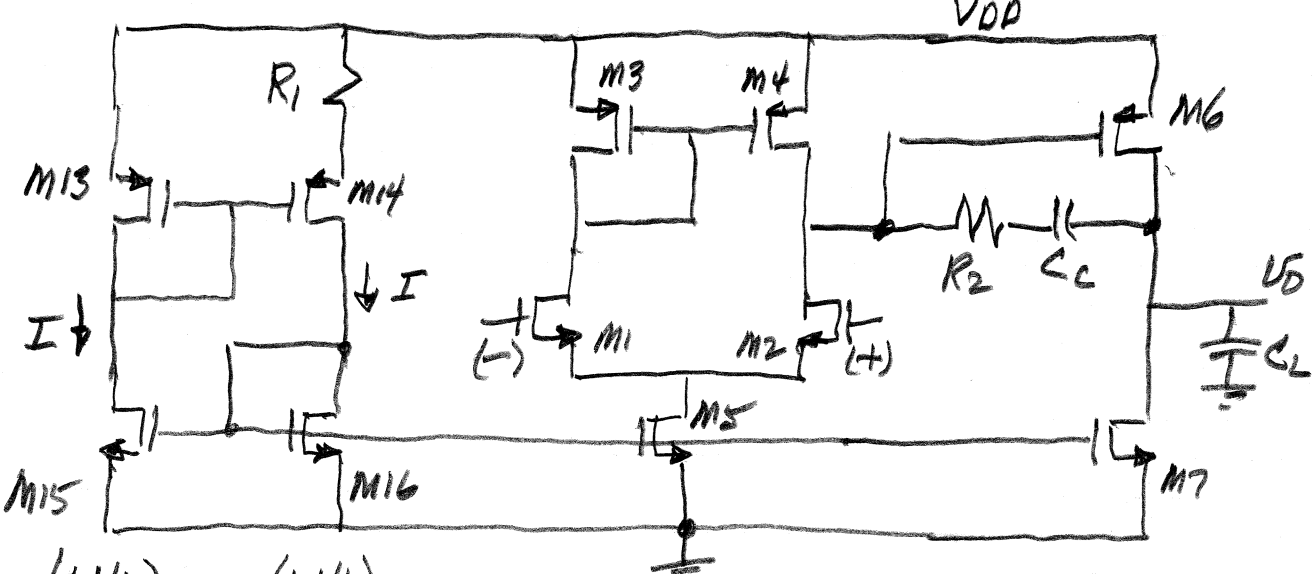
- <u>Some loose ends: k' generator compensation</u>



$(W/L)_{14} > (W/L)_{13}$

$(W/L)_{15} = (W/L)_{16}$

$\therefore$ all currents $= I$

<u>KVL neglecting back-gate effects:</u>

$$I = \frac{2}{k_p' R_1^2} \underbrace{\frac{1}{\left(\frac{1}{\sqrt{(W/L)_{13}}} - \frac{1}{\sqrt{(W/L)_{14}}}\right)^2}}$$

[This is a geometric constant which is insensitive to PVT variations neglecting mismatches]

$$z_1 = -\frac{1}{R_2 C_I}$$

$$p_2 = -\frac{g_{m2}}{C_{II}}$$

Assume $I_7 = I_6 = kI$

$$\therefore p_2 = -\frac{\sqrt{2k_p'(W/L)_6 I_6}}{C_{II}}$$

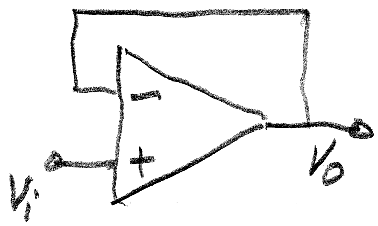
$$= -\frac{\sqrt{2k_p'(W/L)_6\, kI}}{C_{II}} = -\frac{\sqrt{\dfrac{2k_p'(W/L)_6\, k\, 2}{k_p' R_1^2 \left(\frac{1}{\sqrt{(W/L)_{13}}} - \frac{1}{\sqrt{(W/L)_{14}}}\right)^2}}}{C_{II}}$$

$$= -\frac{2}{R_1 C_{II}} \cdot K_1 \quad (K_1 \text{ is another geometric constant})$$

Finally, $z_1 = p_2 \Rightarrow \boxed{R_2 = \frac{R_1}{2K_1}\left(\frac{C_{II}}{C_I}\right)}$ This is a low-voltage solution.

