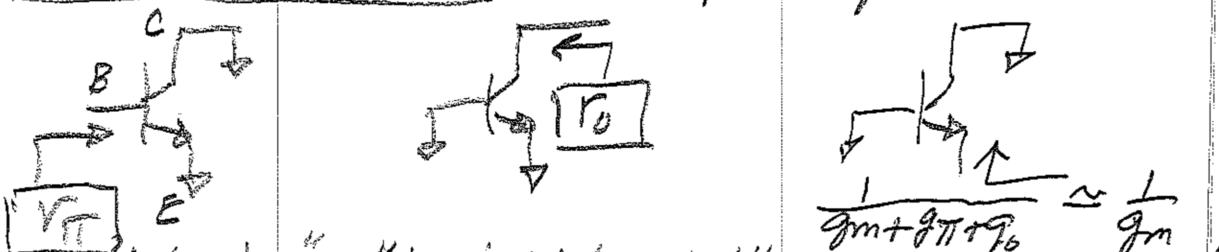
Today: • Inspection analysis of basic stages
• CMOS gain stage analysis

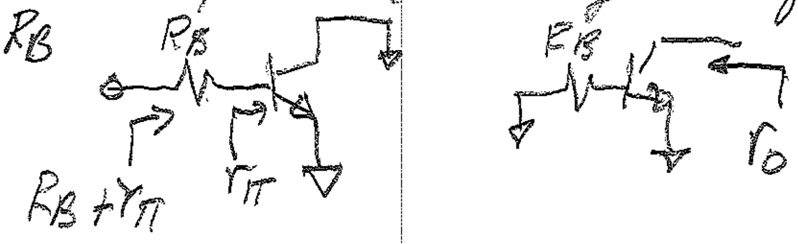
Terminal resistances: (simple single transistor)



"Looking into base" "Looking into collector" "Looking into emitter"

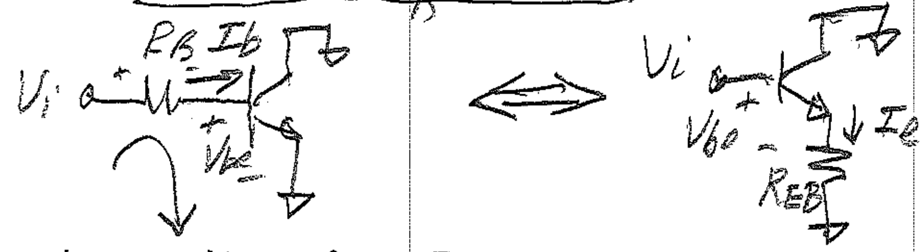
Now, lets extend by adding series resistances

→(A) R_B



• Looking into emitter is slightly more complicated.

Form an equivalence:



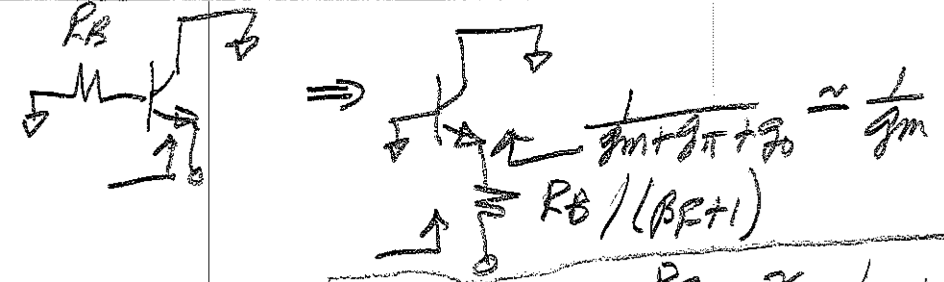
KCL: $V_i = V_{be} + I_b R_B$ KCL: $V_i = V_{be} + I_e R_{EB}$

