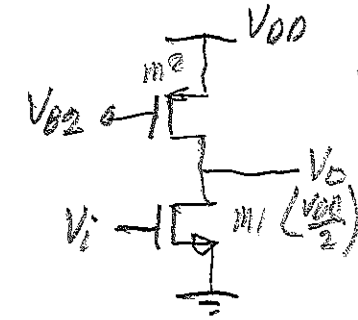
Multi-stage Amplifiers with Active Loads:

(A) PMOS Current Source load for NMOS CS Amp:



Output Swing:

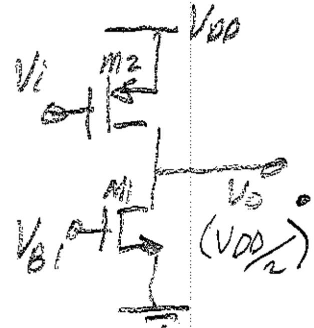
$V_O(\text{min}) = V_{dsat1}$

$V_O(\text{max}) = V_{DD} - |V_{dsat2}|$

$a_v = \dfrac{-g_{m1}}{g_{ds1} + g_{ds2}}$

• DC level shift in pos. direction

(B) NMOS Current Source load for PMOS CS Amp:

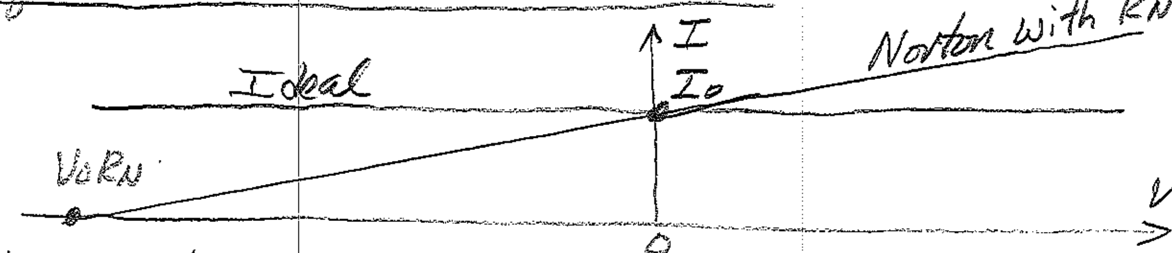


Output swing: Same as (A)

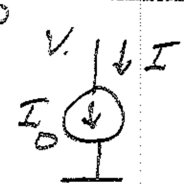
• But, DC level shift in neg. direction

$a_v = \dfrac{-g_{m2}}{g_{ds1} + g_{ds2}}$

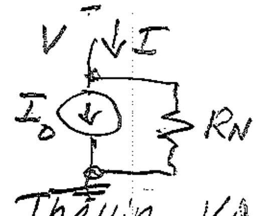
A quick review on current sources:



i) Ideal current source:



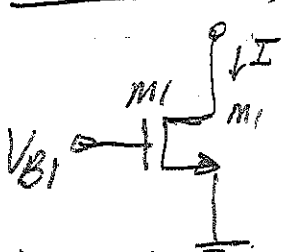
ii) Norton current source:

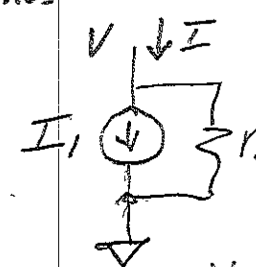


$I = I_0 + \dfrac{V}{R_N}$

$I = 0 \Rightarrow V = -I_0 R_N$ = Thevin voltage is very similar to Early voltage, etc.

