{p_2 = -\frac{1}{R_O C_T}}$

Where R_O is output resistance including negative FB around second stage via C_c; C_T is total capacitance to ground at output.

$$R_O = \frac{R_2}{1+T} \quad \text{where } T \equiv \text{loop gain around stage 2}$$

$$= \frac{R_2}{1 + g_m R_2 f} \simeq \frac{1}{g_m f} \quad \text{with } T = g_m R_2 f \gg 1$$

Note: p_2 is high frequency so evaluate f at high frequency. Thus, capacitive divider sets f:



$$f \simeq \frac{C_c}{C_1 + C_c} \;\&\; R_O \simeq \frac{C_1 + C_c}{g_m C_c}$$

$C_T = C_2$ in parallel with (C_c and C_1 in series)

$\frac{1}{\omega C_1} \ll R_1$

$$C_T = C_2 + \frac{C_1 C_c}{C_1 + C_c} = \frac{C_1 C_2 + C_1 C_c + C_2 C_c}{C_1 + C_c}$$

Same result as from p. 142

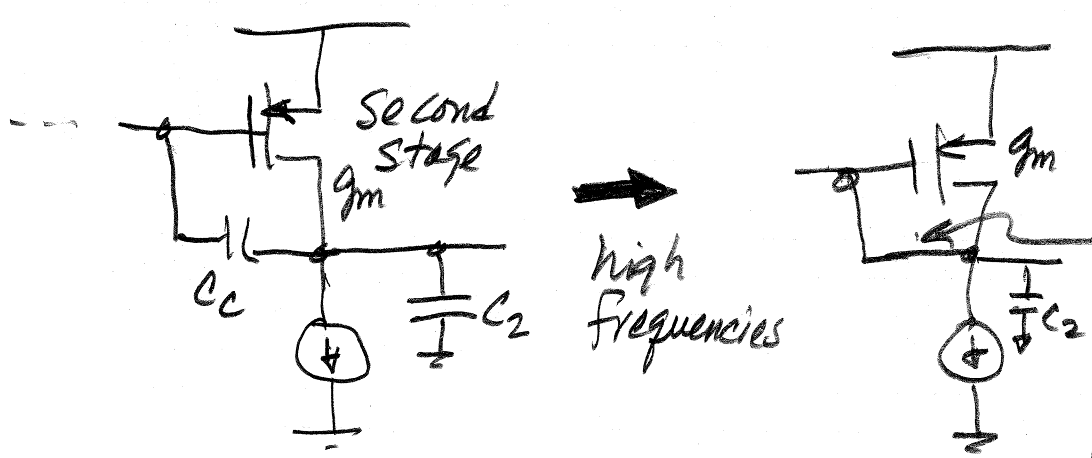
Let's rewrite $p_2 = -\frac{1}{R_O C_T}$ and simplify:

$$p_2 \simeq -\frac{g_m C_c}{\cancel{C_1 + C_c}} \cdot \frac{\cancel{C_1 + C_c}}{C_1 C_2 + C_1 C_c + C_2 C_c}$$

But, in practice, $C_c \simeq C_2 \gg C_1$ (use layout techniques to minimize C_1)

$$\therefore \; p_2 \simeq -\frac{g_m C_c}{\cancel{C_1 C_2} + \cancel{C_1 C_c} + C_2 C_c}$$

$$\boxed{p_2 \simeq -\frac{g_m}{C_2}}$$ Easy to remember



Think of C_c as short circuit so pmos is diode connected with resistance $= 1/g_m$