- <u>Slew-rate in opamps:</u>

Consider a single-pole opamp in unity gain:

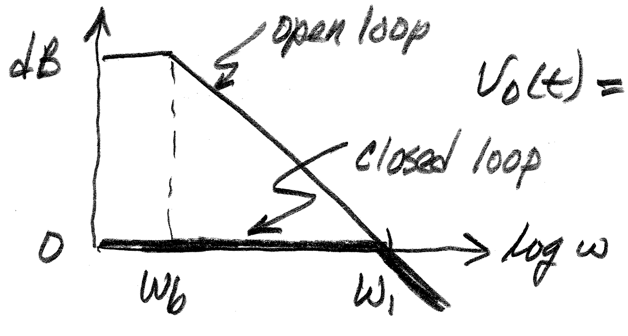


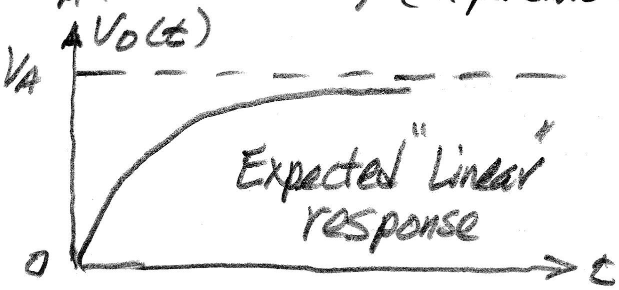
$$V_i(s) = \frac{V_A}{s} \quad \text{(step input)}$$

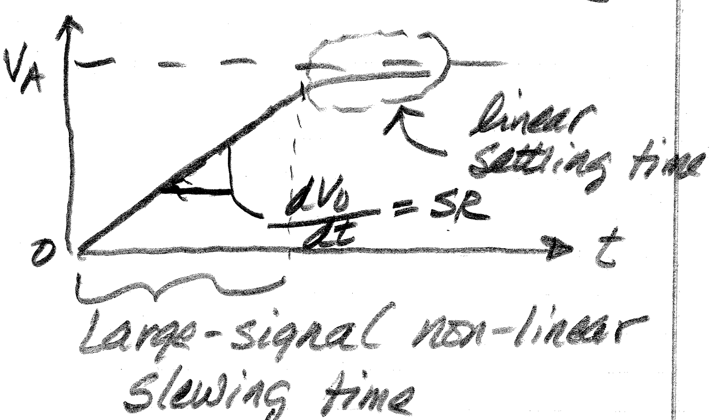
$$V_o(s) = \frac{V_A}{s(1+s\tau_1)} = \frac{V_A}{s} - \frac{V_A}{s+\frac{1}{\tau_1}}$$

$\Updownarrow$ inverse Laplace transform

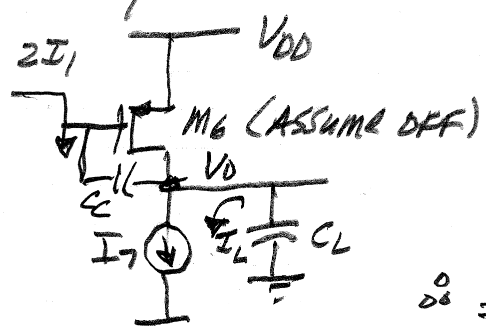
$$V_o(t) = V_A\left(1 - e^{-t/\tau_1}\right) \quad \text{(exponential)}$$







* - For two-stage opamp, SR may be limited at output of first stage and/or output of second stage. From before:

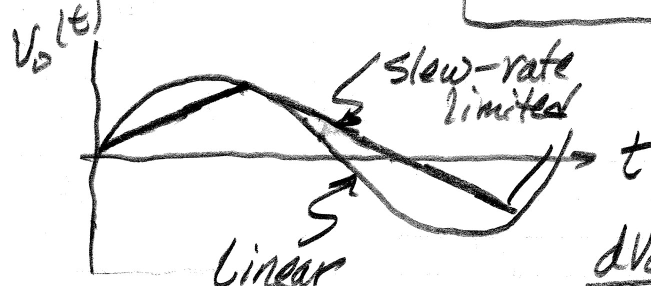


<u>KCL at V_O</u>: $I_7 = I_L + 2I_1$

$\therefore I_L = I_7 - 2I_1 = C_L \frac{dV_o}{dt}$

$$\therefore \underbrace{\frac{I_L}{C_L} = SR_2}_{\text{second stage}} \quad \text{and} \quad \underbrace{\frac{2I_1}{C_C} = SR_1}_{\text{first stage}}$$