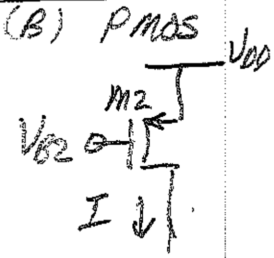
Models: (A) NMOS

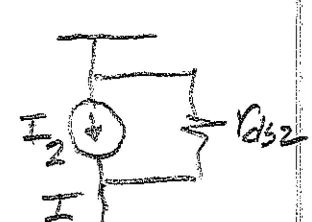




"Sinks" current $\quad r_{ds1} \simeq \dfrac{V_{A1}}{I_1}$

(B) PMOS

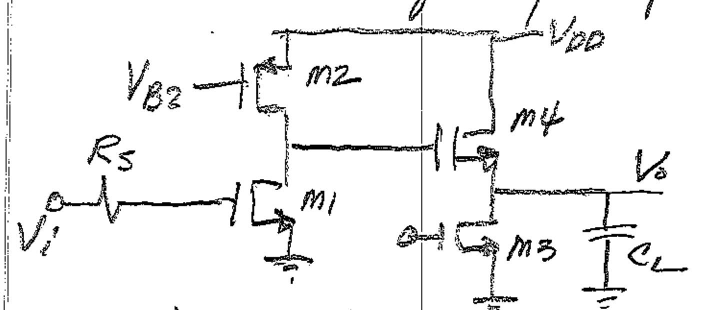


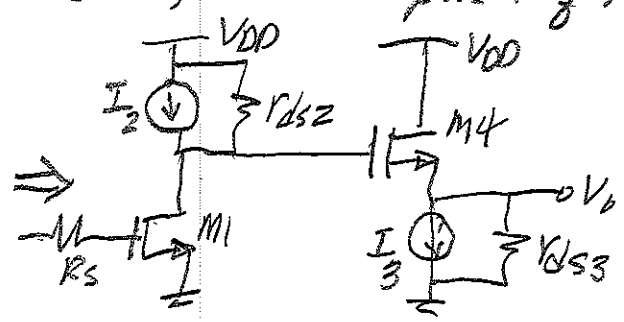


"Sources" current $\quad r_{ds2} \simeq \dfrac{V_{A2}}{I_2}$

Multi-stage Amp with Active loads:

Find: i) Input resistance; ii) Output resistance;
iii) frequency response (i.e., dominant pole freq.)



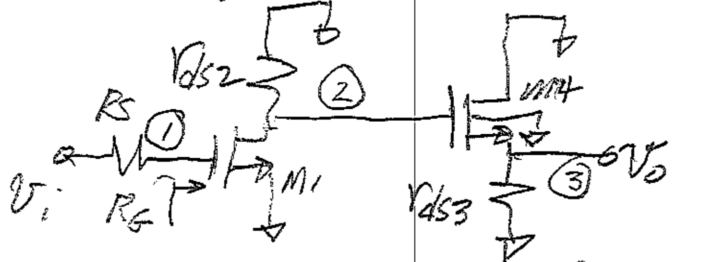


CS stage with PMOS active load

CD stage (i.e., source follower) with NMOS active load)

(Replace current sources with Norton equivalents) M_1 biased at I_2; m4 biased at I_3.

ac equivalent circuit: (low freqs)



total voltage gain:

$$a_v = \frac{V_{①}}{V_i}\cdot\frac{V_{②}}{V_{①}}\cdot\frac{V_o}{V_{②}}$$

$V_{①} = \frac{R_G}{R_G+R_S}V_i$; $V_2 = \frac{-g_{m1}V_{①}}{g_{ds1}+g_{ds2}}$; $V_O = + \frac{g_{m4}}{g_{m4}+g_{mb4}+g_{ds4}+g_{ds3}}$

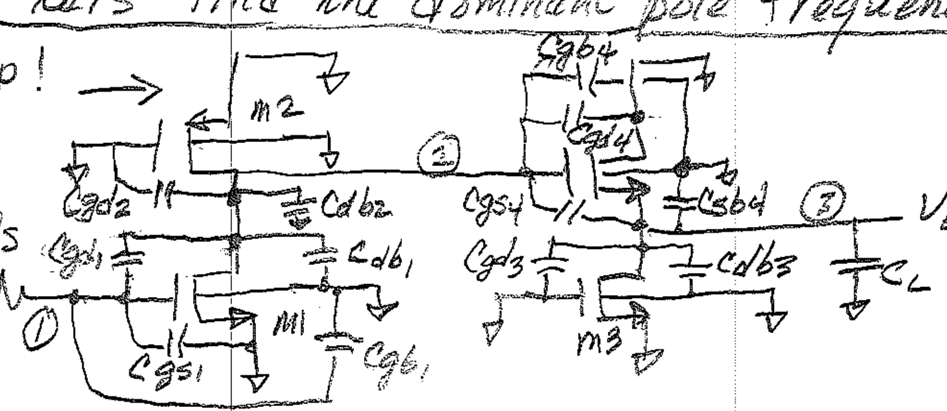
$= V_i \; (R_G = \infty)$

$$\therefore a_v = 1 \cdot \frac{-g_{m1}}{g_{ds1}+g_{ds2}} \cdot \frac{g_{m4}}{g_{m4}+g_{mb4}+g_{ds4}+g_{ds3}}$$

$$a_v \simeq -\frac{g_{m1}}{g_{ds1}+g_{ds2}} \cdot \frac{1}{1+\eta} \quad \text{(neglect } g_{ds4} + g_{ds3} \text{ terms)}$$