$$= V_{be} + (\beta_F+1) I_b R_{EB}$$

Equating $\Rightarrow$ $\boxed{R_{EB} = \dfrac{R_B}{\beta_F+1}}$

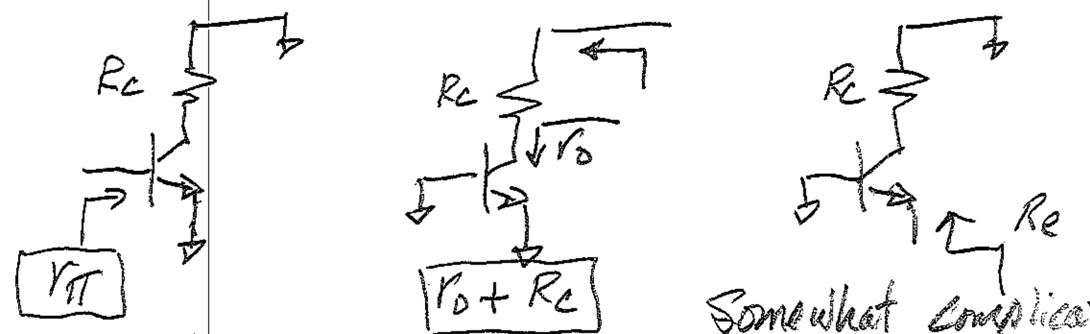
∴ To move a resistance from in series with base to in series with emitter, divide its value by (β_F+1)
Likewise, from Emitter to base, multiply by (β_F+1).

Thus,

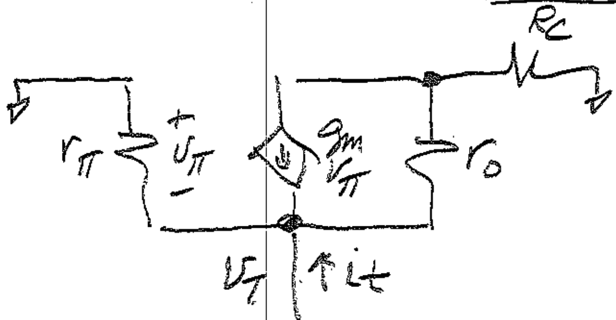


$$\boxed{\frac{1}{g_m+g_\pi+g_o} + \frac{R_B}{\beta_F+1} \simeq \frac{1}{g_m} + \frac{R_B}{\beta_F+1}}$$

(B) R_C



Somewhat complicated



Write two KCL equations and solve:

$$R_e = \frac{g_o + g_c}{g_c(g_m + g_\pi + g_o) + g_o g_\pi} \simeq \frac{1 + g_o/g_c}{g_m + \frac{g_o g_\pi}{g_c}}$$

<u>two cases</u>: (i) R_C large $\simeq r_o$ (i.e., $g_o \simeq g_c$)

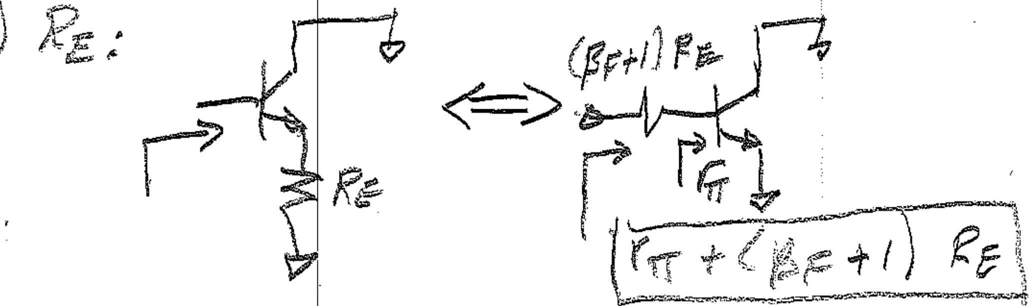
$\therefore$ $\boxed{R_e \simeq \frac{2}{g_m}}$

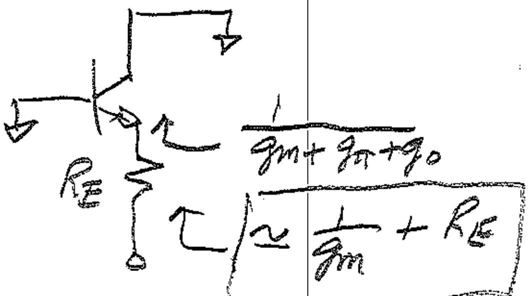
or (ii) R_C small $<< r_o$ (discrete implementation)

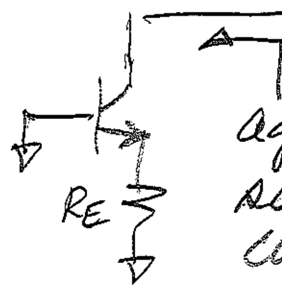
$(g_c >> g_o)$

$\therefore$ $\boxed{R_e \simeq \frac{1}{g_m}}$

(c) R_E:

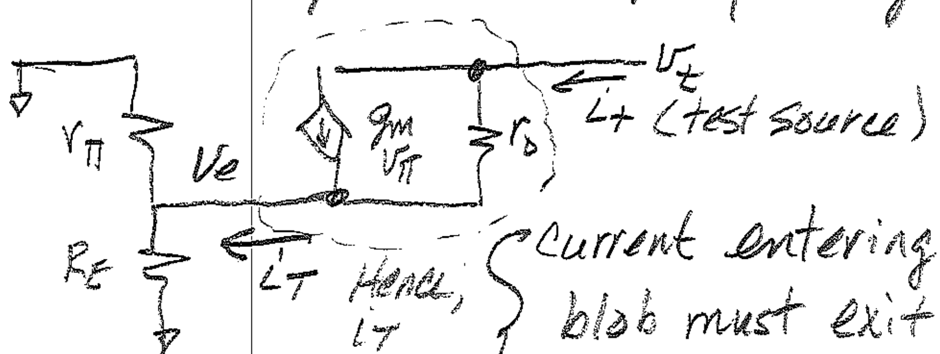






again, somewhat complicated

Draw small-signal model (funny looking way):



current entering this blob must exit this blob.

$\therefore$ by inspection, $v_e = \dfrac{i_T}{(g_\pi + g_E)}$

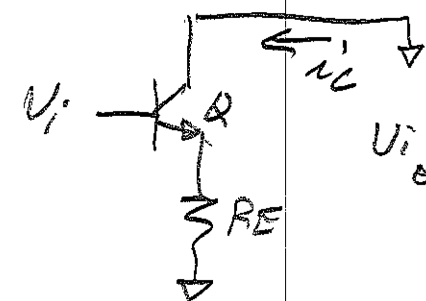
Now, KCL at collector:

$$
\begin{aligned}
i_T &= g_m v_\pi + g_o (v_t - v_e) \\
&= g_m (0 - v_e) + g_o (v_t - v_e) \\
&= -(g_m + g_o) v_e + g_o v_t \\
&= -(g_m + g_o) \frac{i_T}{g_\pi + g_E} + g_o v_t
\end{aligned}
$$

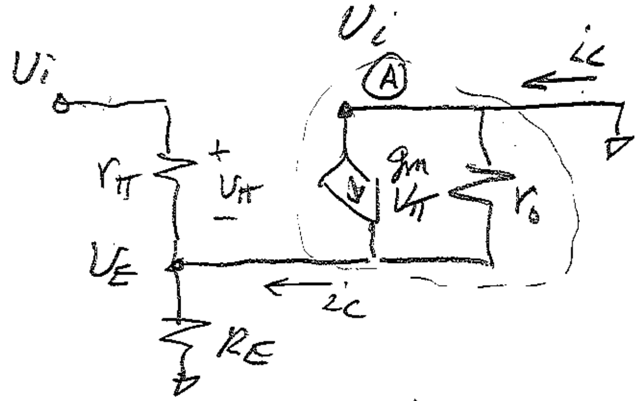
$$
\therefore R_c = \frac{v_t}{i_t} = \left(\frac{g_m + g_\pi + g_o + g_E}{g_\pi + g_E} \right) r_o \simeq \left(\frac{g_m + g_E}{g_\pi + g_E} \right) r_o
$$

$$
= \frac{(1 + g_m / g_E) r_o}{(1 + g_\pi / g_E)} \simeq \frac{(1 + g_m R_E) r_o}{1 + g_m R_E / \beta_F} \simeq \boxed{g_m R_E r_o}
$$

- Small-signal transconductance with emitter degeneration, $G_m \neq g_m$:



$$G_m = \frac{i_c}{v_i}$$



$$v_e = \frac{i_c}{g_\pi + g_E} \quad \text{(By inspection)}$$

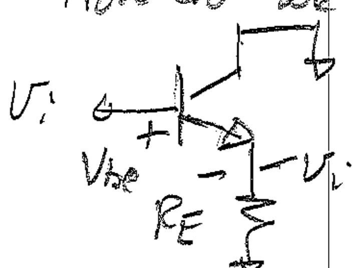
Now, KCL at (A):

$$\begin{aligned} i_c &= g_m v_\pi + g_o(0 - v_e) \\ &= g_m(v_i - v_e) + g_o(0 - v_e) \\ &= g_m v_i - v_e(g_m + g_o) \\ &= g_m v_i - \frac{(g_m + g_o)}{(g_\pi + g_E)} i_c \end{aligned}$$