- For $SR_1 = SR_2 \rightarrow \boxed{I_7 = 2I_1\left(\frac{C_L + C_C}{C_C}\right) \simeq 4I_1}$



<u>Power Bandwidth</u>

$V_o(t) = V_p \sin \omega t$

$$\left.\frac{dV_o}{dt}\right|_{max} = \omega V_p = SR \quad \therefore \omega_{max} = \frac{SR}{V_p}$$

- Consider SR in terms of design variables:

$$SR = \frac{dV_o}{dt} = \frac{I_{XM}}{C_C}(\text{maximum}) = \boxed{\frac{I_{XM}}{G_{mI}} \cdot \omega_{ULT} A_o}$$

$$C_C = \frac{G_{mI}}{\omega_{ULT} A_o} \quad (\omega_{ULT} = \omega @ |T(j\omega)| = 1)$$

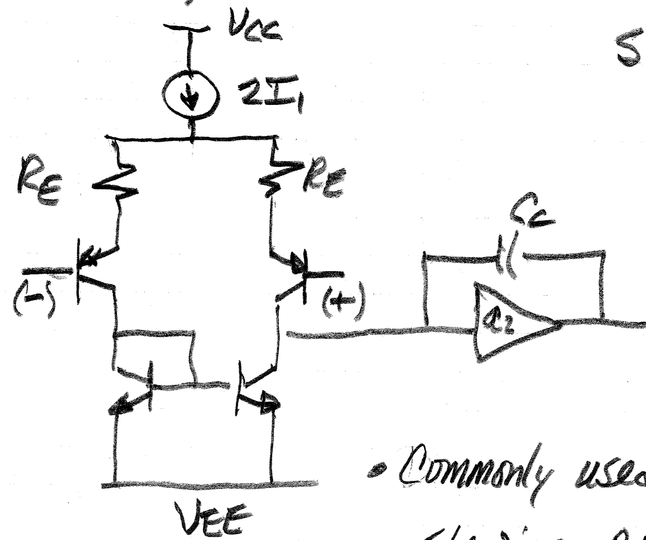
Many performance parameter dependencies:

To increase SR:

1. Decrease G_{mI}
2. Increase ω_{ULT} → increase p_2 (limited by max transistor operating freq)
3. Use a larger A_o if possible by the application.

- Best choice is to ① decrease G_{mI} to increase SR:

(A) Consider BJT design with Emitter (or source for MOS) degeneration:



$$SR = \frac{2I_1}{G_{mI}} \omega_{ULT} A_o$$

But $G_{mI} \simeq \dfrac{g_{m1}}{1 + g_{m1} R_E}$

$$SR = 2I_1 \omega_{ULT} A_o \frac{(1 + g_{m1} R_E)}{g_{m1} R_E}$$

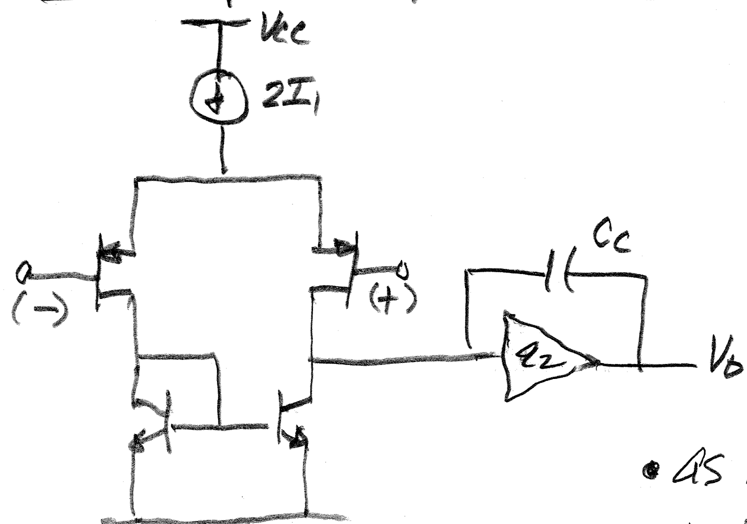
- Commonly used for fast slewing BJT opamps