Now, let's find the dominant pole frequency:

Gasp! →



Now, before we apply ZVTC method to find dominant pole frequency, lets get the capacitance to ground at nodes ①, ② and ③: (use Miller Effect as needed).

node ①: $C_{①} = C_{gs1} + C_{gb1} + C_{M1}$ (Miller multiplied C_{gd1})

$$C_{M1} = C_{gd1}(1 - A_{vo}) = C_{gd1}\left(1 + \frac{g_{m1}}{g_{ds1} + g_{ds2}}\right)$$

$$\simeq \left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) C_{gd1}$$

$$\therefore C_{①} \simeq C_{gs1} + C_{gb1} + C_{gd1}\left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right)$$

node ②: $C_{②} = C_{X1} + C_{db1} + C_{gd2} + C_{db2}$
$+ C_{gd4} + C_{gb4} + C_{M4}$ (Miller mult. C_{gs4})

$$C_{X1} = C_{gd1}\left(1 - \frac{1}{A_{vo}}\right) \simeq C_{gd1}$$

$$C_{M4} = C_{gs4}(1 - A_{vo4})$$

$$\simeq C_{gs4}\left(1 - \frac{g_{m4}}{g_{m4} + g_{mb4}}\right)$$

$$\simeq C_{gs4}\left(\frac{g_{mb4}}{g_{m4} + g_{mb4}}\right) = C_{gs4}\underbrace{\left(\frac{\eta}{1+\eta}\right)}_{\text{Fairly small term.}}$$

$$\therefore C_{②} \simeq C_{gd1} + C_{db1} + C_{gd2} + C_{db2} + C_{gd4} + C_{gb4} + C_{gs4}\left(\frac{\eta}{1+\eta}\right)$$

node ③: $C_{③} = C_{gd3} + C_{db3} + C_{sb4} + C_{X4} + C_L$

$C_{X4} = C_{gs4}\left(1 - \frac{1}{A_{vo4}}\right)$ (we will neglect negative caps)

$$\simeq C_{gd3} + C_{db3} + C_{sb4} + C_L$$

Now, find the driving point resistances at each node:

node ①: By inspection: $R_{①} = R_S \parallel R_{G1}$ $(R_{G1} = \infty)$

$= R_S$

node ②: By inspection: $R_{②} = r_{ds1} \parallel r_{ds2} \parallel R_{G4}$

$= \dfrac{1}{g_{ds1} + g_{ds4}}$ $(R_{G4} = \infty)$

node ③: By inspection: $R_{③} = \dfrac{1}{g_{m4} + g_{mb4} + g_{ds4} + g_{ds3}}$

$\simeq \dfrac{1}{g_{m4} + g_{mb4}}$

$= \dfrac{1}{g_{m4}(1+\eta_4)}$

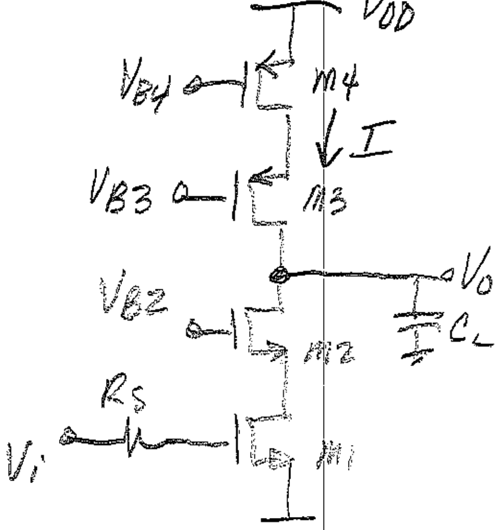
∴ Dominant pole

$\omega_{-3dB} = p_1 = \dfrac{1}{\tau_{①} + \tau_{②} + \tau_{③}}$ where $\tau_1 = R_{①} C_{①}$