$$\therefore G_m \simeq \boxed{\frac{g_m}{1 + g_m R_E}}$$

Usually, $g_m R_E \gg 1 \Rightarrow \boxed{G_m \simeq \frac{1}{R_E}}$

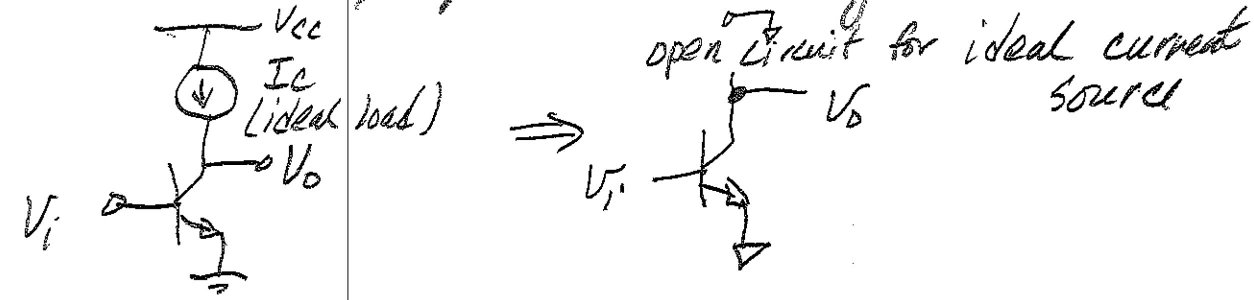
How do we see this by inspection.



If we assume V_{be} is constant at $V_{be} = V_{BE} = 0.7V$, then signal v_i appears across R_E

$\therefore i_E = \frac{v_i}{R_E} \simeq i_c \therefore G_m = \frac{1}{R_E}$

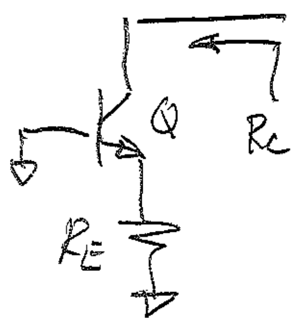
- An important voltage gain limitation = "open-circuit voltage gain" or "self gain"



Quick s.s. analysis: $A_V = \frac{v_o}{v_i} = -\boxed{g_m r_o}$

$$= -\frac{I_C}{V_T} \cdot \frac{V_A}{I_C} = \boxed{-\frac{V_A}{V_T}}$$

- This is extraordinarily useful in inspection analysis:



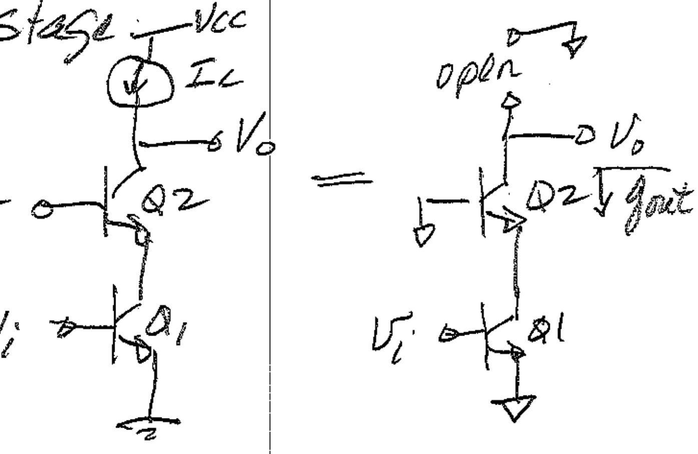
Recall from previous analysis:

$R_C \simeq g_m R_E r_o$ (Bottom page 28)

Rearrange: $\boxed{R_C = g_m r_o \, R_E}$ ← ***

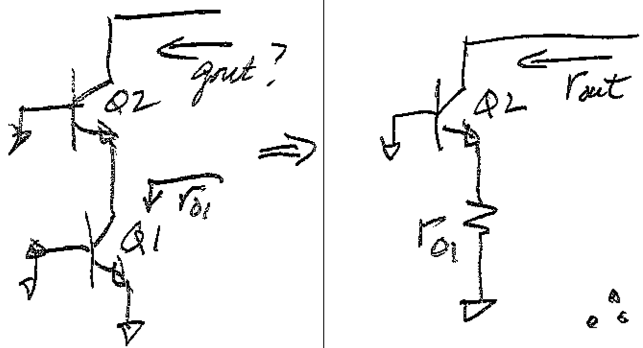
⇒ series resistance is multiplied by self-gain of Q

This is powerful. Consider a cascode gain stage



as we know, $A_V = -g_{m1}/g_{out}$

What is g_{out}?