Disadvantages: ① R_E mismatch → V_{OS} (must limit V_{RE} to limit V_{OS})

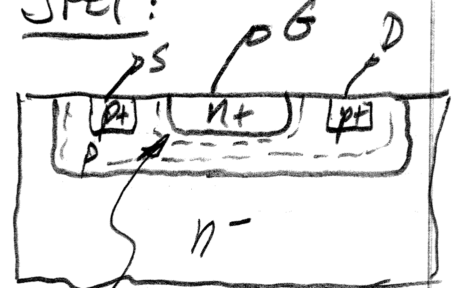
② R_E ↑ gain ↓ (SR-gain tradeoff)

③ R_E contributes thermal noise → must limit size of R_E to preserve opamp noise performance

(B) JFET input stage in BJT technology:



JFET:



• as V_G is made more positive, depletion region widens to pinch off the channel region.

For FETs: $\frac{g_m}{I_D} \simeq \frac{2}{(V_{GS}-V_T)} \simeq \frac{2}{0.2}$

For BJTs: $\frac{g_m}{I_C} = \frac{1}{V_T} = \frac{1}{25\,mV}$

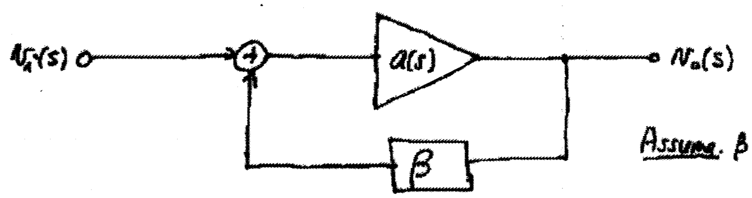
So $\left.\frac{g_m}{I_C}\right|_{BJT} \gg \left.\frac{g_m}{I_D}\right|_{FET}$

$$\therefore \quad \frac{SR|_{FET}}{SR|_{BJT}} = \frac{\frac{I_D}{g_{mF}}\,\omega_{ULT} A_0}{\frac{I_C}{g_{mB}}\,\omega_{ULT} A_0} \simeq \frac{(V_{GS}-V_T)}{2V_T} \geq 5$$

Limitations: ① higher V_{os}

② Increased voltage noise (but decreased current noise)

Obtain expressions for overshoot ($V_{overshoot}$) and settling time (T_s) as functions of phase margin, ϕ_m:



Assume: β = const. w/ freq.

$$A(s) = \frac{v_o(s)}{v_i(s)} = \frac{a(s)}{1+a(s)\beta} = \frac{a_o\omega_1\omega_2}{s^2+(\omega_1+\omega_2)s+\omega_1\omega_2(1+a_o\beta)} = \underbrace{\frac{A_o\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{A_o\omega_o^2}{s^2+\left(\frac{\omega_o}{Q}\right)s+\omega_o^2}}_{\text{General Lowpass Biquad Transfer Functions}} \quad [\omega_o = \omega_n]$$

$$\left[a(s) = \frac{a_o}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} = \frac{a_o\omega_1\omega_2}{(s+\omega_1)(s+\omega_2)}\right]$$

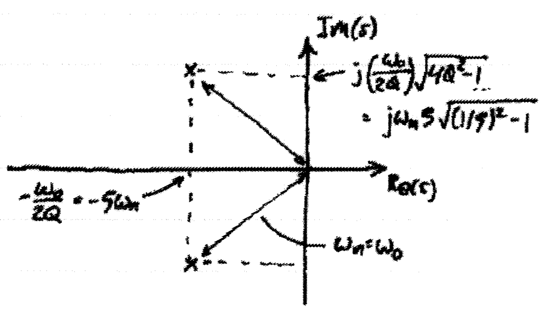
Where by direct comparison:

$$\begin{cases} \omega_n = \sqrt{\omega_1\omega_2(1+a_o\beta)} \\ 2\zeta\omega_n = \omega_1+\omega_2 \longrightarrow \zeta = \dfrac{\omega_1+\omega_2}{2\omega_n} = \dfrac{1}{2}\dfrac{\omega_1+\omega_2}{\sqrt{\omega_1\omega_2(1+a_o\beta)}} \\ A_o\omega_n^2 = a_o\omega_1\omega_2 \longrightarrow A_o = \dfrac{a_o}{1+a_o\beta} \end{cases}$$

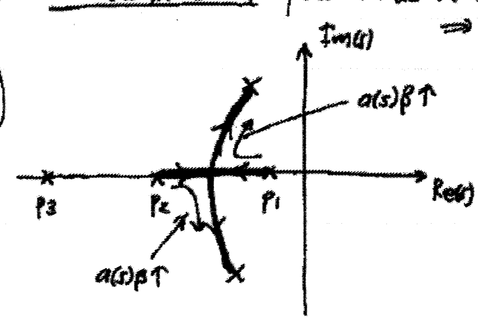
Frequency Response Spectra for various values of ζ on next page

$\boxed{\text{Properties of the General Lowpass Biquad Transfer Function}}$ (a very well-studied function!)

Pole-Zero Diagram:



Root Locus for FB Ckt: poles move as the loop gain increases ⟹ they become complex poles



Time Domain Behavior:

$$v_o(t) = A_oV_i\left[1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t+\phi\right)\right], \quad \text{where } \phi = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

For ζ = small $\rightarrow \phi \simeq \frac{\pi}{2}$

For $\zeta<1$: (for PM < 90°)

$$\%\text{ Overshoot} = \frac{\text{Peak Value} - \text{Final Value}}{\text{Final Value}} = \exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right] \Longrightarrow \boxed{V_{overshoot} = V_o\exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right]}$$

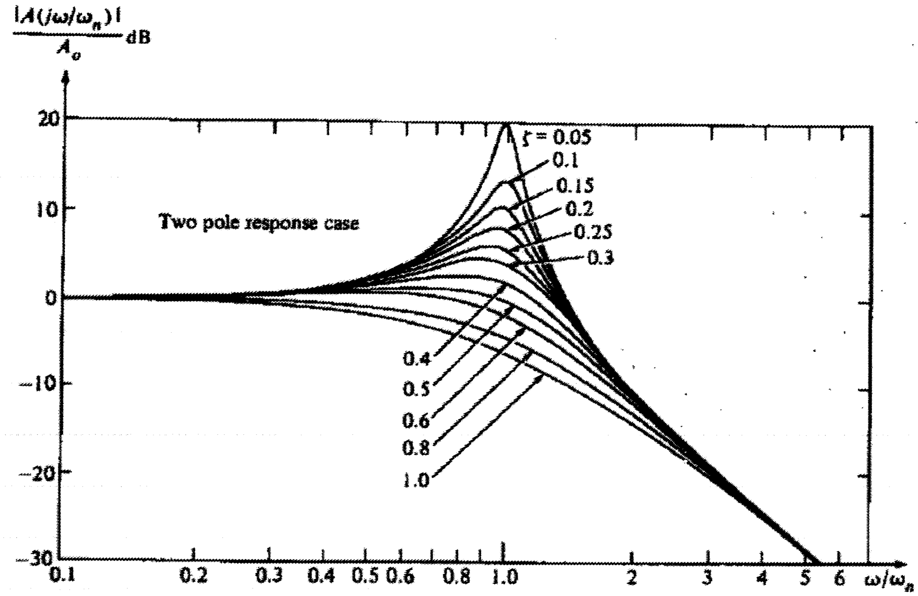
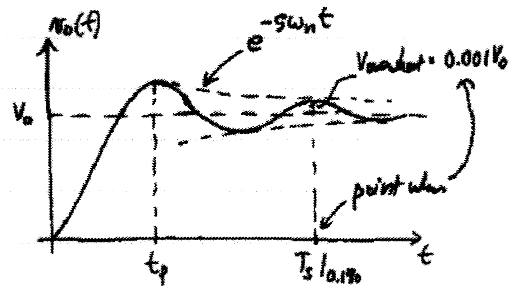


Figure C-2 Gain magnitude response for various values of ζ for a second-order, low-pass system.



Find t_p:

$$\sqrt{1-\zeta^2}\,\omega_n t_p + \phi = \frac{3\pi}{2}$$

$$[\zeta = \text{small} \rightarrow \phi \sim \tfrac{\pi}{2}] \Longrightarrow \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

Approximate Determination of 0.1% Settling Time –

$$V_{overshoot}\, e^{-\zeta\omega_n(T_s-t_p)} = 0.001V_o \longrightarrow \boxed{T_s = t_p - \frac{1}{\zeta\omega_n}\ln\left[\frac{0.001V_o}{V_{overshoot}}\right]}$$

$$\longrightarrow \boxed{\zeta = -\frac{1}{\omega_n(T_s-t_p)}\ln\left[\frac{0.001V_o}{V_{overshoot}}\right]}$$

Determine $\phi_m = f(\zeta)$ –

→ first get an expression for loop transmission:

$$A(s) = \frac{a(s)}{1+a(s)\beta} \longrightarrow a(s) = A(s) + a(s)A(s)\beta \longrightarrow a(s) = \frac{A(s)}{1-\beta A(s)}$$

$$\rightarrow a(s)\beta = \frac{\beta A(s)}{1-\beta A(s)} = \frac{\beta A_o\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2-\beta A_o\omega_n^2} \underset{[\beta \approx \frac{1}{A_o}]}{=} \frac{\omega_n^2}{s^2+2\zeta\omega_n s} = \frac{\omega_n^2}{\underset{\text{pole @ origin}}{s}\,\underset{\text{LHP pole}}{(s+2\zeta\omega_n)}}$$

$\Rightarrow$ get freq. at which $|a(s)\beta| = 1 \rightarrow \omega_u$: $= \omega_{ult}$

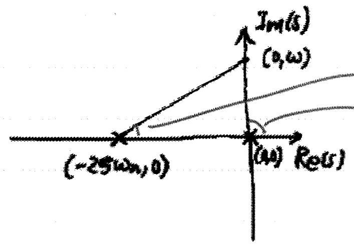
$$a(j\omega)\beta = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega} \Rightarrow |a(j\omega)\beta| = \frac{\omega_n^2}{\sqrt{\omega^4 + \omega^2 4\zeta^2\omega_n^2}}$$

$$\left[|a(j\omega_u)\beta| = 1\right] \Rightarrow \frac{\omega_n^4}{\omega_u^4 + 4\zeta^2\omega_n^2\omega_u^2} = 1 \rightarrow \underbrace{\omega_u^4 + 4\zeta^2\omega_n^2\omega_u^2 - \omega_n^4 = 0}$$

Solve quadratic: $\boxed{\omega_u = \omega_n\left[\sqrt{4\zeta^2+1} - 2\zeta^2\right]^{1/2}}$

Freq. for phase margin determination.

Find expression for phase:



$$\phi = -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2\zeta\omega_n}\right)$$
$$= -90° - \tan^{-1}\left(\frac{\omega}{2\zeta\omega_n}\right)$$

For Phase Margin:

$$\phi_m = 180° + \phi\big|_{\omega=\omega_u} = 90° - \tan^{-1}\left(\frac{\omega_u}{2\zeta\omega_n}\right)$$

(ω_{ult}) (ω_{ult})

$$\tan(90° - \phi_m) = \frac{\omega_u}{2\zeta\omega_n}$$

$$\left[\tan(A+90°) = -\frac{1}{\tan A}\right] \Rightarrow -\frac{1}{\tan(-\phi_m)} = \frac{\omega}{2\zeta\omega_n}$$

$$\left[\tan^{-1}(-x) = -\tan^{-1}x\right] \Rightarrow \boxed{\phi_m = \tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_u}\right) \text{ or } \phi_m = \cos^{-1}\left[\sqrt{4\zeta^4+1} - 2\zeta^2\right]}$$

ω_{ult}

Thus:

$$\zeta = \frac{1}{2}\frac{\omega_{ult}}{\omega_n}\tan\phi_m$$

$$\Rightarrow \boxed{T_s = t_p - \frac{2}{\omega_u \tan\phi_m}\ln\left[\frac{0.001V_o}{V_{overshoot}}\right]}$$ (ignoring effects of slew rate)

ω_{ult}

$\Rightarrow$ But there's more to it than this. See the following papers:

1. B.Y. Kamath, R.G. Meyer, and P.R. Gray, "Relationship between frequency response and settling time of operational amplifiers," IEEE J. of Solid-State Ckts., vol. SC-9, no. 6, pp. 347–352, Dec. 1974.

2. R.J. Apfel and P.R. Gray, "A fast-settling monolithic operational amplifier using doublet compression techniques," IEEE J. of Solid-State Ckts, Vol. SC-9, no. 6, pp. 332–340, Dec. 1974.