$\tau_2 = R_{②} C_{②}$

$\tau_3 = R_{③} C_{③}$

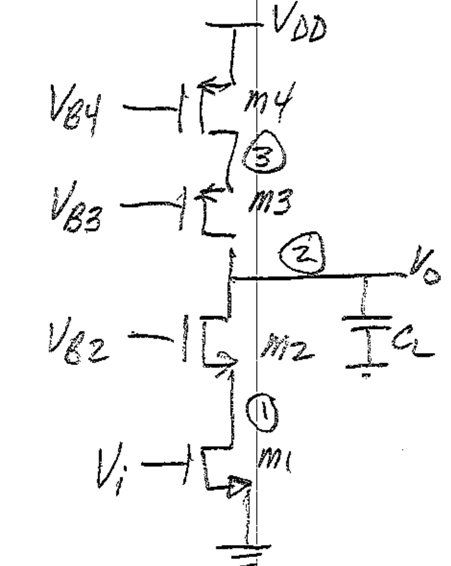
• Now, let's consider a very important gain stage, the cascode CS stage:



Main Advantage: Very high voltage gain [like two cascaded CS stages; i.e., $A_v \sim (g_m r_o)^2$ for only one bias current, I.

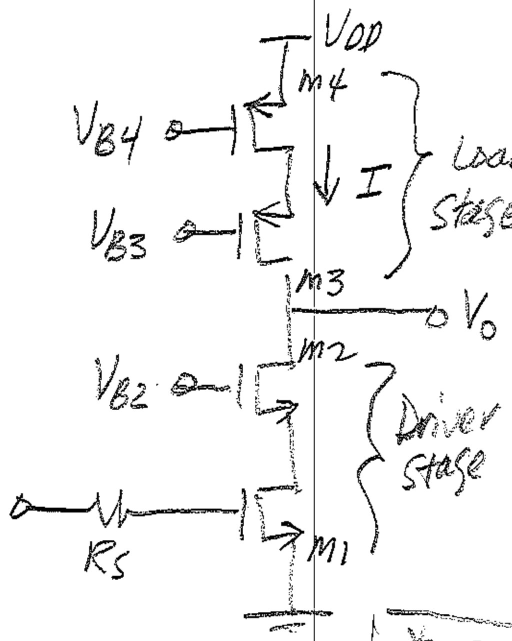
Drawback: Reduced output voltage swing; i.e., greater headroom required.

- Let's consider the output voltage swing limitation:



Observation: If we apply an input signal, v_i, the signal voltage at v_o is large because V_o is a very high impedance (i.e., high gain) node. The voltage swing at 1 and 3 are small as we will see later. So, lets assume zero voltage signals at 1 and 3 for now.

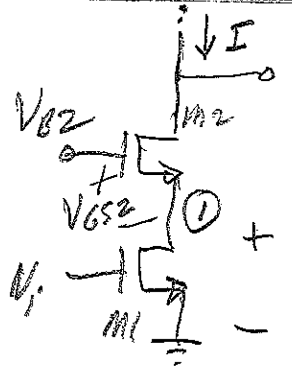
Consider DC bias conditions:



- Must assume $V_o = V_{DD}/2$ because very high impedance node.*
- Must keep $M_1 - M_4$ in saturation as V_o swings from $V_o(min)$ to $V_o(max)$
- Let's consider $M_1 - M_2$ cascode first:

* Recall: for DC bias calculations use $\underline{\underline{I}} = \frac{k_n'}{2}\left(\frac{W}{L}\right)(\underline{V_{GS}} - V_T)^2$

Note: No V_{OS} so we have to assume $V_o(DC) = \frac{V_{DD}}{2}$



- By inspection, $V_o(min) = V_{dsat_1} + V_{dsat_2}$
- But, we are assuming no signal at ① so $V_{ds1} = V_{dsat_1}$ is always true.

- Note that cascode device, M_2, sets the DC at ①:

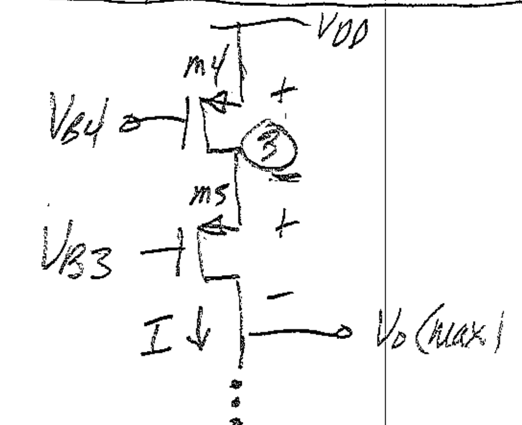
<u>Side note</u>: $V_{GS} = V_T + \underbrace{(V_{GS} - V_T)}_{V_{dsat}} = V_T + V_{dsat}$

$\therefore V_{①} = V_{B2} - V_{GS2} = V_{B2} - V_{T2} - V_{dsat_2}$

But, $V_{①} = V_{dsat_1} \Longrightarrow V_{B2} = V_{T2} + V_{dsat_2} + V_{dsat_1}$

- Design V_{B2} for $V_{OUT}(min) = 2V_{dsat}$ $\longrightarrow$ $\boxed{V_{B2} \simeq V_{T2} + 2V_{dsat}}$ ←

- Now, consider the M3-M4 PMOS Cascode:



- M3-M4 must remain in saturation for $V_O = V_o(max)$
- By inspection, $V_o(max) = V_{DD} - |V_{dsat_3}| - |V_{dsat_4}|$
- Again, assume no voltage swing at node ③:

- Note that M_3 sets DC bias voltage at ③:

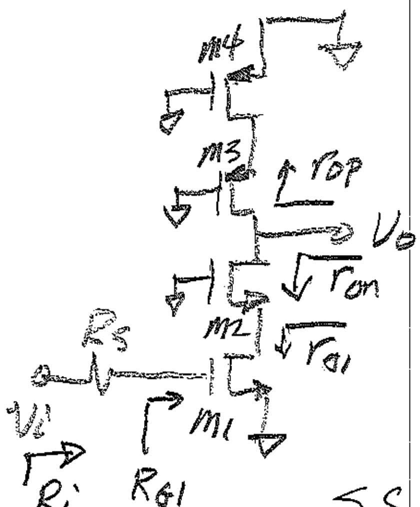
$\therefore V_{③} = V_{B3} + V_{SG3} = V_{B3} + |V_{T3}| + |V_{dsat_3}|$

But, we also want $|V_{ds4}| = |V_{dsat_3}|$

So, solving as before: $\boxed{V_{B3} = V_{DD} - |2V_{dsat}|}$ ←

- These DC designs for V_{B2} and V_{B3} are non trivial!!

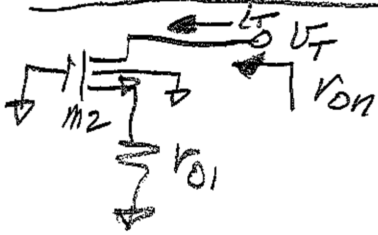
• Now, let's find: (i) Input resistance; (ii) output resistance; (iii) voltage gain and (iv) freq. response



(i) $R_i = R_S + R_{G1} = R_S + \infty = \infty$

(ii) $R_O = r_{on} \,//\, r_{op}$

• Recall this circuit:



S.S. analysis $\Rightarrow r_{on} = r_{o2}\left[1 + (g_{m2} + g_{mb2} + g_{o2}) r_{o1}\right]$

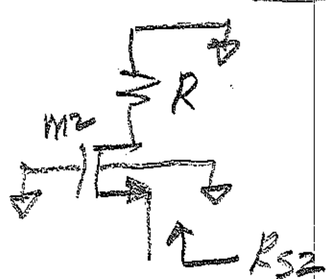
$\simeq r_{o2}\,(g_{m2} + g_{mb2} + g_{o2})\, r_{o1}$

$\simeq r_{o1}\left[(g_{m2} + g_{mb2}) r_{o2}\right]$

$\simeq r_{o1}\,\underbrace{(g_{m2} r_{o2})}_{\text{self-gain of } M2}$

So, $R_o = r_{on} \,//\, r_{op}$
$\simeq r_{o1}(g_{m2} r_{o2}) \,//\, r_{o4}(g_{m3} r_{o3})$
$\therefore$ R_o is very large impedance!

• Let's do one more new circuit:



• S.S. analysis yields:

$$\boxed{R_{S2} = \frac{1 + g_{o2} R}{g_{m2} + g_{mb2} + g_{o2}}}$$

• limiting cases:

i) $R = 0 \Rightarrow R_{S2} = \dfrac{1}{g_{m2} + g_{mb2} + g_{o2}} \simeq \dfrac{1}{g_{m2}}$

(Familiar case)

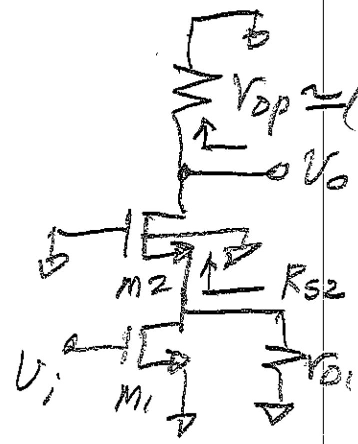
(ii) Let $R = r_o$ (assume $r_o = r_{o2}$ for simplicity)

$$\Rightarrow R_{S2} = \frac{1 + g_{o2} r_{o2}}{g_{m2} + g_{mb2} + g_{o2}} = \frac{2}{g_{m2} + g_{mb2} + g_{o2}} \simeq \frac{2}{g_m}$$

(iii) Let $R = N r_o = N r_{o2}$ (for simplicity

* Neat result to remember. $\Rightarrow R_{S2} = \dfrac{1 + N g_{o2} r_{o2}}{g_{m2} + g_{mb2} + g_{o2}} = \dfrac{N+1}{g_{m2} + g_{mb2} + g_{o2}} \simeq \dfrac{N+1}{g_m}$

Now, let's compute $a_v = \frac{v_o}{v_i}$:



note: The signal current generated by $M1$ is $g_{m1} v_i$. But, not all of that current flows through the high impedance output node because of current division between r_{o1} and R_{S2}. This effect is often misunderstood and usually ignored, but it can be significant. From above:

$$R_{S2} = \frac{1 + g_{o2} r_{op}}{g_{m2} + g_{mb2} + g_{o2}}$$

(Simplify: all $g_{mi} = g_m$, $g_{mbi} = g_{mb}$, $g_{oi} = g_o$, etc.)

$$\rightarrow R_{S2} \simeq \frac{1 + g_o g_m r_o^2}{g_m + g_{mb} + g_o} \simeq \underline{\underline{r_o}}$$

- Thus, only half of $g_{m1} v_i$ flows upwards into M_2:

Now, by inspection:

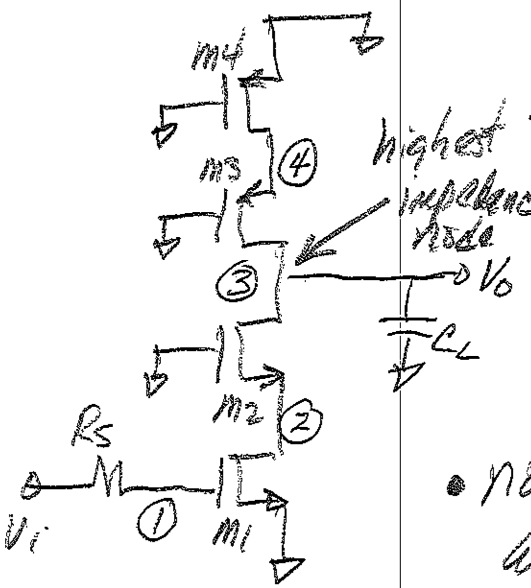
$$a_v = -\frac{1}{2} g_{m1} R_o = -\frac{1}{2} g_{m1} \left[r_{o1} (g_{m2} r_{o2}) \,||\, r_{o4} (g_{m3} r_{o3}) \right]$$

$$= -\frac{1}{2} g_{m1} \left[\frac{1}{2} g_m r_o^2 \right] = -\frac{1}{4} \underbrace{(g_m r_o)^2}_{\text{Very large gain}}$$

- Thus, a_v of cascode gain stage is similar to that of two simple CS stages connected in cascade (i.e., in series)

Now, freq. response starting with dominant pole:

ZVTC Method $\rightarrow \omega_{-3dB} = \dfrac{1}{\tau_1 + \tau_2 + \tau_3 + \tau_4}$



Before we do all of these computations, let's inspect to see if we need to:

- node ①: Driving point resistance, $R_1 = R_S$ (probably small)
- node ②: Based on our previous analysis $R_2 \simeq \frac{r_o}{2}$ (medium value)
- node ④: Same analysis as at node ②: $R_4 \simeq \frac{r_o}{2}$

node ③: $r_{op} \| r_{on} = R_3 \simeq g_{m2} r_{o2} r_{o1} \,\|\, g_{m3} r_{o3} r_{o4}$

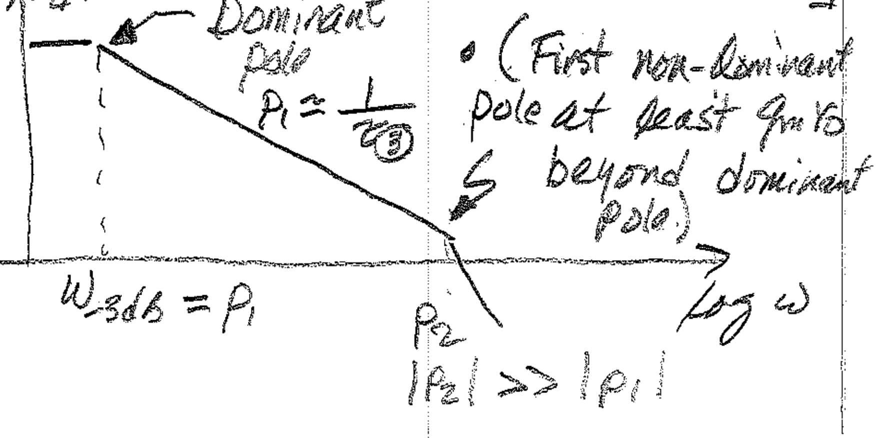
$\simeq \frac{r_o}{2}(g_m r_o)$ Very large Driving pt. Resistance.

- Note also that C_L is usually larger than the transistor capacitances.

Hence, $\omega_{-3dB} \simeq \dfrac{1}{\tau_3}$ *

$$\tau_3 \simeq \left[(g_{m2} r_{o2} r_{o1}) \,\|\, (g_{m3} r_{o3} r_{o4})\right]\left[C_{gd2} + C_{db2} + C_{gd3} + C_{db3} + C_L\right]$$

Bode Plot:



- Now, near P_2, C_L represents a low impedance: <u>Example:</u> let $g_m r_o = 100$, $r_o = 1M\Omega$

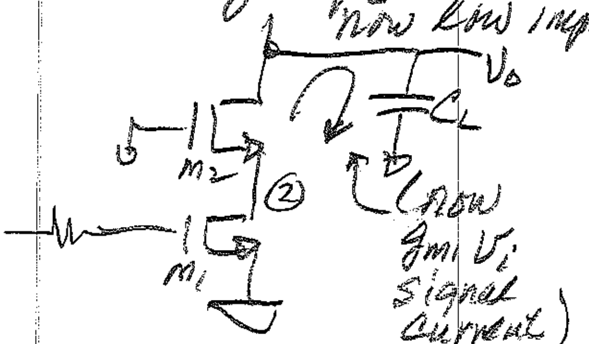
Z_O at low freqs $= r_{op} \parallel r_{on}$

$$\simeq \frac{1}{2}(g_m r_o) r_o = \frac{1}{2}(100)(1M\Omega) = 50M\Omega$$

$|Z_c| = \left|\frac{1}{j\omega C_L}\right|$ at $\omega_2 = 10^8$ rad/sec with $C_L = 5pF$

$\therefore |Z_c| = 2K\Omega$ ← Now, V_o very low impedance!

$\therefore$ Compared to r_{op}, $|Z_c|$ is very small at P_2. Thus, we can analyze the following circuit to get P_2:



Because of low impedance at V_o:

$$R_{②} \simeq \frac{1}{g_{m2} + g_{mb2} + g_{ds2} + g_{ds1}} \simeq \frac{1}{g_{m2}}$$

$\therefore$ $\tau_{②} = R_{②} C_{②}$

$$= \frac{1}{g_{m2}}\left[C_{X1}^{*} + C_{db1} + C_{gs2} + C_{sb2}\right]$$

$$\therefore \boxed{P_2 \simeq \frac{g_{m2}}{2C_{gd1} + C_{db1} + C_{gs2} + C_{sb2}}}$$

- First non-dominant pole. (see bode plot on previous page)

$$* \quad \frac{V_{②}}{V_{①}} = \frac{-g_{m1}}{g_{②}} \simeq \frac{-g_{m1}}{g_{m2}} \simeq -1 \text{ V/V}$$

$$\therefore C_{M1} = C_{X1} \simeq 2C_{gd1}$$