$r_{out} \simeq (g_{m2} r_{o2})\, r_{o1}$

$\therefore\ A_V = -g_{m1} g_{m2} r_{o2} r_{o1}$

$= -(g_{m1} r_{o1})(g_{m2} r_{o2})$

$\therefore$ Cascode gives gain of two simple stage but only uses one shared bias current

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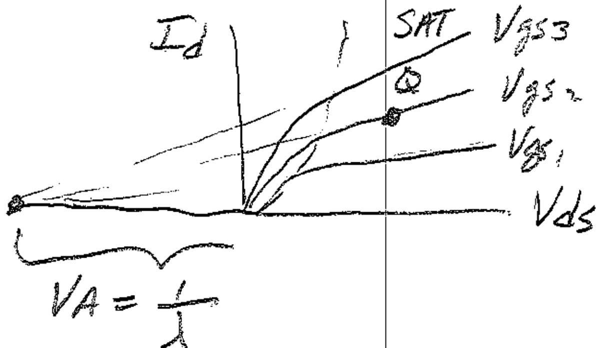
MOSFET small-signal models:



Saturation:

$V_{gs} > \underbrace{V_T}_{\text{threshold voltage not thermal voltage}}$ and $V_{ds} > (V_{gs} - V_T)$

$$I_d = \frac{k_n'}{2}\left(\frac{W}{L}\right)(V_{gs} - V_T)^2(1 + \lambda V_{ds})$$

$$g_{ds} = \frac{\partial I_d}{\partial V_{ds}}\bigg|_Q = \frac{\lambda I_D}{1 + \lambda V_{DS}} \simeq \boxed{\lambda I_D}$$

Otherwise, including for DC calculations, neglect channel-length modulation term:

$$I_d = \frac{k_n'}{2}\left(\frac{W}{L}\right)(V_{gs} - V_T)^2$$

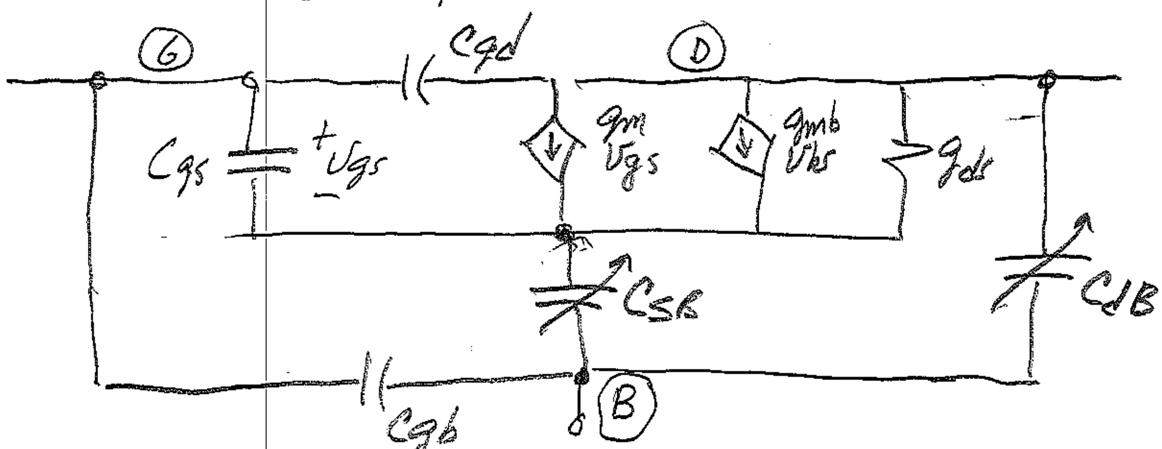
$$g_m = \frac{\partial I_d}{\partial V_{gs}}\bigg|_Q = \boxed{k_n'\left(\frac{W}{L}\right)(V_{GS} - V_T)}$$ #1

$$= \boxed{\frac{2 I_D}{(V_{GS} - V_T)}}$$ #2 (most like BJT)

$$= \sqrt{2 k_n' \left(\frac{W}{L}\right) I_D}$$ #3

$$g_{mb} = \frac{\partial I_d}{\partial V_{bs}}\Big|_Q = \text{Back-gate transconductance}$$

$$= \frac{\gamma}{2\sqrt{V_{SB}+2\phi_f}} \cdot g_m$$



Complete MOSFET small-signal model: